Minimal cooperation in polarizationless P systems with active membranes. Complexity aspects

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DP: class of "differences" of any two languages in **NP** (Papadimitriou and Yannakakis, 1984).

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#P: counting problems associated with polynomially balanced polynomial-time decidable relations (Valiant, 1979).

- $\mathbf{PP} \prec \#\mathbf{P} \subseteq \mathbf{PSPACE}$ and and $\mathbf{PH} \subseteq \mathbf{P}^{\#\mathbf{PP}}$.
- The `#SAT` problem.

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$\Pi = (\Gamma, H, \mu, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{out})$ of degree $q \geq 1$:

* Γ is a finite alphabet whose elements are called objects;
* H is a finite alphabet such that $H \cap \Gamma = \emptyset$ whose elements are called labels;
* μ is a labelled rooted tree consisting of q nodes injectively labeled by elements of H;
* $\mathcal{M}_1, \ldots, \mathcal{M}_q$ are multisets over Γ;
* $\mathcal{R}$ is a finite set of rules, of the following forms:
    - (a_0) $[\, a \rightarrow u \,]_h$ (**object evolution rules**).
    - (b_0) $a\, [\ \ ]_h \rightarrow [\, b \,]_h$ (**send–in communication rules**).
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It is well known that $\mathbf{PMC}_{\mathcal{NAM}^0} = \mathbf{P}$.

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Theorem: `Subset-Sum` $\in \mathbf{PMC}_{\mathcal{DAM}^0(+e,+c,+d,+n)}$ (2005).

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Are necessary **division rules for non-elementary membranes**?

Păun's conjecture

[1] Gh. Păun. Further twenty six open problems in membrane computing. In M.A. Gutiérrez et al. Third Brainstorming Week on Membrane Computing, Report RGNC 01/2005, Fénix Editora, Sevilla, 2005, pp. 249–262.

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At the beginning of 2005, Gh. Păun wrote (problem **F** from [][1]):

> *My favorite question (related to complexity aspects in P systems with active membranes and with electrical charges) is that about the number of polarizations.* **Can the polarizations be completely avoided?** **The feeling** *is that this* **is not possible** *– and such a result would be rather sound: passing from no polarization to two polarizations amounts to passing from non–efficiency to efficiency.*

This so–called Păun's conjecture can be formally formulated as follows:

$$\boxed{\mathbf{PMC}_{\mathcal{DAM}^0(+e,+c,+d,-n)} = \mathbf{P}}$$

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A *negative answer*: provide a borderline between tractability and intractability (assuming that $\mathbf{P} \neq \mathbf{NP}$).

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What syntactical ingredients are enough to solve **NP**-complete problems in an efficient way, by using the frameworks $\mathcal{DAM}^0(\mathbf{-d}, \mathbf{-n})$ or $\mathcal{SAM}^0(\mathbf{-d}, \mathbf{-n})$?

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Dissolution: An apparently **innocent** rule.

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The class $\mathcal{DAM}^0(\alpha, +c, -d, \pm n)$, where $\alpha \in \{\text{mc}, \text{pmc}, \text{bmc}, \text{mcmp}\}$.

RGNC

CCIA

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- ⋆ New frontier of the efficiency in the framework $\mathcal{AM}^0(bmc,+c,-d,-n)$: **separation** versus **division**.

- ⋆ New frontier of the efficiency in the framework $\mathcal{DAM}^0(*,+c,-d,-n)$: **non-cooperation** in object evolution rules versus **bmc** in object evolution rules.

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Theorem: $\texttt{SAT} \in \mathbf{PMC}_{\mathcal{DAM}^0(\mathbf{mcmp},+c,-\mathbf{d},-\mathbf{n})}$.

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Theorem: $\mathbf{PMC}_{\mathcal{SAM}^0(\mathbf{mcmp},+c,\mathbf{-d},\mathbf{+n})} = \mathbf{P}$.

Counting membrane systems

Decision problems: abstract problem that has a `yes` or `no` answer.

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- Counting membrane systems: inspired from recognizer membrane systems but the boolean answer of these systems is replaced by an **answer** encoded by a **natural number expressed in a binary notation**.
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Minimal cooperation and minimal production in communication rules

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⋆ **mcmp in send-in and send-out communication rules** ($\mathbf{mcmp_{in-out}}$):

$$\left.\begin{array}{rcl} a\,[\,]_h & \longrightarrow & [\,b\,]_h \\ a\ b\,[\ ]_h & \longrightarrow & [\,c\,]_h \\ [\,a\,]_h & \longrightarrow & b\ [\ ]_h \\ [\,a\,b\,]_h & \longrightarrow & c\,[\ ]_h \end{array}\right\} \text{for } h \in H \text{ and } a, b, c \in \Gamma$$

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The class $\mathcal{DAM}^0(+\mathbf{e}, \beta, \pm\mathbf{d}, \pm\mathbf{n})$, $\beta \in \{\text{mcmp}_{in-out}, \text{mcmp}_{in}, \text{mcmp}_{out}\}$.

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Are necessary **division rules for non-elementary membranes**?

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THANK YOU

FOR YOUR ATTENTION!