P systems with costs

and their relation to priced timed automata, and priced timed Petri nets

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Outline

- Introduction, motivation
- P sytsems with costs
- Priced timned automata
- Priced timed Petri nets
- Conclusions

Introduction

- In this talk we deal with the (evolution) costs in a membrane system.
- A P system with costs is essentially a simple membrane system in which object storage costs per evolution step are assigned to membranes, and execution costs are assigned to rules.
- Notice that here we consider the cost only as an external/observer variable, and thus whether a rule is applicable only depends on available resources (not cost value).

Introduction

- This extension could be useful in tackling an open problem[a]:

  "Such [cells] are cooperators paying an individual cost to produce a public good [. . .] The overall computing power [. . .] is then linked to the fate of those cells that contribute [. . .]. However, the public goods nature of the computation is potentially threatened by the endogenous appearance of cheaters, cells that [. . .] do not contribute [. . .] avoiding paying the costs associated to it."

[a] M.Cavaliere, A.Sanchez: Evolutionary Resilience of Membrane Computations. Bulletin of the Int'l Membrane Computing Society no.**2**, 159–160 (2016)

P Systems with Costs

The set $\mathcal{M}(\Pi)$ of membranes for a P system with costs Π, and the membrane structures are inductively defined as follows:

- if L is a label, c is a cost and w is a multiset over $O \cup (O \times \{here\}) \cup (O \times \{out\}) \cup \{\delta\}$, then $\langle L ; c \,|\, w \rangle \in \mathcal{M}(\Pi)$; $\langle L ; c \,|\, w \rangle$ is called simple (or elementary) membrane, and it has the structure $\langle\rangle$;
- if L is a label, c is a cost and w is a multiset over $O \cup (O \times \{here\}) \cup (O \times \{in_{L(M_j)} | j \in [n]\}) \cup (O \times \{out\}) \cup \{\delta\}$, $M_1, \ldots, M_n \in \mathcal{M}(\Pi)$, $n \geq 1$, where each membrane M_i has the structure μ_i, then $\langle L ; c \,|\, w \,;\, M_1, \ldots, M_n \rangle \in \mathcal{M}(\Pi)$; $\langle L ; c \,|\, w \,;\, M_1, \ldots, M_n \rangle$ is called a composite membrane having the structure $\langle \mu_1, \ldots, \mu_n \rangle$.

P Systems with Costs

- Each P system with costs has an initial configuration which is characterized by the initial multiset of objects for each membrane and the initial membrane structure of the system.
- For two configurations C_1 and C_2 of Π, we say that there is a transition from C_1 to C_2 with cost c, and write $C_1 \Rightarrow_c C_2$, if the following steps are executed in the given order:
  - maximal parallel rewriting step;
  - parallel communication of objects through membranes;
  - parallel membrane dissolving of the membranes containing δ.

Example

Consider a small example containing only of a single membrane labelled by 1 and with associated cost 2. This membrane contains three objects A and three rules with different assigned costs (rule R_0 with cost 10, rule R_1 with cost 8 and rule R_2 with cost 12).

$$A + A + A$$

$$R_0 : A \xrightarrow{10} B$$

$$R_1 : A \xrightarrow{8} C$$

$$R_2 : C \xrightarrow{12} D$$

2

1

There are different possible evolutins with different costs:

- $(A + A + A) \Rightarrow_{30} (B + B + B)$
- $(A + A + A) \Rightarrow^{*}_{40} (B + B + D)$
- $(A + A + A) \Rightarrow^{*}_{50} (B + D + D)$
- $(A + A + A) \Rightarrow^{*}_{60} (D + D + D)$

Priced Timed Automata

Definition (Priced Timed Automata)

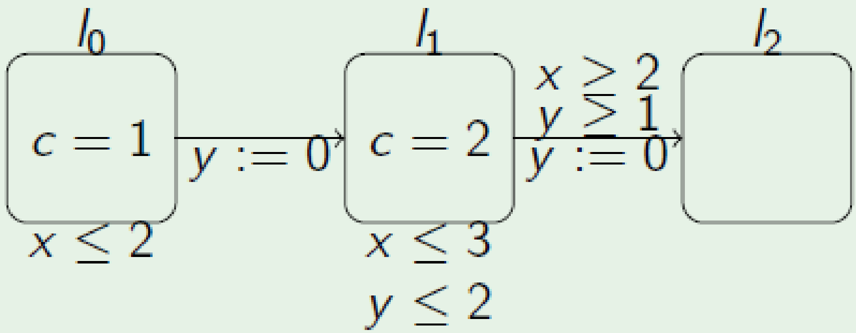
Let X be a set of clocks and Act a set of actions. A priced timed automaton over X and Act is a tuple $A = (L, E, l_0, I, P)$, where:

- L is a set of locations,
- $E \subseteq L \times \mathcal{B}(X) \times Act \times 2^X \times L$ is a set of edges,
- $l_0 \in L$ is the initial location,
- $I : L \to \mathcal{B}(X)$ assigns invariants to locations, and
- $P : L \cup E \to \mathbb{N}_0$ assigns cost rates and costs to locations and edges, respectively.

G. Behrmann, A. Fehnker, T. Hune, K. G. Larsen, P. Pettersson, J. Romijn, F. Vaandrager. Minimum-Cost Reachability for Priced Timed Automata. *Lecture Notes in Computer Sciences* **2034**, 147–161 (2001).

Priced Timed Automata

Example



Consider the priced timed automaton A having two clocks x and y, a single goal location l_2 and two locations l_0 and l_1 with cost rate 1 and 2 respectively. Below we offer three sample traces of A:

$$\alpha_0 = (l_0, x = 0, y = 0) \rightarrow_0 (l_1, x = 0, y = 0) \xrightarrow{2}_4 (l_1, x = 2, y = 2)$$
$$\rightarrow_0 (l_2, x = 2, y = 0)$$
$$\alpha_1 = (l_0, x = 0, y = 0) \xrightarrow{2}_2 (l_0, x = 2, y = 2) \rightarrow_0 (l1, x = 2, y = 0)$$
$$\xrightarrow{1}_2 (l1, x = 3, y = 1) \rightarrow_0 (l2, x = 3, y = 0)$$
$$\alpha_2 = (l0, x = 0, y = 0) \xrightarrow{1}_1 (\iota 0, x = 1, y = 1) \rightarrow_0 (l1, x = 1, y = 0)$$
$$\xrightarrow{1}_2 (11, x = 2, y = 1) \rightarrow_0 (l2, x = 2, y = 0)$$

Definition (Priced Timed Petri Nets)

Is a tuple $N = (Q, P, T, Cost)$ where

- Q is a finite set of control-states;
- P is a finite set of places.
- T is a finite set of transitions;
- $Cost : P \cup T \to \mathbb{N}$ is the cost function assigning firing costs to transitions and storage costs to places.

It should be noticed that priced timed Petri nets subsume priced timed automata[a].

[a] P.A.Abdula, R.Mayr: Priced Timed Petri Nets. *Logical Methods in Computer Science* **9**, 1–51 (2013)

- We defined P systems with costs by assigning storage costs to membranes, as well as and execution costs to rules.
- Investigate the connections between P systems with costs and priced timed automata (and priced timed Petri nets).
- We intend to deal with the Cost Problems in the framework of membrane systems by considering two variants of the cost problem, namely:
  - Cost-Threshold Problem: can we obtain an evolution cost under a certain threshold value?
  - Cost-Optimality Problem: compute the minimal evolution cost.
- We also intend to study how different evolution strategies influence the computed cost of reaching a desired configuration.

Thank you.

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