Tissue P systems with evolutional communication rules. Complexity aspects

Mario J. Pérez Jiménez

Research Group on Natural Computing
Dpto. Computer Science and Artificial Intelligence
University of Seville, Spain
Academia Europaea (The Academy of Europe)

www.cs.us.es/~marper marper@us.es

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Basic tissue P systems with symport/antiport rules

$$\Pi = (\Gamma, \Sigma, \mathcal{E}, \mathcal{M}_1, \dots, \mathcal{M}_q, \mathcal{R}, i_{in}, i_{out})$$

- ⋆ Γ, Σ are finite *alphabets*.
- ⋆ $\mathcal{E} \subseteq \Gamma$.
- ⋆ $\mathcal{M}_1, \dots, \mathcal{M}_q$ are finite multisets over $\Gamma \setminus \Sigma$.
- ⋆ $\mathcal{R}$ is a finite set of rules of types
  - $(\mathbf{i}, \mathbf{u}/\lambda, \mathbf{j})$ (symport rule)
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  where $0 \le i, j \le q$, $i \neq j$, $u, v \in M_f^+(\Gamma)$.
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Length of the symport/antiport rule $(\mathbf{i}, \mathbf{u}/\mathbf{v}, \mathbf{j})$: $|\mathbf{u}| + |\mathbf{v}|$

Tissue P systems with symport/antiport rules and cell division or cell separation

Tissue P systems with symport/antiport rules and cell division or cell separation

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- The sets

  $\mathbf{TC}$, $\mathbf{TDC}$, $\mathbf{TSC}$, $\widehat{\mathbf{TDC}}$, $\widehat{\mathbf{TSC}}$

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Frontiers of the efficiency

Non − Efficiency	*Efficiency*	
TC	**TDC**	(*adding rules*)
TC	**TSC**	(*adding rules*)
TDC(1)	**TDC**(2)	(*length*)
TSC(2)	**TSC**(3)	(*length*)
$\widehat{\textbf{TSC}}$	**TSC**	(*environment*)
$\widehat{\textbf{TSC}}(3)$	**TSC**(3)	(*environment*)
TSC(2)	**TDC**(2)	(*kind of rules*)
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Length of an evolutional communication rule ......

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  $\mathbf{TDEC}(k)$, $\mathbf{TSEC}(k)$, for each $k \geq 1$

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Let $\mathbf{X} \in \{\mathbf{S}, \mathbf{D}\}$.

TXC ⊆ TXEC

- ⋆ Any symport rule $(\mathbf{i}, \mathbf{u}/\lambda, \mathbf{j})$ can be viewed as $[\ \mathbf{u}\ ]_i\ [\ \ ]_j \longrightarrow [\ \ ]_i\ [\ \mathbf{u}\ ]_j$
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$\boxed{\text{If } k_1 \geq 2 \text{ then } \mathbf{TXEC}(k_1) \subseteq \mathbf{TXEC}(k_1, k_1 - 1)}$

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Theorem: $\mathbf{PMC}_{\mathbf{TDEC}(1)} = \mathbf{PMC}_{\mathbf{TDEC}(2)} = \mathbf{P}$.

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$PMC_{TDEC(k)}$

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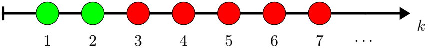
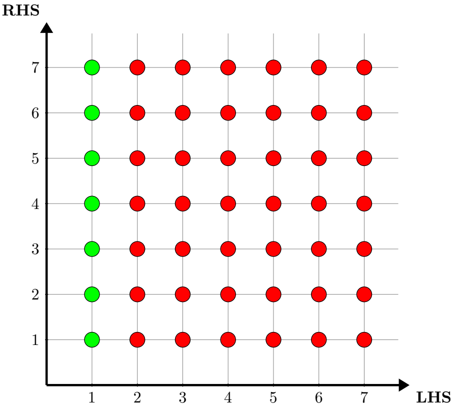


$\mathbf{P} = \mathbf{PMC}_{\mathcal{R}}$ (green) $\mathbf{DP} \subseteq \mathbf{PMC}_{\mathcal{R}}$ (red)

$\mathbf{PMC}_{\mathbf{TDEC}(\mathbf{k_1},\mathbf{k_2})}$

$\mathbf{PMC}_{\mathbf{TDEC}(k_1,k_2)}$



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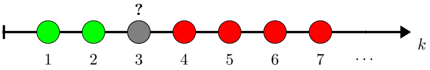

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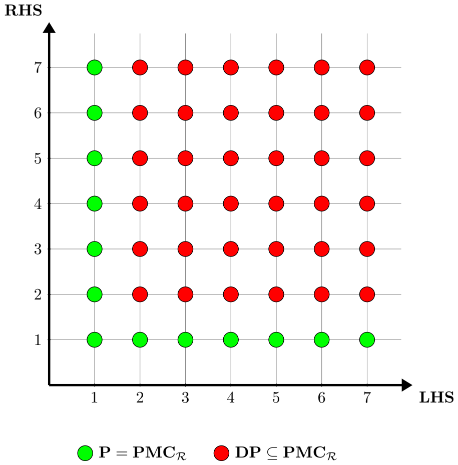
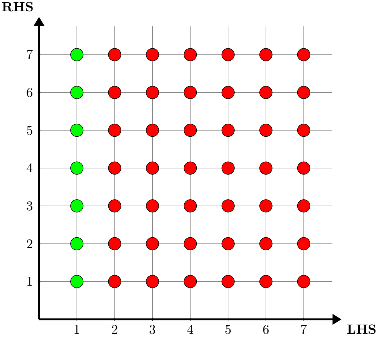
$\mathbf{PMC_{TSEC(k_1,k_2)}}$

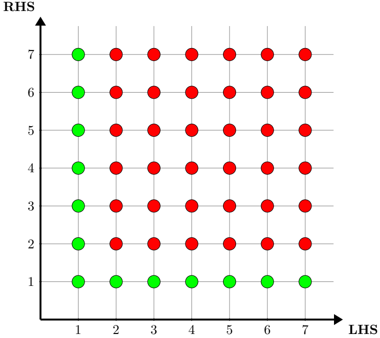
$PMC_{TSEC(k_1,k_2)}$

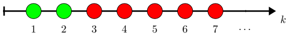


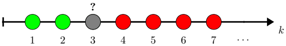
$\mathbf{PMC_{TDEC(k_1,k_2)}}$ versus $\mathbf{PMC_{TSEC(k_1,k_2)}}$

$\mathbf{PMC}_{\mathbf{TDEC}(k_1,k_2)}$ versus $\mathbf{PMC}_{\mathbf{TSEC}(k_1,k_2)}$









$\mathbf{P} = \mathbf{PMC}_{\mathcal{R}}$ $\mathbf{DP} \subseteq \mathbf{PMC}_{\mathcal{R}}$

References

References

1. L.F. Macías-Ramos, M.A. Martínez-del-Amor, M.J. Pérez-Jiménez, A. Riscos-Núñez, L. Valencia-Cabrera. The role of the direction in tissue P systems with cell separation. *Journal of Automata, Languages and Combinatorics*, 19, 1-4 (2014), 185-199.
2. L. Pan, B. Song, L. Valencia-Cabrera, M.J. Pérez-Jiménez. The computational complexity of tissue P systems with evolutional symport/antiport rules. *Complexity*, in press, 2017.
3. M.J. Pérez-Jiménez, P. Sosík. An optimal frontier of the efficiency of tissue P systems with cell separation. *Fundamenta Informaticae*, 138, 1-2 (2015), 45-60.
4. M.J. Pérez-Jiménez, A. Riscos, M. Rius, F.J. Romero. A polynomial alternative to unbounded environment for tissue P systems with cell division. *International Journal of Computer Mathematics*, 90, 4 (2013), 760-775
5. B. Song, C. Zhang, L. Pan, Tissue-like P systems with evolutional symport/antiport rules, *Information Sciences*, 378 (2017) 177–193.

THANK YOU

FOR YOUR ATTENTION!