Membrane Computing Applications in Computational Economics

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Motivation

- Success of MC modeling biological systems
- Translation to unexplored field: Economic Modeling
- Replication of Păun's Producer - Retailer Problem results:
  - Selection of the proper type of P System
  - Economic processes modeling
  - Implementation in P-Lingua & MeCoSim
  - Simulate & discuss results
- Extension of the original model with new economic processes:
  - Identification and modeling of processes
  - Implementation & simulation
- Further developments

Why not extend to other fields?

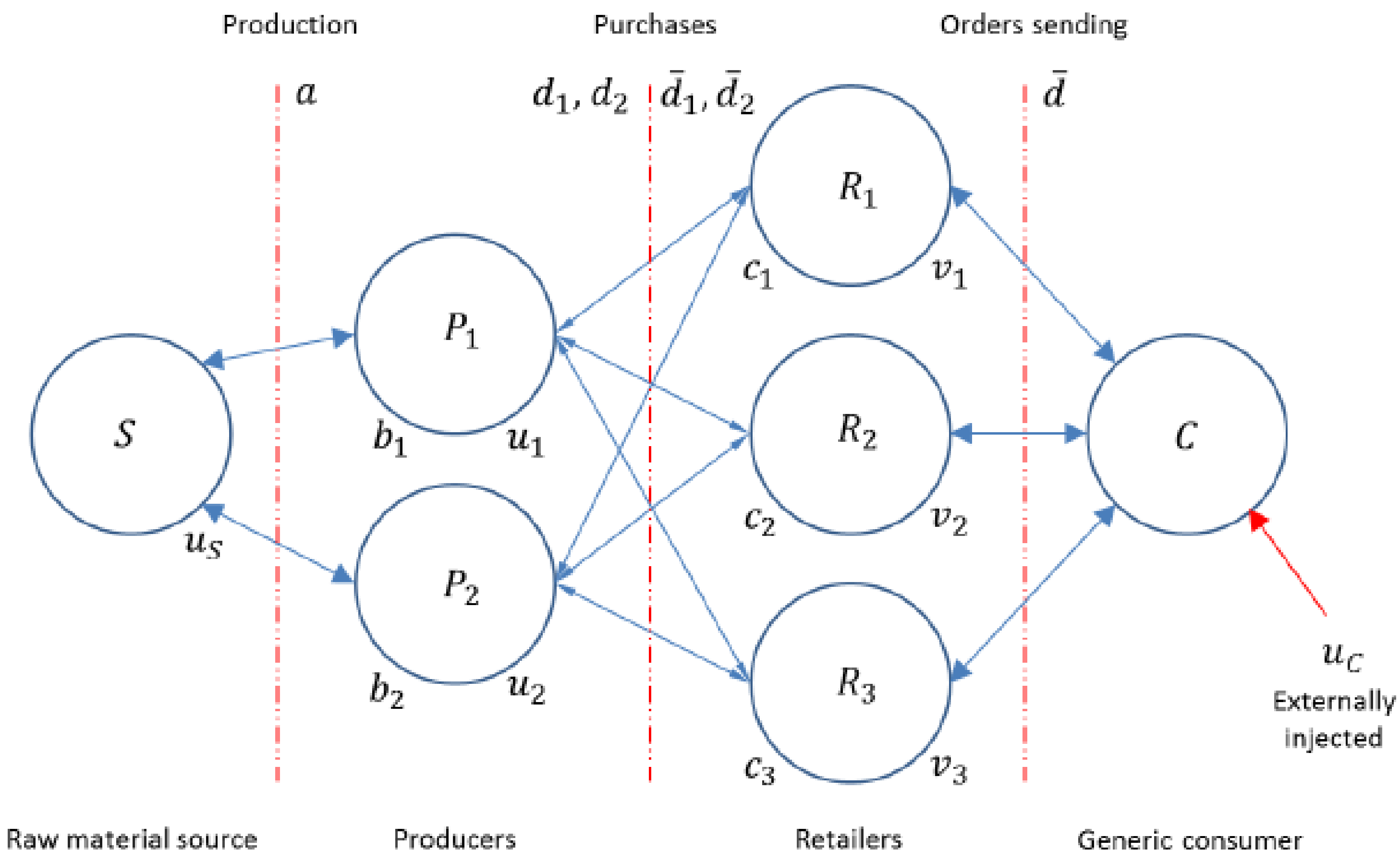
- Computational economics:
  - Computational modeling of economic systems (ODEs, ABM, …)

- Up-to-date efforts:
  - Polish authors: Korczynski (2005)
  - Păun's efforts:
    - Membrane computing as a framework for modeling economic processes. In *Proc. SYNASC 05*, Timisoara, Romania, IEEE Press, 2005, 11–18 Păun Gh. and Păun R. (2005)
    - Păun Gh. and Păun R. Membrane Computing and Economics. In Păun Gh., Rozenberg G., Salomaa, eds. (2010) A. *Handbook of Membrane Computing.* Oxford University Press, 2010, 632-644

Păun's proposals

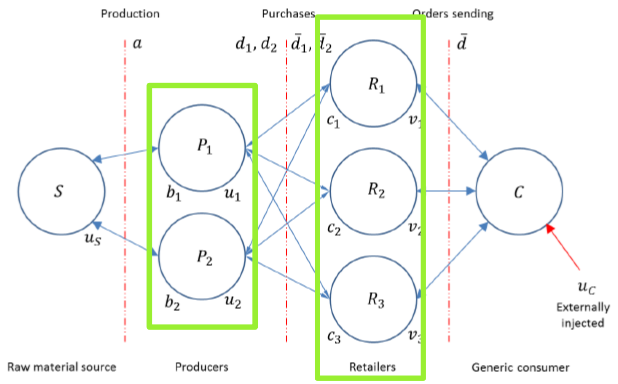
- Encourage researchers of other areas to use P Systems.
- Suggests modeling of some processes:
  - Production of goods
  - Order of goods
  - Purchase transactions:
    - Preferences between pairs (producer, retailer)
    - Geographical barriers
    - No distinction between counterparts
  - Monetary unit exchange
  - Capacity increase

Producer – Retailer Problem

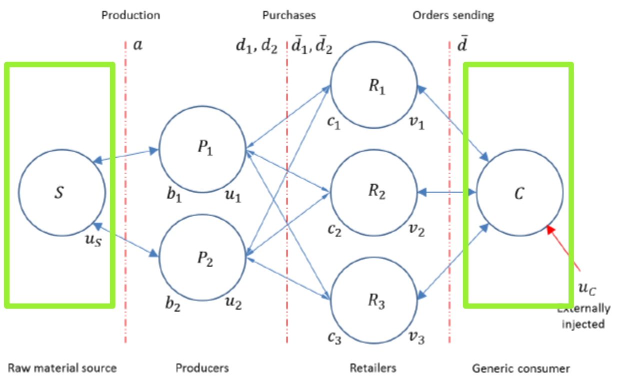


Model Entities

- Actors:
  - Producers:
    - (b_i, u_i) -> (capacity, money)
  - Retailers
    - (c_j, v_j) -> (capacity, money)

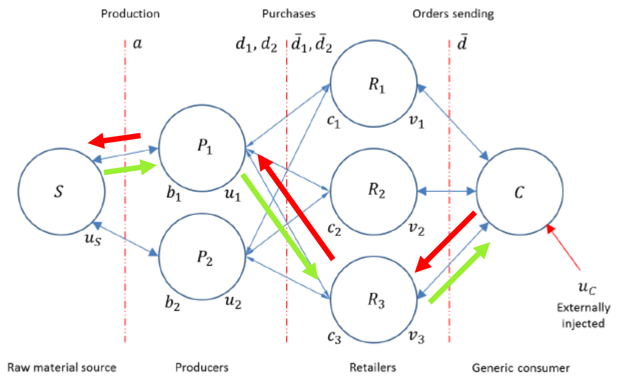


- Generic sources:
  - Of raw material (u_S, generation rate)
  - Of demand or Generic consumer (u_C, demand rate)



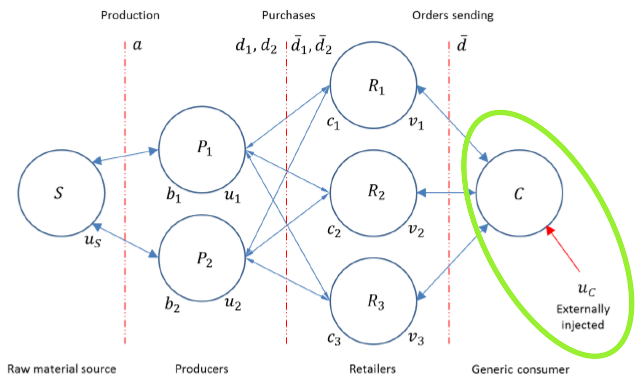
Model interactions

- Good’s and order’s flows:
  - Producers generate good d from raw material.
  - Retailers receive order $\bar{d}$ from generic consumer.
  - d and $\bar{d}$ are matched
  - Purchase $P_{i,j} = P(producer\ i, retailer\ j)$

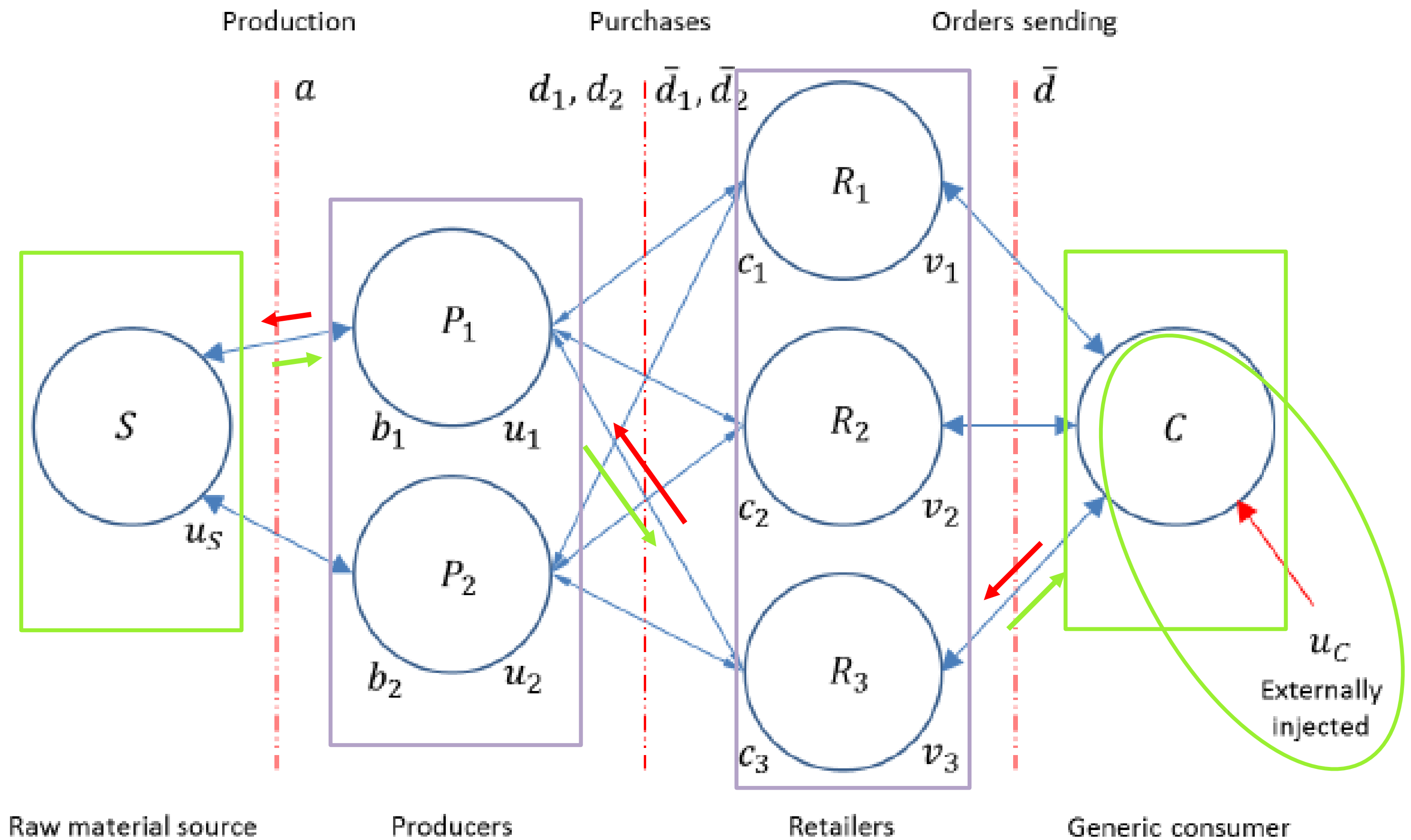


CYCLIC PROCESS

- Monetary flows:
  - Monetary unit exchange (u_S, u_C, u_i, u_j)
  - A set of prices.
- External monetary injection:
  - Key role for system evolving.

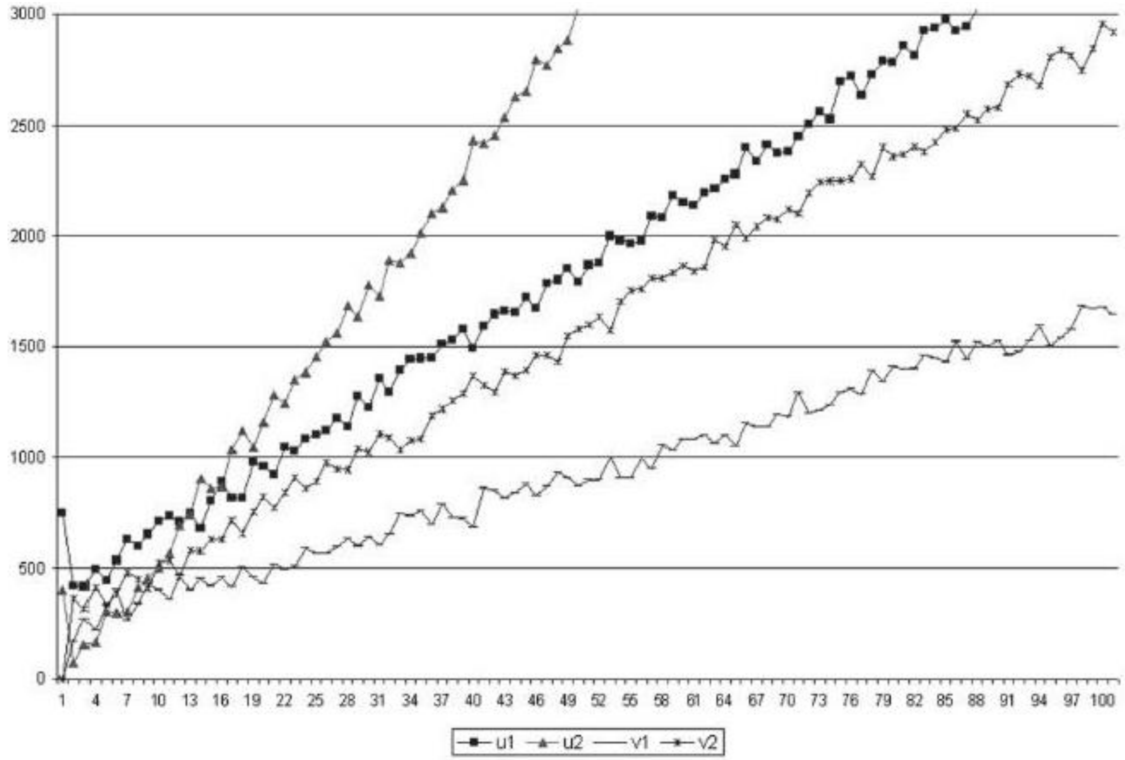


Summarized actors & interactions



Păun's proposed system dynamic

- Presents a system behavior simulation:



Proposal - drawbacks

- Păun sketches the model:
  - No indications about:
    - Type of P System to be used.
    - The sequence of steps of the cyclic behavior.
    - The competing set of rules to be used.
  - Probabilities associated to rules in a strange way.

$$[\, d_1 \bar{d}_1 v_1{}^{price}]_2 \xrightarrow{p11} [\, b_1 c_1 u_1{}^{price}]_2$$

$$[\, d_2 \bar{d}_1 v_1{}^{price}]_2 \xrightarrow{p12} [\, b_2 c_1 u_2{}^{price}]_2$$

$$[\, d_3 \bar{d}_1 v_1{}^{price}]_2 \xrightarrow{p13} [\, b_3 c_1 u_3{}^{price}]_2$$

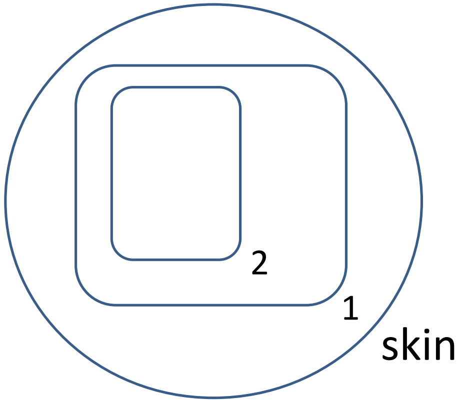
$$\sum_i p_{ij} = 1$$

Non-as-usual

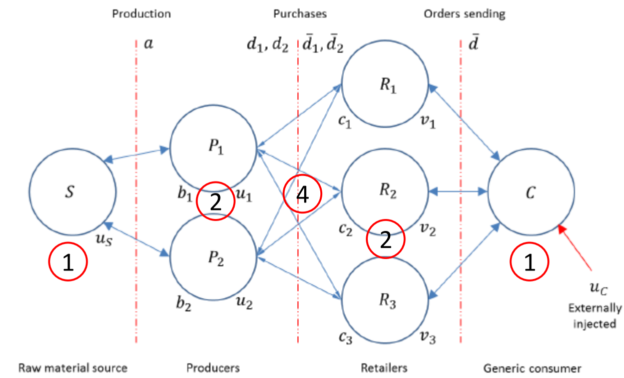
  - Randomness introduced in a naive way.

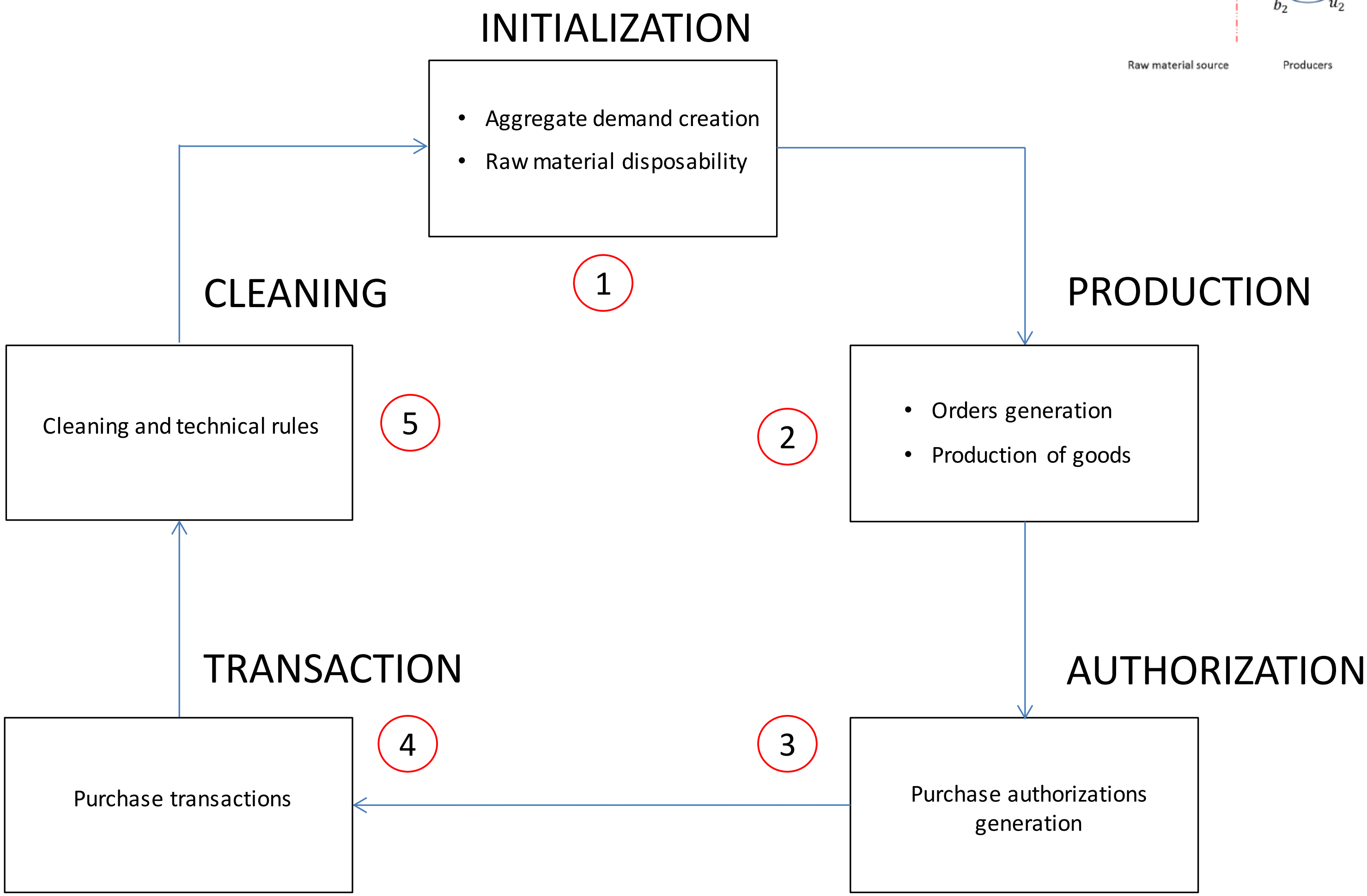
Reproducing Păun's system evolution

- Define a so-called: Initial Model
- Steps:
  - Select a type of P System -> PDP System.
    - Probabilities associated to rules.
    - Success in ecosystem modeling.
  - Define the steps of the cycle:
    - Associated to the transactions.
  - Formalize the model.
  - Stablish the set of rules:
    - Following Păun's guidelines.
    - Avoid problems associated to "strange" probabilities.



Defining steps of cycle





Model Formalization (I)

$\Pi = (\mathrm{G}, \Gamma, \Sigma, \mathrm{T}, \mathcal{R}_{\mathrm{E}}, \mu, \mathcal{R}_{\Pi}, \{\mathrm{f}_{\mathrm{r}} \in \mathcal{R}_{\Pi}\}, M_1, M_2)$ PDP System of degree (2,1)

Where:

- $G = (V, E)$ with $V = \{e_1\}$ and $E = \{(e_1, e_1)\}$.
- Working alphabet: $\Gamma = \{b_i, d_i, u_i, c_j, \bar{d}_j, v_j, \bar{e}_j, f_{i,j} : 1 \leq i \leq k_1, 1 \leq j \leq k_2\} \cup \{R_1, R_2\} \cup \{C, S, \bar{d}, a, u_C, u_S\}$

Where:

C: aggregate generic consumer.

S: raw material supplier.

$\bar{d}$: unit of aggregate demand from C.

a: unit of supplied raw material provided by S.

u_C: monetary unit owned by C.

u_S: monetary unit owned by S.

b_i: unit of production capacity of producer $i . 1 \leq i \leq k_1$.

d_i: unit of good supplied by producer i. $1 \leq i \leq k_1$.

u_i: monetary unit owned by producer $i . 1 \leq i \leq k_1$.

c_j: unit of capacity of retailer $j . 1 \leq j \leq k_2$.

$\bar{d}_j$: unit of good demanded by retailer j. $1 \leq j \leq k_2$.

v_j: monetary unit owned by retailer $j . 1 \leq j \leq k_2$.

$\bar{e}_j$: unit of good demanded by retailer and authorized for transaction unit of $\bar{d}_j$. $1 \leq j \leq k_2$.

$f_{i,j}$: authorization for $\bar{d}_j$ to be exchange with d_i . $1 \leq i \leq k_1, 1 \leq j \leq k_2$.

R_1, R_2: for technical reasons.

Model Formalization (II)

- $\Sigma = \emptyset$.
- $R_E = \emptyset$.
- $\Pi = \{\Gamma,\ \mu,\ M_1,\ M_2, \mathcal{R}_\Pi\}$ where:
  - Membrane structure: $\mu = [\ [\ ]_2\ ]_1$.
  - $M_1 = \{C, S, R_1, R_2\} \cup \{b_i^{k_{i,1}}, u_i^{k_{i,2}} : 1 \le i \le k_1\} \cup \{c_j^{k_{j,3}} : 1 \le j \le k_2\}$

Initial multisets contain basically:

- $b_i^{k_{i,1}}, u_i^{k_{i,2}}$: producers' initial parameters.
- $c_j^{k_{j,3}}$: retailers' initial capacities.

Where:

$k_{i,1}$: initial production capacity of producer i. $1 \le i \le k_1$.

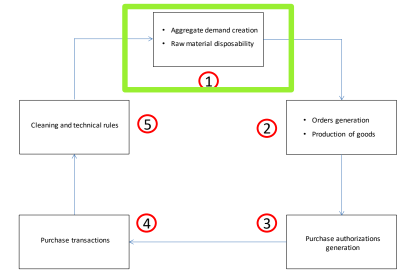
$k_{i,2}$: initial monetary units of producer i. $1 \le i \le k_1$.

$k_{j,3}$: initial capacity of retailer j. $1 \le j \le k_2$.

Model Parameters

- Goal: maximize model parametrization
  - k_1: total number of producers.
  - k_2: total number of retailers.
  - k_3: units of raw material inserted into the system by S.
  - k_4: allowed deviation from k_3.
  - k_5: units of aggregate demand inserted into the system by C.
  - k_6: allowed deviation from k_5.
  - k_7: price fixed by S for each unit of a.
  - k_8: price fixed by C as an estimation of each order of good.
  - $k_{i,1}$: initial production capacity of producer i. $1 \leq i \leq k_1$.
  - $k_{i,2}$: initial monetary units of producer i. $1 \leq i \leq k_1$.
  - $k_{j,3}$: initial capacity of retailer j. $1 \leq j \leq k_2$.
  - $k_{m,4}$: discrete prob. distribution of units of raw material inserted into the system by S. $1 \leq m \leq 3$.
  - $k_{m,5}$: discrete prob. distribution of units of aggregate demand inserted into the system by C. $1 \leq m \leq 3$.
  - $k_{i,6}$: price fixed by producer i for each unit of d_i. $1 \leq i \leq k_1$.
  - $k_{j,7}$: price fixed by retailers j for each order of good. $1 \leq j \leq k_2$.

Set of rules – Initialization



- Step 1.a: raw material disposability

$$r_1 \equiv R_1\ s[\ ]_2 \xrightarrow{k_{1,4}} a^{k_3+k_4}\ s[\ R_1]_2^+$$

$$r_2 \equiv R_1\ s[\ ]_2 \xrightarrow{k_{2,4}} a^{k_3}\ s[\ R_1]_2^+$$

$$r_3 \equiv R_1\ s[\ ]_2 \xrightarrow{k_{3,4}} a^{k_3-k_4}\ s[\ R_1]_2^+$$

$$r_4 \equiv R_1\ s[\ ]_2 \xrightarrow{1-k_{1,4}-k_{2,4}-k_{3,4}} a^{k_3-2*k_4}\ s[\ R_1]_2^+$$

k_3: units of raw material inserted into the system by S.

k_4: allowed deviation from k_3.

$k_{m,4}$: discrete prob. distr. of units of raw material inserted.

k_5: units of aggregate demand inserted by C.

k_6: allowed deviation from k_5.

$k_{m,5}$: discrete prob. distr. of units of aggr. demand inserted.

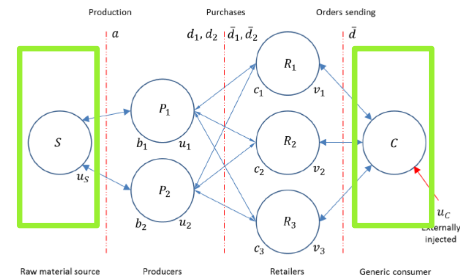
- Step 1.b: generic demand creation

$$r_5 \equiv R_2\ c[\ ]_2 \xrightarrow{k_{1,5}} \bar{d}^{k_5+k_6}\ {u_C}^{(k_5+k_6)*k_8} c[\ R_2]_2^+$$

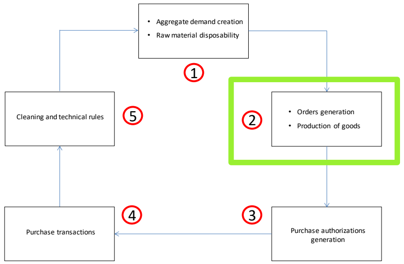
$$r_6 \equiv R_2\ c[\ ]_2 \xrightarrow{k_{2,5}} \bar{d}^{k_5}\ {u_C}^{k_5*k_8} c[\ R_2]_2^+$$

$$r_7 \equiv R_2\ c[\ ]_2 \xrightarrow{k_{3,5}} \bar{d}^{k_5-k_6}\ {u_C}^{(k_5-k_6)*k_8} c[\ R_2]_2^+$$

$$r_8 \equiv R_2\ c[\ ]_2 \xrightarrow{1-k_{1,5}-k_{2,5}-k_{3,5}} \bar{d}^{k_5-2*k_6}\ {u_C}^{(k_5-2*k_6)*k_8} c[\ R_2]_2^+$$



Set of rules – Production



- Step 2.a: producer operation

$$r_9 \equiv a\, b_i\, {u_i}^{k_7} [\ ]_2^+ \rightarrow {u_S}^{k_7} [\, d_i]_2^0 \quad 1 \leq i \leq k_1$$

- Step 2.b: retailer operation

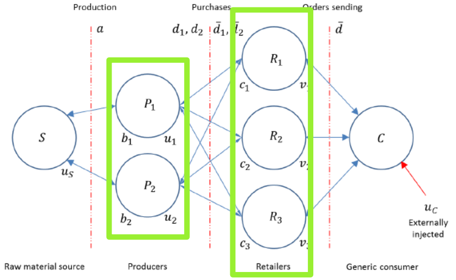
$$r_{10} \equiv \bar{d}\, c_j\, {u_C}^{k_{j,7}} [\ ]_2^+ \rightarrow \left[\bar{d}_j {v_j}^{k_{j,7}} \right]_2^0 \quad 1 \leq j \leq k_2$$

k_1: total number of producers.

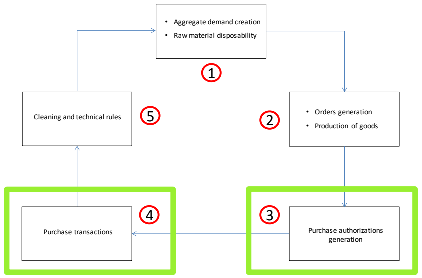
k_2: total number of retailers.

k_7: price fixed by S for each unit of a.

$k_{j,7}$: price fixed by retailers j for each order of good.



Set of rules – Auth. & Trans.



- Step 3: Purchase auth. generation

$$r_{14} \equiv [\,\bar{d}_1]_2 \rightarrow [\ \bar{e}_1 f_{1,1}]_2$$
$$r_{15} \equiv [\,\bar{d}_1]_2 \xrightarrow{0} [\ \bar{e}_1 f_{1,2}]_2$$

Geo-barriers

$$r_{16} \equiv [\,\bar{d}_2]_2 \xrightarrow{0.5} [\ \bar{e}_2 f_{2,1}]_2$$
$$r_{17} \equiv [\,\bar{d}_2]_2 \xrightarrow{0.5} [\ \bar{e}_2 f_{2,2}]_2$$

Non-preferences

$$r_{18} \equiv [\,\bar{d}_3]_2 \xrightarrow{0.15} [\ \bar{e}_3 f_{3,1}]_2$$
$$r_{19} \equiv [\,\bar{d}_3]_2 \xrightarrow{0.85} [\ \bar{e}_3 f_{3,2}]_2$$

Preferences

k_1: total number of producers.

k_2: total number of retailers.

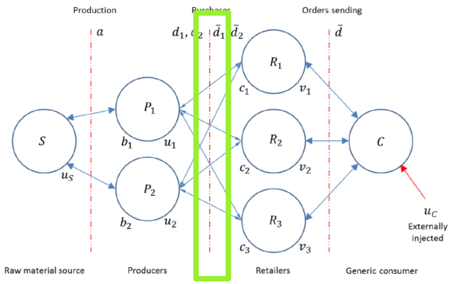
$k_{i,6}$: price fixed by producer i for each unit of d_i.

$$[\ d_1 \bar{d}_1 v_1{}^{price}]_2 \xrightarrow{p11} [\ b_1 c_1 u_1{}^{price}]_2$$
$$[\,d_2 \bar{d}_1 v_1{}^{price}]_2 \xrightarrow{p12} [\ b_2 c_1 u_2{}^{price}]_2$$
$$[\,d_3 \bar{d}_1 v_1{}^{price}]_2 \xrightarrow{p13} [\ b_3 c_1 u_3{}^{price}]_2$$

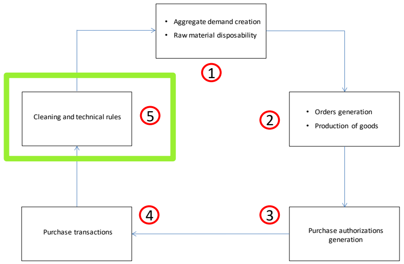
Solution: $f_{i,j}$ follows the probability distribution of the desired transactions probabilities.

- Step 4: Purchase transactions

$$r_{20} \equiv [\,d_i \bar{e}_j f_{j,i} v_j{}^{k_{i,6}}]_2^0 \rightarrow [b_i c_j u_i{}^{k_{i,6}}]_2^- \quad 1 \leq i \leq k_1, 1 \leq j \leq k_2$$



Set of rules – Cleaning



- Step 5: cleaning rules

Eliminate non-exhausted authorizations:

$$r_{26} \equiv [\, f_{i,j}]_2^{-} \rightarrow [\quad]_2^0 \qquad 1 \leq i \leq k_1, 1 \leq j \leq k_2$$

Unauthorize non-exhausted $\bar{e}_j$:

$$r_{27} \equiv [\bar{e}_j]_2^{-} \rightarrow \bar{d}_j [\quad]_2^0 \quad 1 \leq j \leq k_2$$

Signaling a new cycle:

$$r_{30} \equiv [\, r_1, r_2]_2^{-} \rightarrow r_1, r_2 \,[\quad]_2^0$$

k_1: total number of producers.

k_2: total number of retailers.

P - Lingua

- Set of rules has been implemented in P – Lingua.
- An example for each set of rules:
  - Initialization:

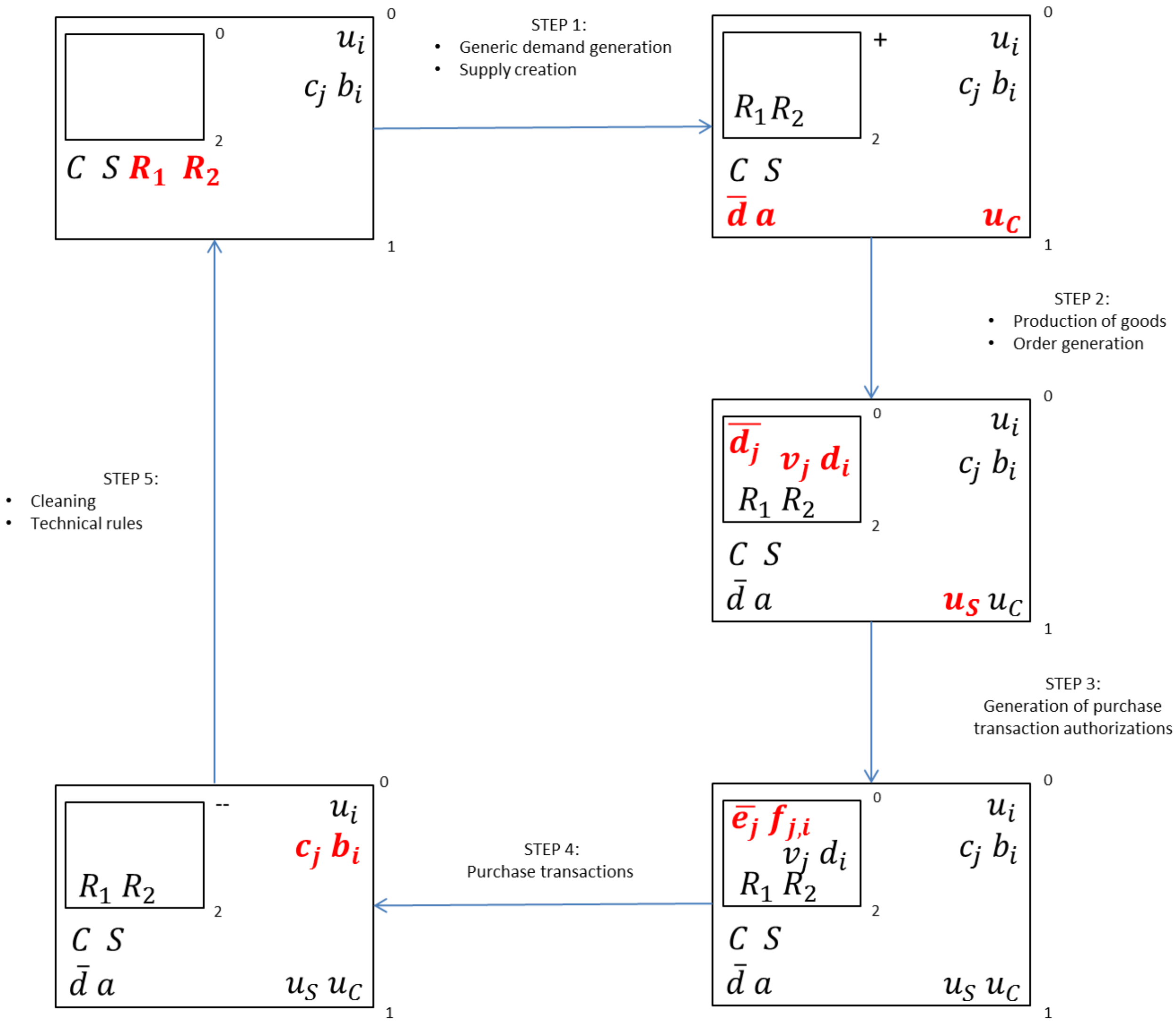
    $/* \; r2 \; */ \; s, R_1[\,]'2 \; -\!\!\rightarrow \; s, a * k\{3\} + [R_1]'2 \;\; :: k_{2,4};$
  - Production:

    $/* \; r9 \; */ \;\; b\{i\}, a, u\{i\} * k\{7\} + [\,]'2 \; -\!\!\rightarrow \; us * k\{7\}[\, d\{i\}]'2 \;\; :: 1 : 1 \leq i \leq k\{1\}$
  - Authorization:

    $/* \; r18 \; */ \;\; [dn\{3\}\,]'2 \; -\!\!\rightarrow \; [en\{3\}, f\{3{,}1\}\,]'2 \;\; :: 0.15$
  - Transaction:

    $/* \; r20 \; */ \;\; [d\{i\}, en\{j\}, f\{j, i\}, v\{j\} * k\{i, 6\}\,]'2 \; -\!\!\rightarrow \; -[\, b\{i\}, c\{j\}, u\{i\} * k\{i, 6\}\,]'2 \;\; :: 1 \; 1 \leq i \leq k\{1\}, 1 \leq j \leq k\{2\}$

Simplified trace



Simulation parameters

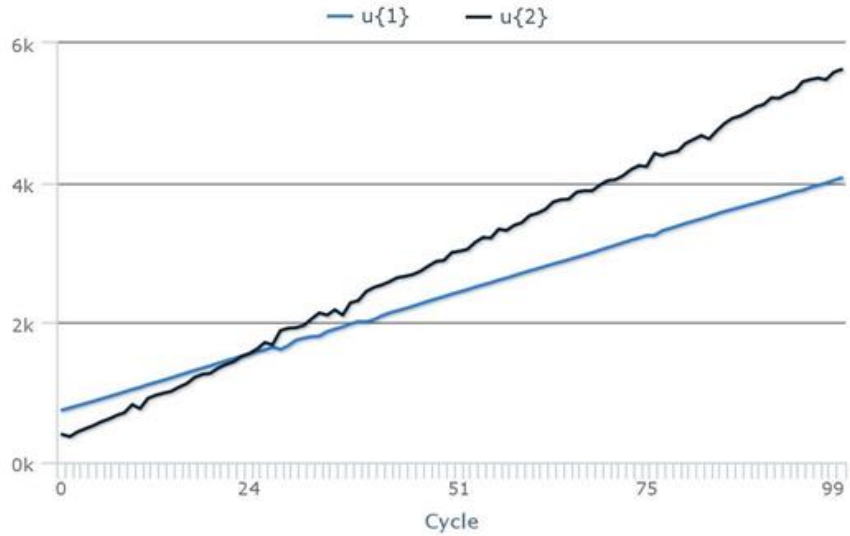
- Simulation tool: MeCoSim
- Parameters: same as Păun’s paper

Parameter	Value/s	Description
k_1	2	Total number of producers
k_2	3	Total number of retailers
k_3	60	Units of raw material inserted into the system by S
k_4	1	Deviation from k_3
k_5	60	Units of aggregate demand inserted into the system by C
k_6	1	Deviation from k_5
k_7	11	Price fixed by S for each unit of a
k_8	14	Price fixed by C as an estimation of each order of good
$k_{i,1}$	(65,35)	Initial production capacity of producer i. $1 \leq i \leq k_1$
$k_{i,2}$	{750,400)	Initial monetary units of producer i. $1 \leq i \leq k_1$
$k_{j,3}$	(50,30,20)	Initial capacity of retailer j. $1 \leq j \leq k_2$
$k_{m,4}$	(0.01,0.95,0.03)	Values of discrete probability distribution of units of raw material inserted into the system by S
$k_{m,5}$	(0.03,0.90,0.04)	Values of discrete probability distribution of units of aggregate demand inserted into the system by C
$k_{i,6}$	(12,13)	Price fixed by producer i for each unit of d_i
$k_{j,7}$	(13,14,15)	Price fixed by retailer j for each order of good j. $1 \leq j \leq k_2$

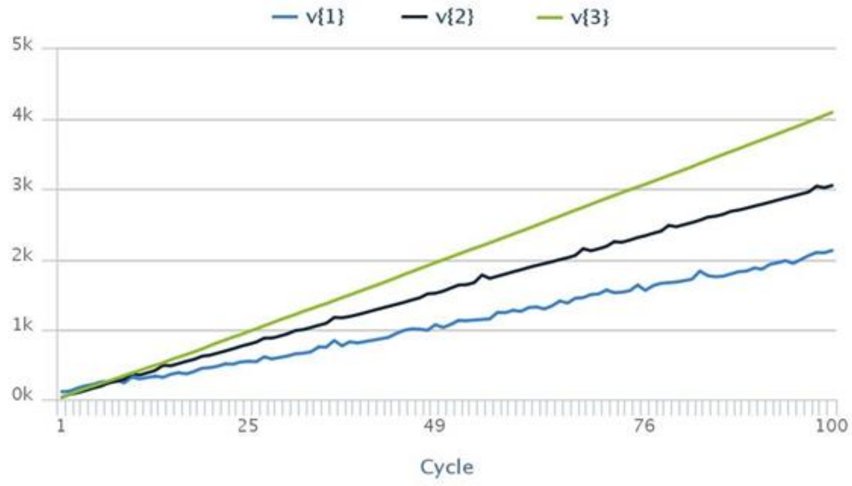
MeCoSim definition

Parameter	Value	Description
k_1	<@r,1> Index 1 = 1	Captures number of producers based on the number of rows in table Producer_input
k_2	<@r,8> Index 2 = 2	Captures number of retailers based on the number of rows in table Retailer_input
k_3	<9,1-2,2> Index 1 = [3..<@r,9>+2]	Units of raw material inserted into the system by S
k_4		Deviation from k_3
k_5		Units of aggregate demand inserted into the system by C
k_6		Deviation from k_5
k_7		Price fixed by S for each unit of a
k_8		Price fixed by C as an estimation of each order of good
$k_{i,1}$	<1,1,2+3> Index 1 = [1..k{1}] Index 2 = [1..2]	Initial production capacity of producer i. $1 \leq i \leq k_1$
$k_{i,2}$		Initial monetary units of producer i. $1 \leq i \leq k_1$
$k_{j,3}$	<8,1,4> Index 1 = [1..k{2}] Index 2 = 3	Initial capacity of retailer j. $1 \leq j \leq k_2$
$k_{m,4}$	<10,1,2-3> Index 1 = [1..<@r,10>] Index 2 = [4..5]	Values of discrete probability distribution of units of raw material inserted into the system by S
$k_{m,5}$		Values of discrete probability distribution of units of aggregate demand inserted into the system by C
$k_{i,6}$	<1,1,6> Index 1 = [1..k{1}] Index 2 = 6	Price fixed by producer i for each unit of d_i
$k_{j,7}$	<8,1,5> Index 1 = [1..k{2}] Index 2 = 7	Price fixed by retailer j for each order of good j. $1 \leq j \leq k_2$

Simulation results – monetary units

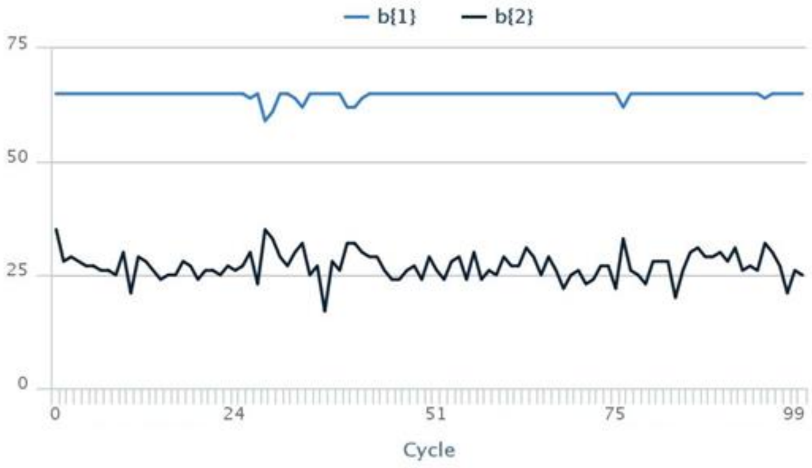


Producers' monetary units

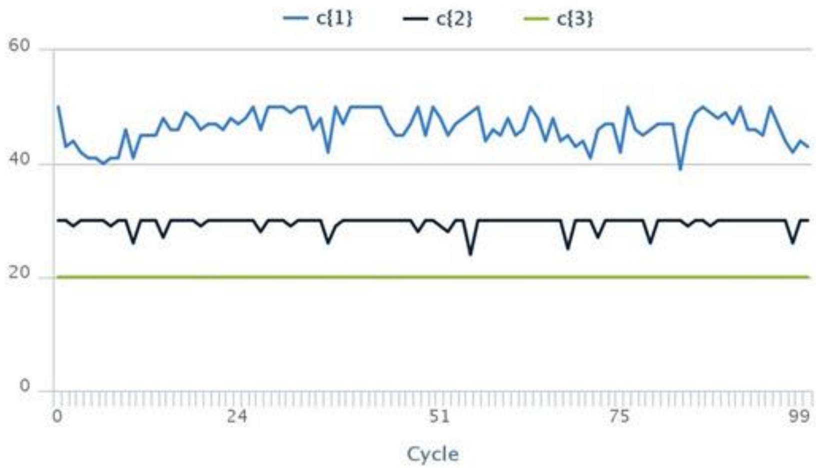


Retailers' monetary units

Simulation results - capacities

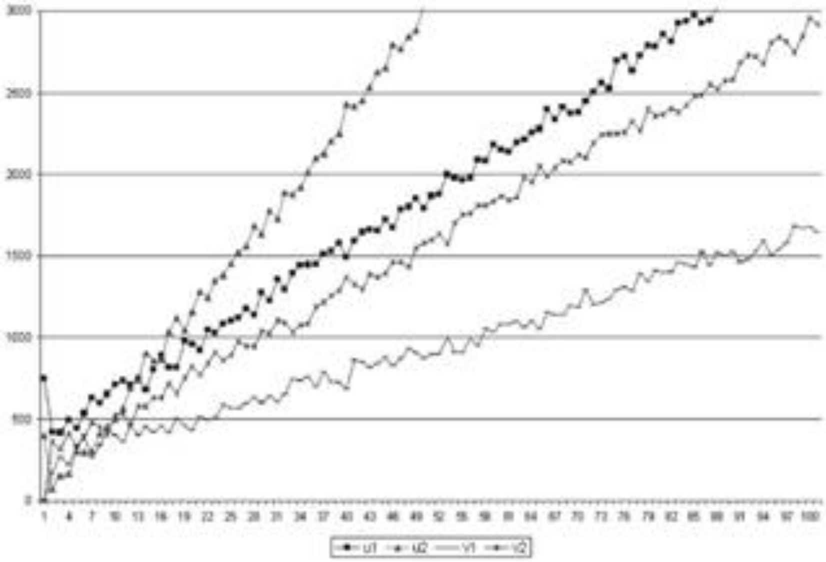


Producers' capacities

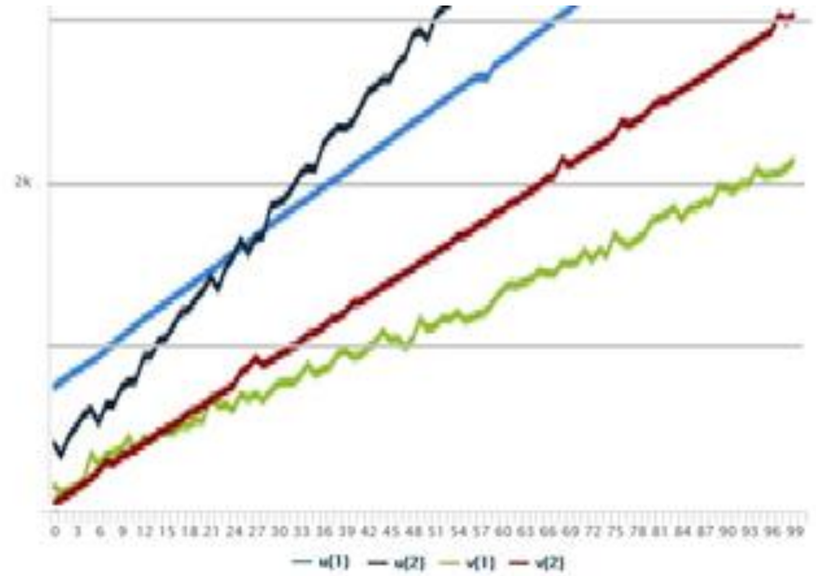


Retailers' capacities

Simulation results - comparison



Păun's evolution



Initial model evolution

Enhanced Model

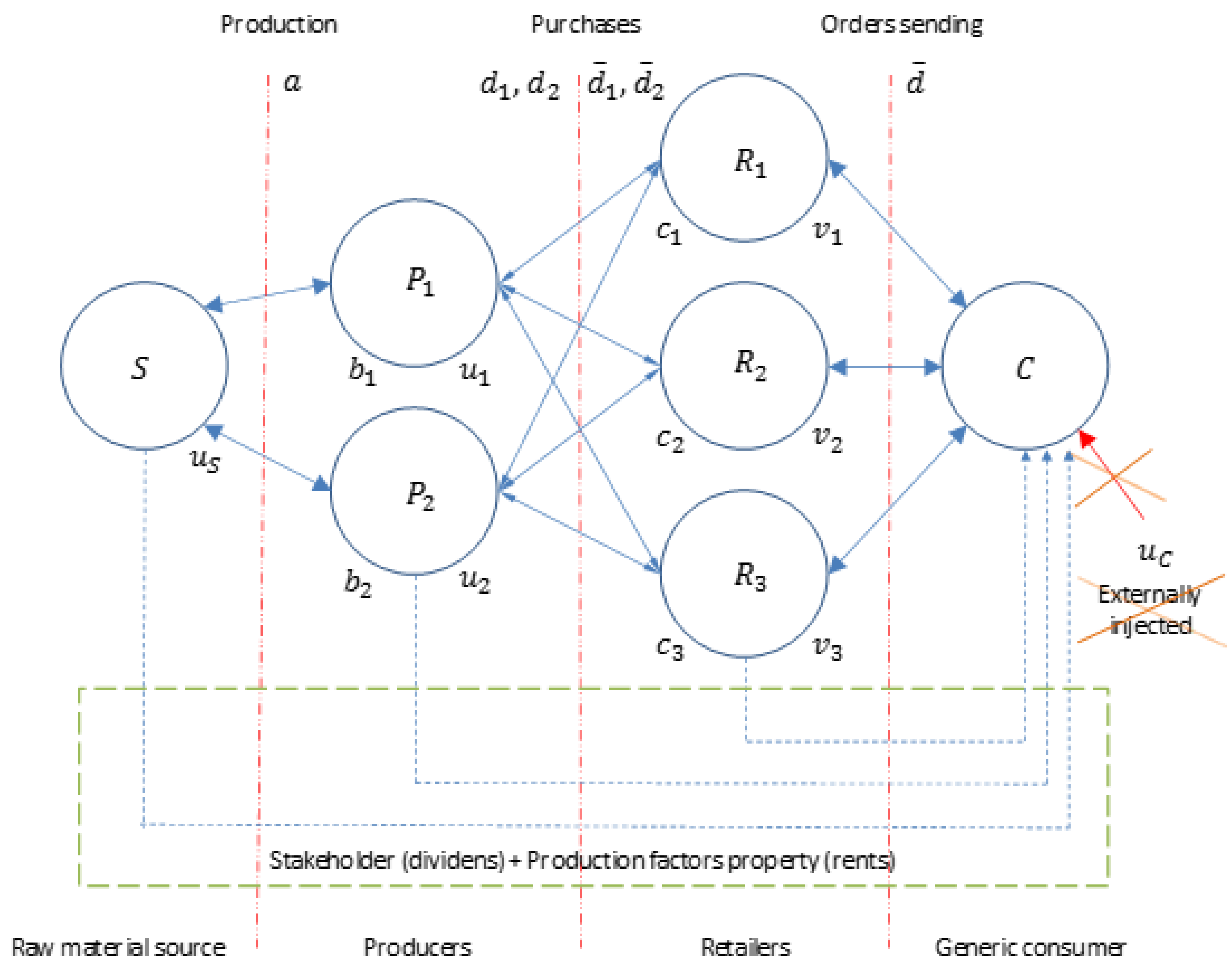
- Summarized behavior of Initial Model:
  - A steady increase of monetary units owned by producers, retailers and generic consumer.
  - Nearly stable producer's and retailer's capacities.
  - Monetary units obtained by raw source of material get out of circulation.

- Why?
  - Producers' & retailers' capacities are fixed and no changes are allowed.
  - Raw material and aggregate demand are initially settled and remain unchanged during the system evolution.
  - Artificial exogenous injection of monetary units into consumer C at the beginning of each cycle. This flow is necessary to maintain system evolving.

Enhanced Model

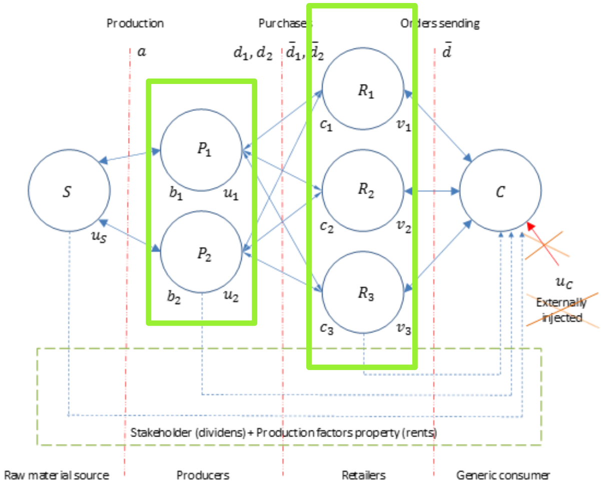
- Getting closer to real situations:
  - Allowing variations of producers' and retailers' capacities:
    - Capital stock depreciation.
    - Investment or capital increase decision.
  - Remove external injection of monetary units:
    - Payment of rents to the owners of the production factors.
    - Raw material source is owned by the aggregate consumer.
    - Aggregate consumer is stakeholder of producers and retailers, thus implying dividends payments.
  - Inclusion of randomness in a PDP-way:
    - Raw material generation.
    - Aggregate demand generation.
    - Mechanism of capacity increase decision.

Producer – Retailer Enhanced Model



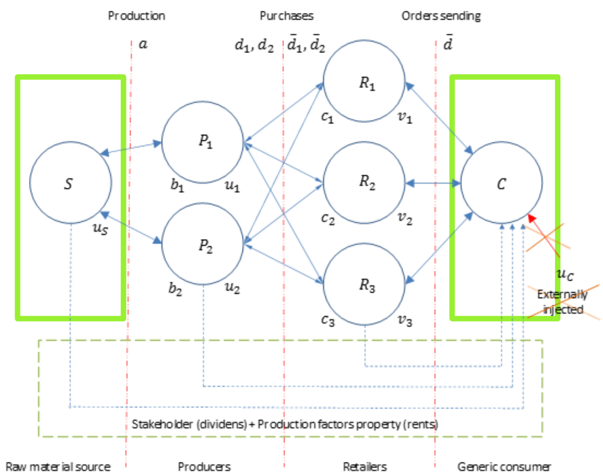
Model Entities

- Actors:
  - Producers:
    - (b_i, u_i) -> (capacity, money)
  - Retailers
    - (c_j, v_j) -> (capacity, money)



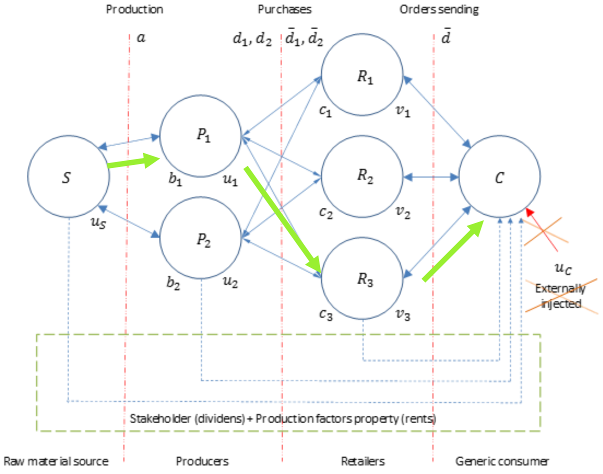
- Generic sources:
  - Of raw material (u_S, generation rate)
  - Of demand or Generic consumer

    (u_C, demand rate)



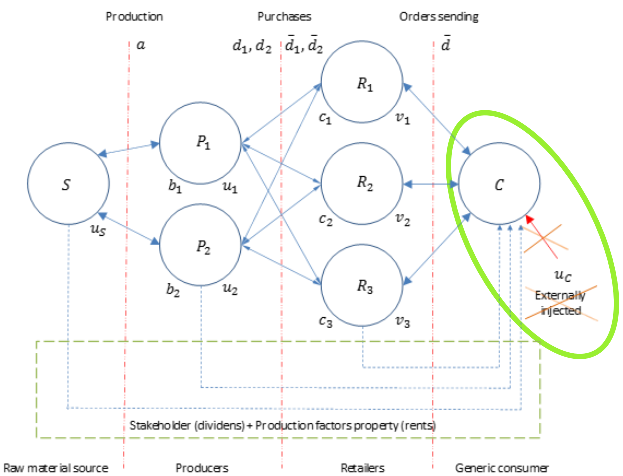
Model interactions

- Good's and order's flows:
  - Producers generate good d from raw material.
  - Retailers receive order $\bar{d}$ from generic consumer.
  - d and $\bar{d}$ are matched
  - Purchase $P_{i,j} = P(producer\ i, retailer\ j)$



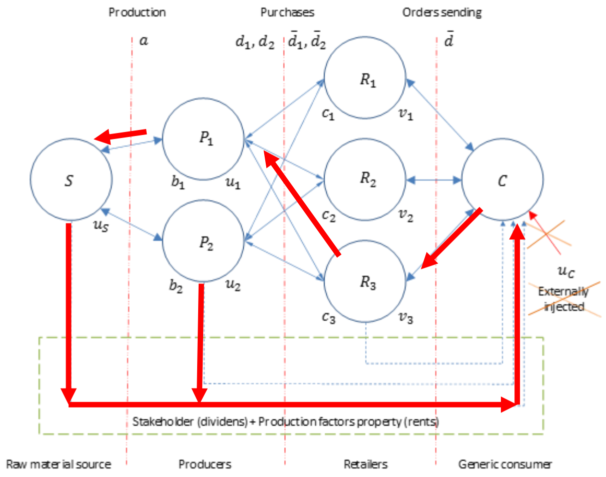
CYCLIC PROCESS

- External monetary injection:
  - Removed.



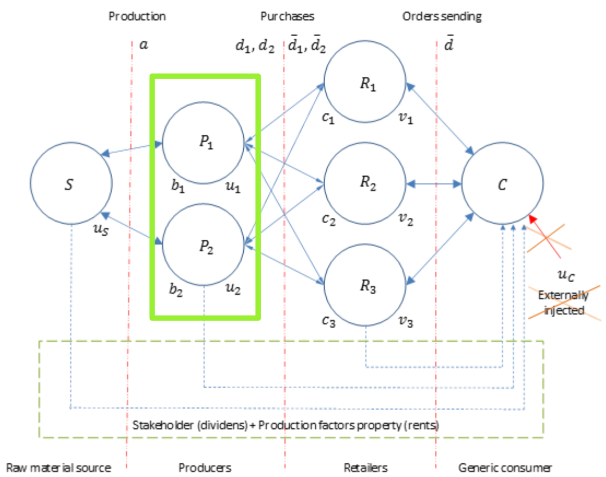
Additional interactions

- Monetary flows:
  - Initial Model monetary exchange due to prices.
  - Rents payments to owners: Generic Consumer.
  - Dividends payments to stakeholders: Generic Consumer.
  - Raw material source owners: Generic Consumer.

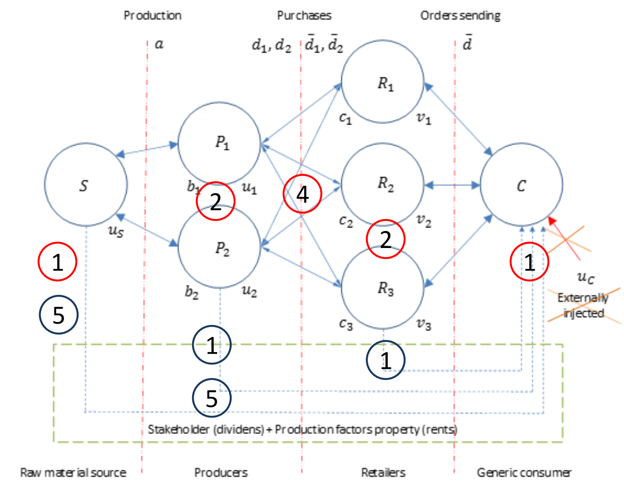


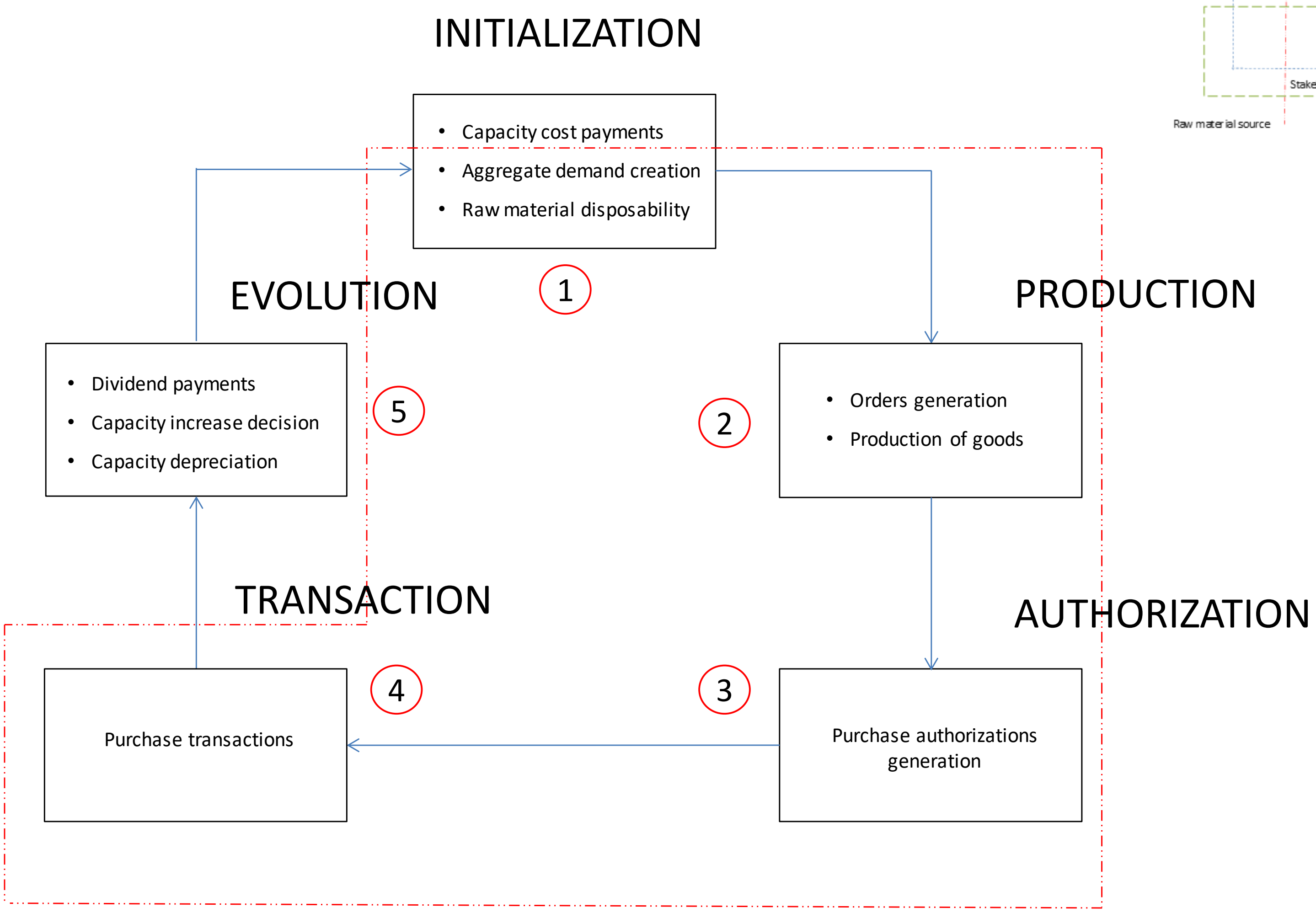
CYCLIC PROCESS

- Capacity variations:
  - Producers' capacity depreciation.
  - Producers' capacity increase decision: non-satisfied demand from retailers.



Defining steps of cycle





Model Formalization (I)

$\Pi = (\mathrm{G}, \Gamma, \Sigma, \mathrm{T}, \mathcal{R}_{\mathrm{E}}, \mu, \mathcal{R}_{\Pi}, \{\mathrm{f_r} \in \mathcal{R}_{\Pi}\}, M_1, M_2)$ PDP System of degree (2,1)

Where:

- $G = (V, E)$ with $V = \{e_1\}$ and $E = \{(e_1, e_1)\}$.
- Working alphabet: $\Gamma_{enhanced} = \Gamma_{initial} / \{u_S, R_2\} \cup \{g_i, y_i, , m_i, z_i, h_i : 1 \leq i \leq k_1\} \cup \{p, q\}$

Where:

C: aggregate generic consumer.

S: raw material supplier.

$\bar{d}$: unit of aggregate demand from C.

a: unit of supplied raw material provided by S.

u_C: monetary unit owned by C.

b_i: unit of production capacity of producer $i.1 \leq i \leq k_1$.

d_i: unit of good supplied by producer i. $1 \leq i \leq k_1$.

u_i: monetary unit owned by producer $i.1 \leq i \leq k_1$.

c_j: unit of capacity of retailer $j.1 \leq j \leq k_2$.

$\bar{d}_j$: unit of good demanded by retailer j. $1 \leq j \leq k_2$.

v_j: monetary unit owned by retailer $j.1 \leq j \leq k_2$.

$\bar{e}_j$: unit of good demanded by retailer and authorized for transaction unit of $\bar{d}_j$. $1 \leq j \leq k_2$.

$f_{i,j}$: authorization for $\bar{d}_j$ to be exchange with d_i $.1 \leq i \leq k_1, 1 \leq j \leq k_2$.

R_1: for technical reasons.

p: randomness generator for a provision by S.

q: randomness generator for $\bar{d}$ generation by C.

h_i : unit of production capacity of producer i before depreciation.$1 \leq i \leq k_1$.

y_i: unit (in idle state) of aborted purchase transactions considered for capacity increase. $1 \leq i \leq k_1$.

m_i: randomness generator for y_i. $1 \leq i \leq k_1$.

z_i : activated unit of aborted purchase transactions considered for capacity increase. $1 \leq i \leq k_1$.. $1 \leq i \leq k_1$.

g_i: for technical reasons. $1 \leq i \leq k_1$

Model Formalization (II)

- $\Sigma = \emptyset$.
- $R_E = \emptyset$.
- $\Pi = \{\Gamma,\ \mu,\ M_1,\ M_2, \mathcal{R}_\Pi\}$ where:
  - Membrane structure: $\mu = [\,[\,]_2\,]_1$.
  - $M_1 = \{C, S, R_1\} \cup \left\{g_i, u_i^{k_{i,1}*k_{10}*7} : 1 \le i \le k_1\}, \{v_j^{k_{j,3}*k_{10}*7} : 1 \le j \le k_2\right\}$
  - $M_2 = \left\{c_j^{k_{j,3}} :\ 1 \le j \le k_2\right\} \cup \{b_i^{k_{i,1}} :\ 1 \le i \le k_1\}$

Initial multisets contain basically:

- $b_i^{k_{i,1}}, u_i^{k_{i,1}*k_{10}*7}$: producers' initial parameters.
- $c_j^{k_{j,3}}, v_j^{k_{j,3}*k_{10}*7}$: retailers' initial parameters.

They need same initial amount of monetary units to pay initial capacity costs. Where:

$k_{i,1}$: initial production capacity of producer i. $1 \le i \le k_1$.

$k_{j,3}$: initial capacity of retailer j. $1 \le j \le k_2$.

Model Parameters

- Goal: maximize model parametrization
  - k_1: total number of producers.
  - k_2: total number of retailers.
  - k_3: raw material inserted into the system by S – minimum value of range
  - k_4: raw material inserted into the system by S – maximum value of range.
  - k_5: aggregate demand inserted into the system by C – minimum value of range.
  - k_6: aggregate demand inserted into the system by C – maximum value of range.
  - k_7: price fixed by S for each unit of a.
  - k_8: number of failed purchases considered for the analysis of increasing capital stock – minimum value.
  - k_9: number of failed purchases considered for the analysis of increasing capital stock – maximum value.
  - k_{10}: cost of capital stock per cycle.
  - k_{11}: depreciation rate of capital stock.
  - k_{12}: step of capacity increase.
  - k_{13}: dividend percentage.
  - $k_{i,1}$: initial production capacity of producer i. $1 \leq i \leq k_1$.
  - $k_{i,2}$: price fixed by producer i for each unit of d_i. $1 \leq i \leq k_1$.
  - $k_{j,3}$: initial capacity of retailer j. $1 \leq j \leq k_2$.
  - $k_{i,6}$: price fixed by retailers j for each order of good. $1 \leq j \leq k_2$.

Set of rules – Initialization

- From Naïve randomness:

$$r_5 \equiv R_2\, c[\,]_2 \xrightarrow{k_{1,5}} \bar{d}^{k_5+k_6} c[\,R_2]_2^+$$
$$r_6 \equiv R_2\, c[\,]_2 \xrightarrow{k_{2,5}} \bar{d}^{k_5}\, c[\,R_2]_2^+$$
$$r_7 \equiv R_2\, c[\,]_2 \xrightarrow{k_{3,5}} \bar{d}^{k_5-k_6}\, c[\,R_2]_2^+$$
$$r_8 \equiv R_2\, c[\,]_2 \xrightarrow{1-k_{1,5}-k_{2,5}-k_{3,5}} \bar{d}^{k_5-2*k_6}\, c[\,R_2]_2^+$$

Generates $\bar{d}$ around k_5

- To a PDP-way: raw material disposability & generic demand creation:

$$r_1 \equiv R_1\, s\, c[\,]_2 \;\rightarrow\; a^{k_3}\, p^{k_4-k_3}\, \bar{d}^{k_5}\, q^{k_6-k_5} s\, c[\,R_1]_2^+$$
$$r_2 \equiv p\,[\;]_2^- \xrightarrow{0.5} [\;]_2^+$$
$$r_3 \equiv p\,[\;]_2^- \xrightarrow{0.5} a\,[\;]_2^+$$
$$r_4 \equiv q\,[\;]_2^- \xrightarrow{0.5} [\;]_2^+$$
$$r_5 \equiv q\,[\;]_2^- \xrightarrow{0.5} \bar{d}\,[\;]_2^+$$

Generates $[\bar{d}^{k_5}, \bar{d}^{k_6}]$

Generates $[a^{k_3}, a^{k_4}]$

Set of rules – Capacity costs

- Rents for capacity:
  - Generic consumer is the owner of production factors.
  - Agents have enough monetary units to pay for capacity:

$$r_9 \equiv {u_i}^{k_{10}} \, [b_i\,]_2 \;\rightarrow\; b_i \;\; {u_C}^{k_{10}} \, [\;\,]_2^{+} 1 \leq i \leq k_1$$
$$r_{10} \equiv {v_j}^{k_{10}} \, \left[c_j\,\right]_2 \;\rightarrow\; c_j \;\; {u_C}^{k_{10}} \, [\;\,]_2^{+} 1 \leq j \leq k_2$$

  - Agents are not able to pay for capacity:

$$r_{11} \equiv \; [b_i\,]_2^{+} \;\rightarrow\; {u_C}^{k_{10}} \, [\;\,]_2 1 \leq i \leq k_1$$
$$r_{12} \equiv \; \left[c_j\,\right]_2^{+} \;\rightarrow\; {u_C}^{k_{10}} \, [\;\,]_2 1 \leq j \leq k_2$$

k_1: total number of producers.

k_2: total number of retailers.

k_{10}: cost of capital stock per cycle.

Set of rules – Operations

- Main changes:
  - Generic consumer is the owner of raw material source

- Producer operation:

$$r_{14} \equiv a\ b_i\ {u_i}^{k_7}[\ ]_2^+ \rightarrow {u_C}^{k_7}[\ d_i]_2^0 \quad 1 \leq i \leq k_1$$

- Retailer operation:

$$r_{15} \equiv \bar{d}\ c_j\ {u_C}^{k_{j,6}}[\ ]_2^+ \rightarrow \left[\bar{d}_j {v_j}^{k_{j,6}}\ \right]_2^0 \quad 1 \leq j \leq k_2$$

k_1: total number of producers.

k_2: total number of retailers.

k_7: price fixed by S for each unit of a.

$k_{i,6}$: price fixed by retailers j for each order of good.

$k_{j,7}$: price fixed by retailers j for each order of good.

- Unused capacities:

$$r_{16} \equiv b_i\ [\ ]_2 \rightarrow [\ b_i\ ]_2 \quad 1 \leq i \leq k_1$$

$$r_{17} \equiv c_j\ [\ ]_2 \rightarrow \left[\ c_j\ \right]_2 \quad 1 \leq j \leq k_2$$

Retired from the operational membrane waiting for their depreciation.

Set of rules – Auth. & Transactions

- Purchase authorization generation

$$r_{18} \equiv [\,\bar{d}_1]_2 \rightarrow [\;\bar{e}_1 f_{1,1}]_2$$
$$r_{19} \equiv [\,\bar{d}_1]_2 \xrightarrow{0} [\;\bar{e}_1 f_{1,2}]_2$$

Geo-barriers

$$r_{20} \equiv [\,\bar{d}_2]_2 \xrightarrow{0.5} [\;\bar{e}_2 f_{2,1}]_2$$
$$r_{21} \equiv [\,\bar{d}_2]_2 \xrightarrow{0.5} [\;\bar{e}_2 f_{2,2}]_2$$

Non-preferences

$$r_{22} \equiv [\,\bar{d}_3]_2 \xrightarrow{0.15} [\;\bar{e}_3 f_{3,1}]_2$$
$$r_{23} \equiv [\,\bar{d}_3]_2 \xrightarrow{0.85} [\;\bar{e}_3 f_{3,2}]_2$$

Preferences

k_1: total number of producers.

k_2: total number of retailers.

$k_{i,2}$: price fixed by producer i for each unit of d_i.

- Purchase transactions

$$r_{24} \equiv [\,d_{\mathrm{i}} \bar{e}_{\mathrm{j}} f_{\mathrm{j,i}} v_{\mathrm{j}}{}^{k_{\mathrm{i},2}}]_2^0 \;\rightarrow\; u_{\mathrm{i}}{}^{k_{\mathrm{i},2}} [h_{\mathrm{i}}\; c_{\mathrm{j}}]_2^- \quad 1 \le i \le k_1, 1 \le j \le k_2$$

b_{i} are retired as h_{i} from the operational membrane waiting for their depreciation.

Set of rules - Evolution

- Dividend payment:

$$r_{25} \equiv [\, v_{\mathrm{j}} \,]_2^{-} \rightarrow v_{\mathrm{j}} [\ ]_2^{0} \quad 1 \leq j \leq k_2$$

$$r_{26} \equiv u_{\mathrm{i}} [\ ]_2^{-} \xrightarrow{k_{13}} u_{\mathrm{C}} [\ ]_2^{0} \quad 1 \leq i \leq k_1$$

$$r_{27} \equiv u_{\mathrm{i}} [\ ]_2^{-} \xrightarrow{1-k_{13}} u_{\mathrm{i}} [\ ]_2^{0} \quad 1 \leq i \leq k_1$$

Both blocks of rules only applied to producers

- Capacity depreciation:

$$r_{31} \equiv [h_{\mathrm{i}}\ ]_2^{-} \xrightarrow{1-k_{11}} [\, b_{\mathrm{i}}\ ]_2^{0} \quad 1 \leq i \leq k_1$$

$$r_{32} \equiv [h_{\mathrm{i}}\ ]_2^{-} \xrightarrow{k_{11}} [\ ]_2^{0} \quad 1 \leq i \leq k_1$$

k_1: total number of producers.

k_2: total number of retailers.

k_{11}: depreciation rate of capital stock.

k_{13}: dividend percentage.

Set of rules – capacity increase

- When strictly necessary only
- Trigger: non-exhausted $f_{j,i}$
  - Case a: Enough producer capacity:

$$r_{28} \equiv \left[f_{j,i}\, d_i\right]_2^- \rightarrow [\, d_i\,]_2^0 \qquad 1 \le i \le k_1, 1 \le j \le k_2$$

$$r_{29} \equiv \left[f_{j,i}\, h_i\right]_2^- \xrightarrow{1-k_{11}} [b_i\,]_2^0 \qquad 1 \le i \le k_1, 1 \le j \le k_2$$

$$r_{30} \equiv \left[f_{j,i}\, h_i\right]_2^- \xrightarrow{k_{11}} [\;]_2^0 \qquad 1 \le i \le k_1, 1 \le j \le k_2$$

  - Case b: Not enough producer capacity:

$$r_6 \equiv g_i\,[\;]_2^0 \rightarrow \left[g_i\, y_i^{\,k_8}\, m_i^{\,(k_9-k_8)}\right]_2^+ \qquad 1 \le i \le k_1$$

$$r_7 \equiv [m_i\,]_2^+ \xrightarrow{0.5} [\;\;]_2^0 \qquad 1 \le i \le k_1$$

$$r_8 \equiv [m_i\,]_2^+ \xrightarrow{0.5} [\,y_i\,]_2^0 \qquad 1 \le i \le k_1$$

Generates $[y_i^{\,k_8}, y_i^{\,k_9}]$

$$r_{33} \equiv [y_i\,]_2^- \rightarrow [\,z_i\,]_2^0 \qquad 1 \le i \le k_1$$

$$r_{34} \equiv \left[f_{j,i}\, z_i\right]_2^0 \rightarrow b_i^{\,k_{12}}[\;]_2^+ \qquad 1 \le i \le k_1, 1 \le j \le k_2$$

k_1: total number of producers.

k_2: total number of retailers.

k_8: number of failed purchases considered for the analysis of increasing capital stock – min value.

k_9: number of failed purchases considered for the analysis of increasing capital stock – max value.

k_{11}: depreciation rate of capital stock.

.

Set of rules – Cleaning

- Cleaning rules and technical rules
  - Eliminate non-exhausted authorizations:

$$r_{35} \equiv [f_{\mathrm{j,i}}]_2^+ \rightarrow [\ ]_2^0 \qquad 1 \leq i \leq k_1, 1 \leq j \leq k_2$$

$$r_{36} \equiv [z_{\mathrm{i}}]_2^+ \rightarrow [\ ]_2^0 \qquad 1 \leq i \leq k_1$$

k_1: total number of producers.

k_2: total number of retailers.

  - Unauthorize non-exhausted $\bar{e}_j$:

$$r_{13} \equiv v_j [\ \ ]_2^+ \rightarrow [v_j]_2^0 \qquad 1 \leq j \leq k_2$$

$$r_{37} \equiv [\bar{e}_j]_2^+ \rightarrow [\bar{d}_j]_2^0 \quad 1 \leq j \leq k_2$$

  - Signaling a new cycle:

$$r_{38} \equiv [\,r_1]_2^- \rightarrow r_1 [\ \ ]_2^0$$

$$r_{39} \equiv [\,g_i]_2^- \rightarrow g_i [\ \ ]_2^0 \qquad 1 \leq j \leq k_2$$

P - Lingua

- Set of rules has been implemented in P – Lingua.
- An example for each set of rules:
  - Initialization:

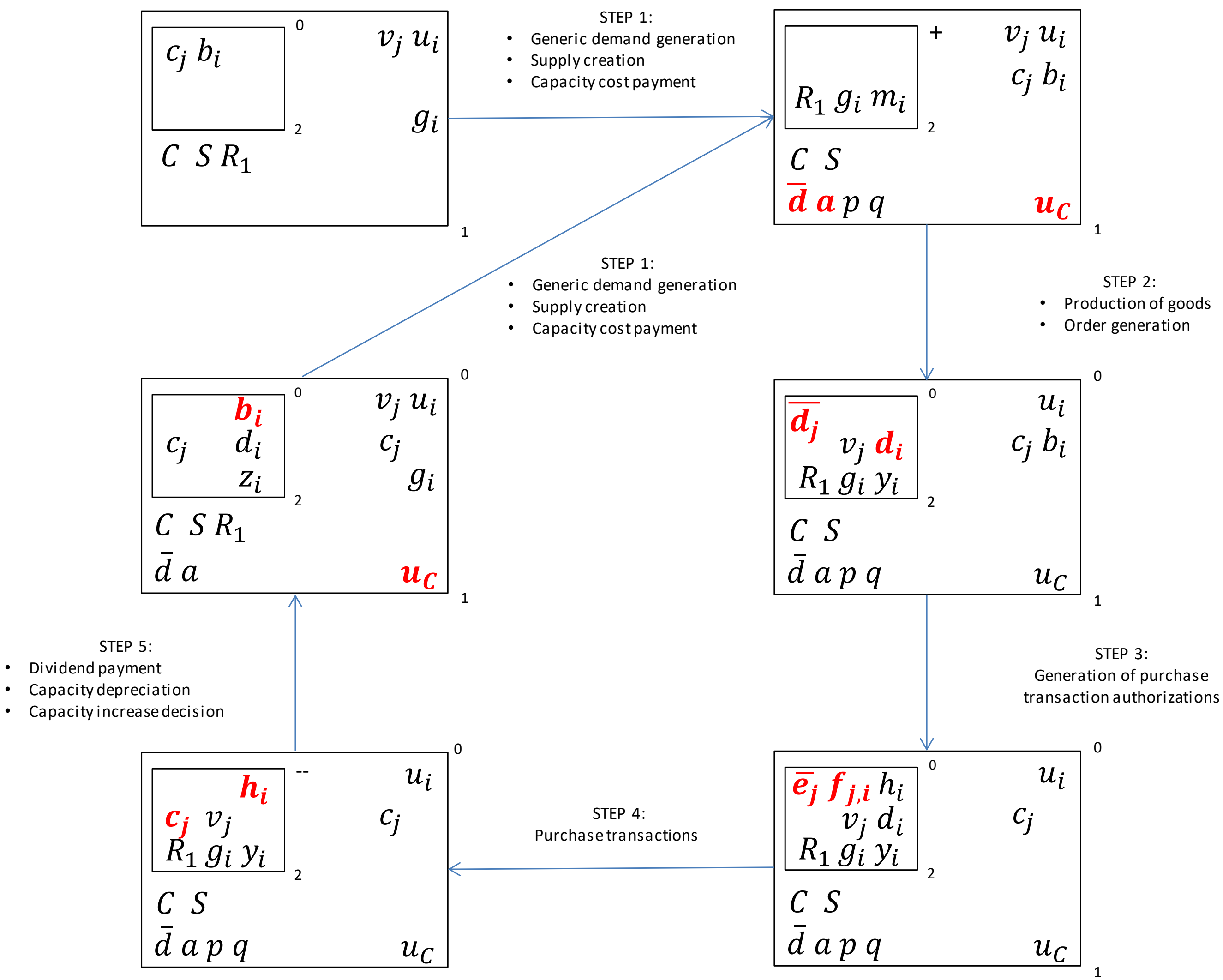
    $/* \, r1 \, */ \quad s, c, r1 \, [\quad]'2 \;\rightarrow\; s, c, a * k\{3\}, p * (k\{4\} - k\{3\}), dn * k\{5\}, q * (k\{6\} - k\{5\})$
    $+[r1]'2::1$
  - Production:

    $/* \, \mathrm{r9} \, */ \quad u\{i\} * k\{10\} \, [b\{i\}]'2 \;\;{-}{\rightarrow}\;\; b\{i\}, uc * k\{10\} + [\,]'2 :: 1 : 1 <= i <= k\{1\};$
  - Transaction:

    $/* \, r24 \, */ \quad [d\{i\}, en\{j\}, f\{j, i\}, v\{j\} * k\{i, 2\}\,]'2 \;{-}{\rightarrow}\; u\{i\} * k\{i, 2\} - [\, h\{i\}, c\{j\},]'2 \;\; :: 1 : 1 \leq$
    $i \leq k\{1\}, 1 \leq j \leq k\{2\}$
  - Capacity increase:

    $/* \, r34 \, */ \quad [\, f\{j, i\}, z\{i\}]'2 \;{-}{\rightarrow}\; b\{i\} * k\{12\} + [\quad]'2 \quad :: 1 \; : 1 \leq i \leq k\{1\}, 1 \leq j \leq k\{2\}$

Simplified trace



Simulation parameters

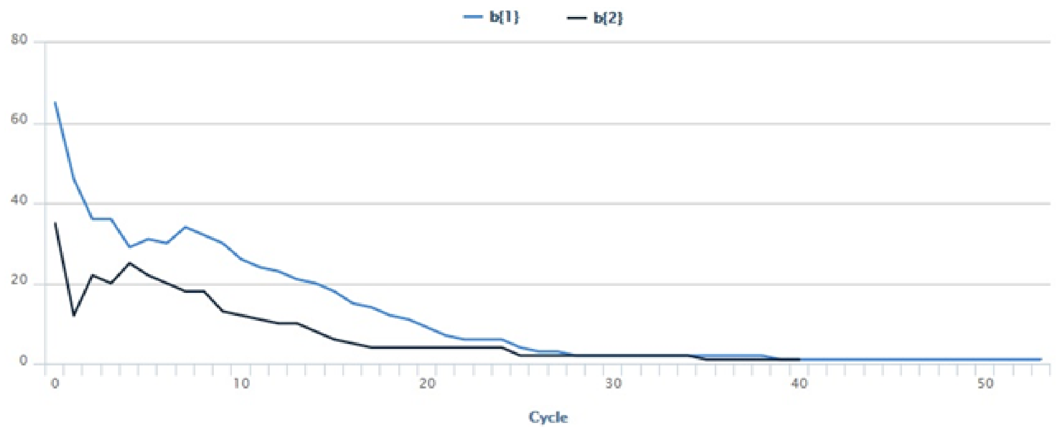
- Parameters: similar to Păun's paper

Parameter	Value	Description
k_1	2	Total number of producers
k_2	3	Total number of retailers
k_3	59	Units of raw material inserted into the system by S– minimum value of range
k_4	62	Units of raw material inserted into the system by S– maximum value of range
k_5	59	Units of aggregate demand inserted into the system by C – minimum value of range
k_6	62	Units of aggregate demand inserted into the system by C – maximum value of range
k_7	11	Price fixed by S for each unit of a
k_8	3	# failed purchases considered for the analysis of increasing capital stock – minimum value.
k_9	5	# failed purchases considered for the analysis of increasing capital stock – maximum value.
k_{10}	2	cost of capital stock per cycle
k_{11}	0.1	depreciation rate of capital stock
k_{12}	1	step of capacity increase
k_{13}	0.01	Dividend percentage
$k_{i,1}$	(65,35)	Initial production capacity of producer i. $1 \leq i \leq k_1$
$k_{i,2}$	{13,13)	Price fixed by producer i for each unit of d_i. $1 \leq i \leq k_1$
$k_{j,3}$	(50,30,20)	Initial capacity of retailer j. $1 \leq j \leq k_2$
$k_{i,6}$	(15,15,15)	Price fixed by retailer j for each order of good j. $1 \leq j \leq k_2$

MeCoSim definition

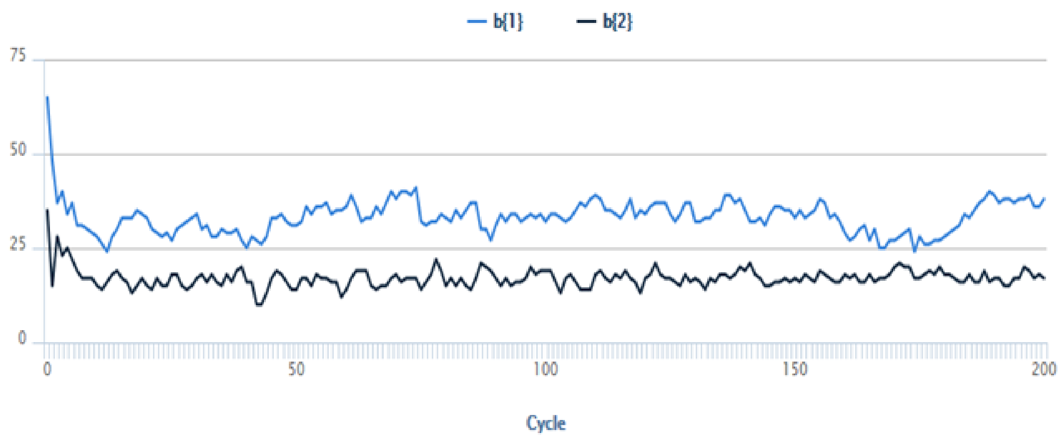
Parameter	Value	Description
k_1	<@r,1> Index 1 = 1	Captures number of producers based on the number of rows in table Producer_input
k_2	<@r,8> Index 2 = 2	Captures number of retailers based on the number of rows in table Retailer_input
k_3	<9,1-2,2> Index 1 = [3..<@r,9>+2]	Units of raw material inserted into the system by S– minimum value of range
k_4		Units of raw material inserted into the system by S– maximum value of range
k_5		Units of aggregate demand inserted into the system by C – minimum value of range
k_6		Units of aggregate demand inserted into the system by C – maximum value of range
k_7		Price fixed by S for each unit of a
k_8		# failed purchases considered for the analysis of increasing capital stock – minimum value.
k_9		# failed purchases considered for the analysis of increasing capital stock – maximum value.
k_{10}		Cost of capital stock per cycle
k_{11}		Depreciation rate of capital stock
k_{12}		Step of capacity increase
k_{13}		Dividend percentage
$k_{i,1}$	<1,1,2+1> Index 1 = [1..k{1}] Index 2 = [1..2]	Initial production capacity of producer i. $1 \leq i \leq k_1$
$k_{i,2}$		Price fixed by producer i for each unit of d_i. $1 \leq i \leq k_1$
$k_{j,3}$	<8,1,2> Index 1 = [1..k{2}] Index 2 = 3	Initial capacity of retailer j. $1 \leq j \leq k_2$
$k_{i,6}$	<8,1,3> Index 1 = [1..k{2}] Index 2 = 6	Price fixed by retailer j for each order of good j. $1 \leq j \leq k_2$

Simulation results – capacities



Producers' capacities with:

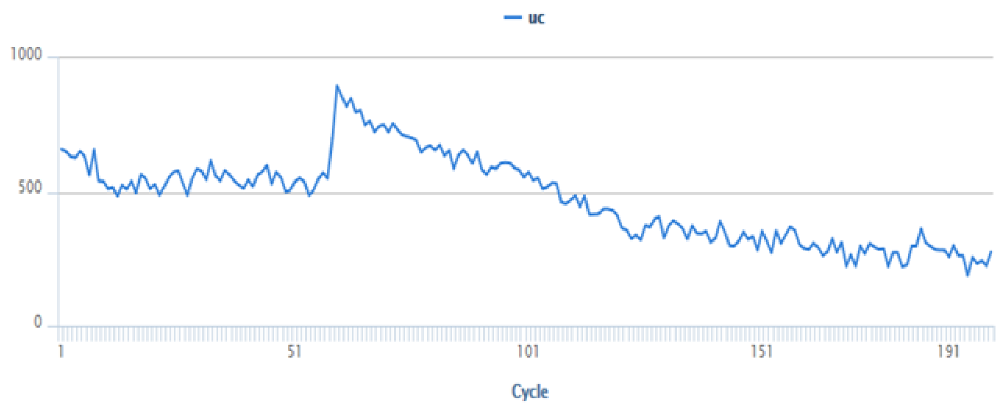
- Depreciation rate = 0.1
- Deactivated capacity increase mechanism.



Producers' capacities with:

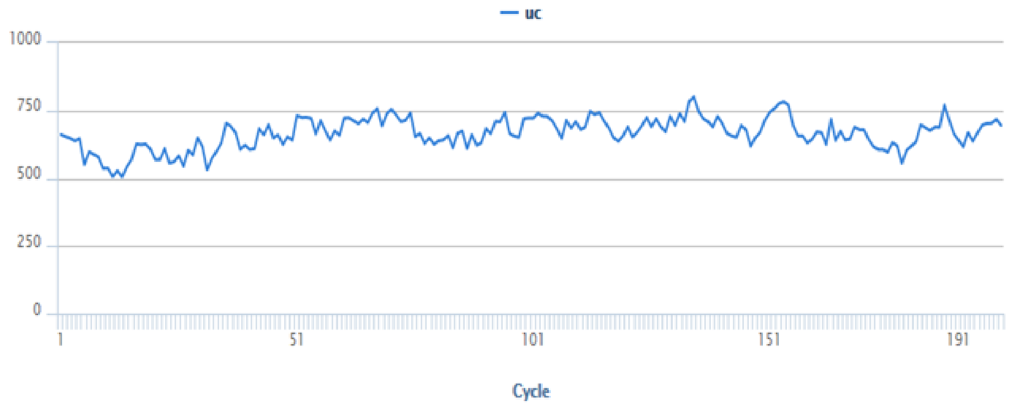
- Depreciation rate = 0.1
- Activated capacity increase mechanism.

Simulation results – dividends



Generic consumer monetary units:

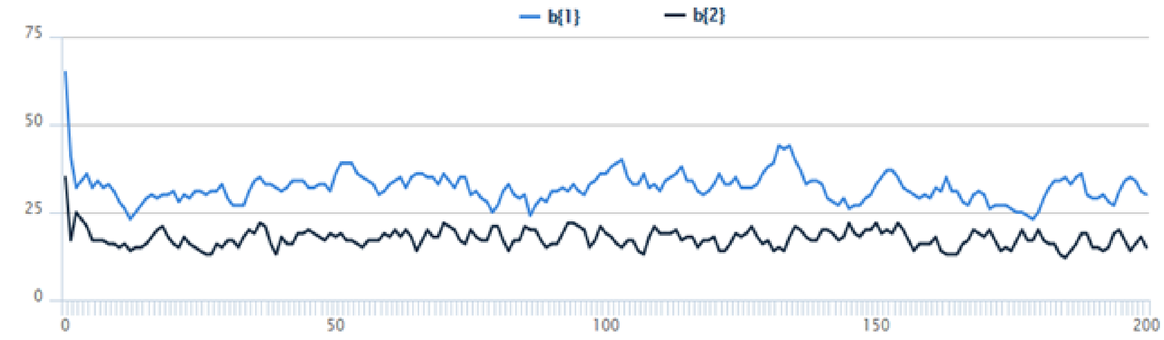
- Deactivated dividend payment.



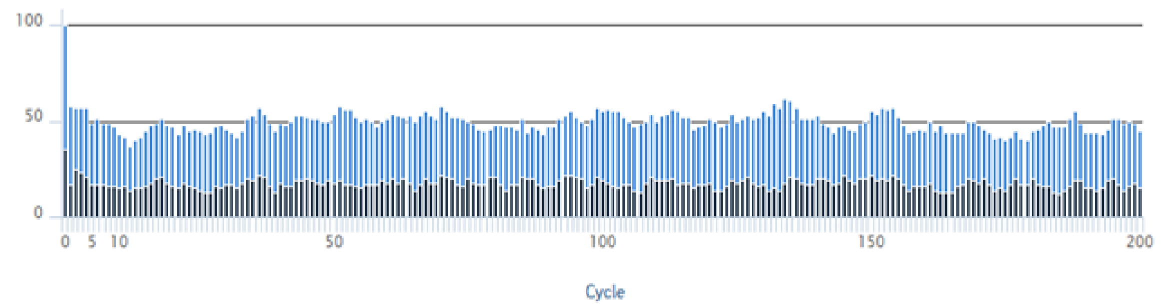
Generic consumer monetary units:

- Restored dividend payment.

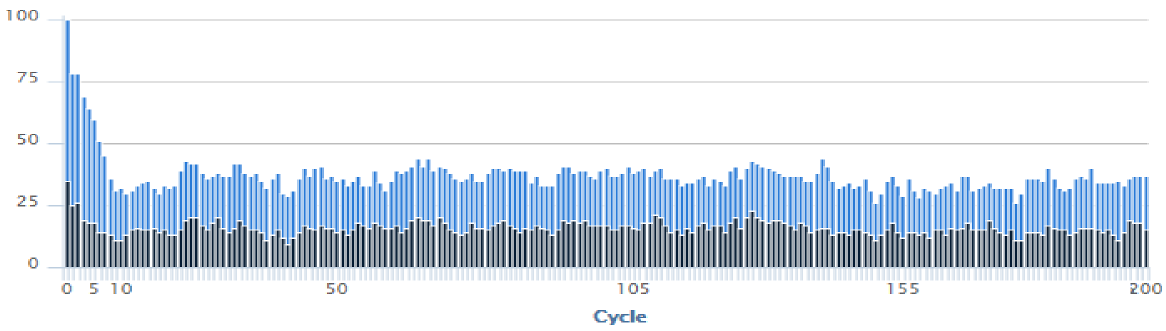
Simulation results – producer



- Initial distribution of capacities: 65 + 35



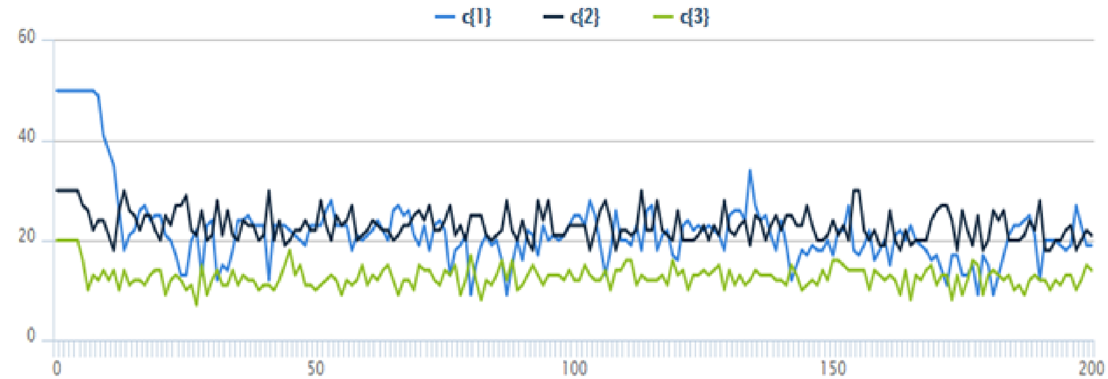
- Raw material generation rate [59,62]

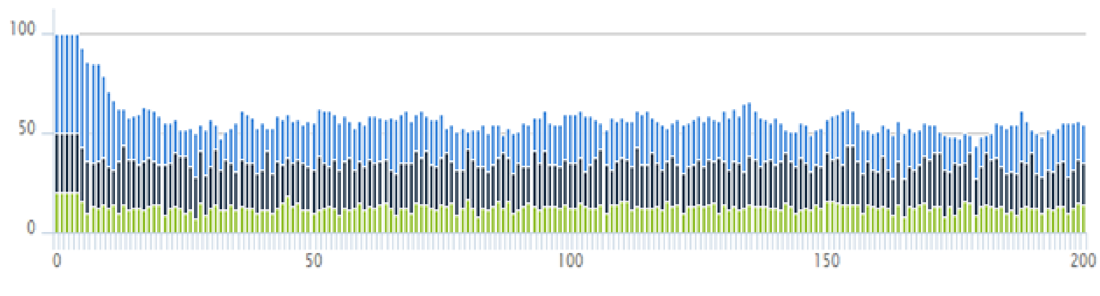


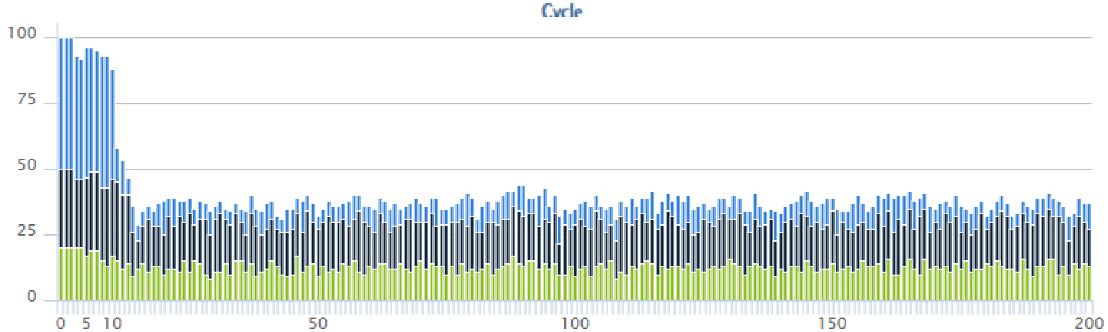
- Raw material generation rate [40,43]

System tries to reach an equilibrium point in function of parameters of *S*

Simulation results – retailer







- Initial distribution of capacities: 50 + 30 + 20
- Generic demand generation rate [59,62]
- Generic demand generation rate [40,43]

System tries to reach an equilibrium point in function of parameters of *C*

CONCLUSIONS

- Initial model:
  - We have been able to reproduce Păun’s results using:
    - PDP systems.
    - P – Lingua & MeCoSim framework.
    - Inference engine DNDP4.
- Enhanced model:
  - Initial model has been extended including several real world economic processes.
    - Cost of production factors, dividend payment.
    - Capacity depreciation, capacity increase mechanisms.
    - Removing external injection of monetary units.
  - Model evolves autonomously around an equilibrium point different from the initial conditions.

FURTHER DEVELOPMENTS

- Complete the enhanced model:
  - Macroeconomics interest: behavior of system under perturbation around equilibrium.
  - Introduce mechanisms to adjust prices to some stimulus.
  - Investigate if different patterns of randomness could be generated easily.
- Future Case of Study:
  - SDGE (Stochastic Dynamic General Equilibrium).
  - Previous techniques can be utilized in this problem.
  - Challenge: generate an emergent optimization behavior.

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