Non-cooperative polymorphic P systems with "finitely representable" regions

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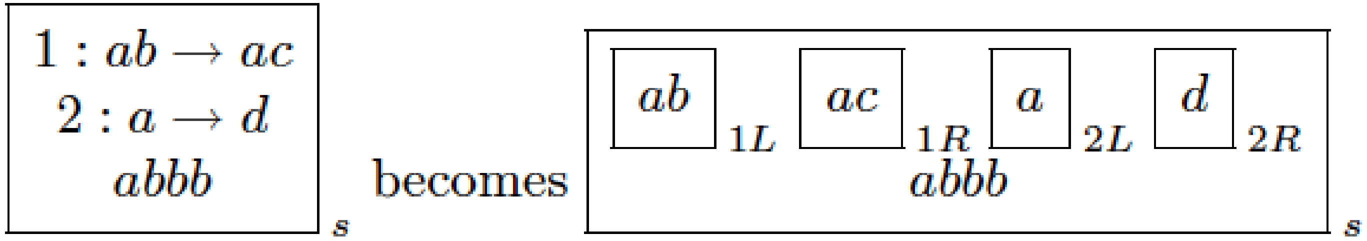
Polymorphic P systems - The idea

- To manipulate the **rules** during a computation: **represent** them **as data**



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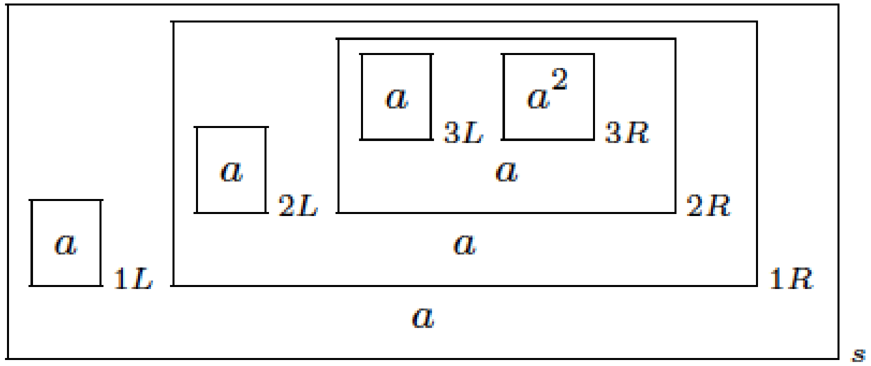
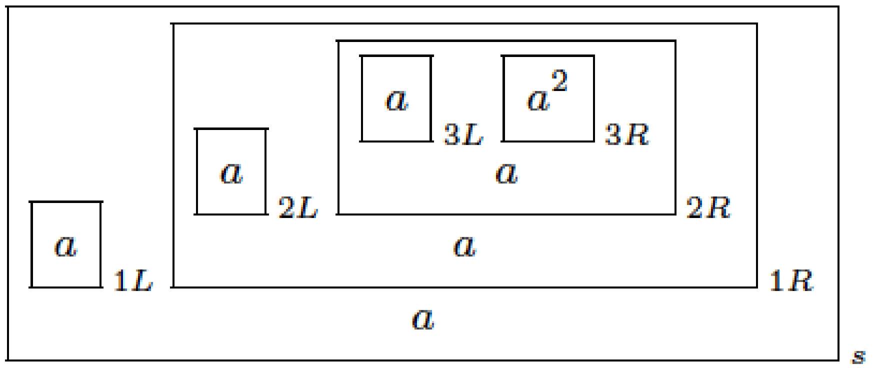
Artiom Alhazov, Sergiu Ivanov, Yurii Rogozhin: Polymorphic P Systems.
In: CMC 2010, Vol. 6501 of LNCS, pp. 81-94, 2010

For example



$3 : a \to a^2$ in $2R$
$2 : a \to a$ in $1R$
$1 : a \to a$ in s
$\Rightarrow$

For example

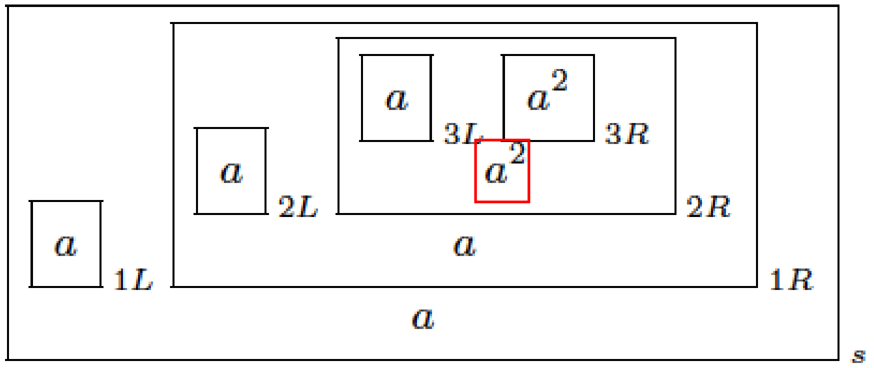


$3 : a \rightarrow a^2 \text{ in } 2R$

$2 : a \rightarrow a \text{ in } 1R$

$1 : a \rightarrow a \text{ in } s$

$\Rightarrow$



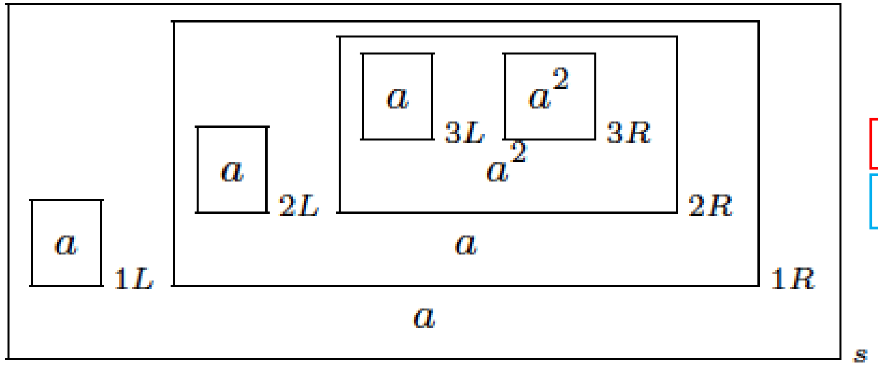
$3 : a \rightarrow a^2 \text{ in } 2R$

$2 : a \rightarrow a^2 \text{ in } 1R$

$1 : a \rightarrow a \text{ in } s$

$\Rightarrow$

For example

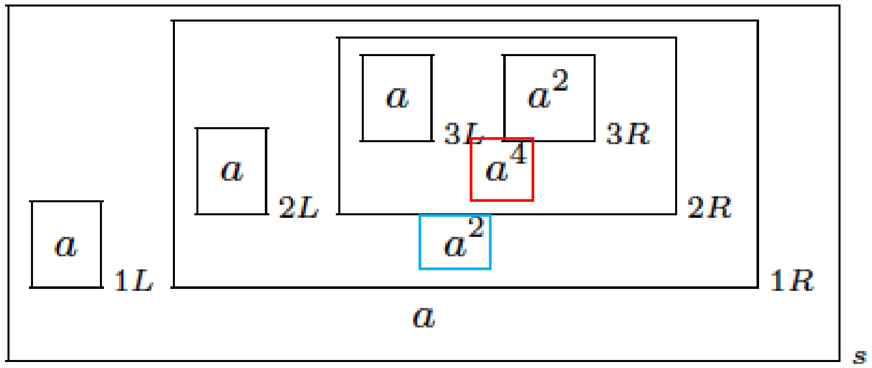


$3 : a \rightarrow a^2$ in $2R$

$2 : a \rightarrow a^2$ in $1R$

$1 : a \rightarrow a$ in s

$\Rightarrow$



$3 : a \rightarrow a^2$ in $2R$

$2 : a \rightarrow a^4$ in $1R$

$1 : a \rightarrow a^2$ in s

$\Rightarrow$

Systems with non-cooperative rules

- Artiom Alhazov, Sergiu Ivanov, Yurii Rogozhin: Polymorphic P Systems. In: *CMC 2010*, Vol. 6501 of *LNCS*, pp. 81-94, 2010
- Sergiu Ivanov: Polymorphic P Systems with Non-cooperative Rules and No Ingredients.
  In: *CMC 2014*, Vol. 8961 of *LNCS*, pp. 258-273, 2014

What is “no ingredients”?

Non-cooperative polymorphic P systems with limited depth

Theorem: $PsET0L \subseteq \mathcal{L}(NOP^3(polym, ncoo))$.

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Systems with finite sets of instances of dynamic rules

- **Non-cooperative** rules → Left-membranes have **finitely many** possible **membrane contents** in any computation

→left-membranes are always “**finitely representable**”

- What about “finitely representable” **right-membranes**?

Sergiu Ivanov: Polymorphic P Systems with Non-cooperative Rules and No Ingredients.
In: CMC 2014, Vol. 8961 of LNCS, pp. 258-273, 2014

Finite representabilty

$$\Pi = (O, T, \mu, w_s, \langle w_{1L}, w_{1R} \rangle, \ldots, \langle w_{nL}, w_{nR} \rangle, h_o)$$

- If w_h is the contents of **region *h*** after the ***j*-th step** of a computation, and w'_h can be obtained from w_h in the **next** computational **step:**

$$w'_h \in \sigma_{j,h}(w_h)$$

- and $\sigma^0_{j,h}(w_h) = w_h$,

  $\sigma^{k+1}_{j,h} = \sigma_{j+k,h}(\sigma^k_{j,h}(w_h))$ (set of contents obtainable in k+1 steps)

- $\sigma^*_{j,h} = \bigcup_{k \geq 0} \sigma^k_{j,h}(w_h)$

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Finite representability

Region ***h*** **is FIN-representable** if the **set of successor multisets** of the initial contents w_h of region *h* is **finite**.

→ if $\sigma^*_{0,h}(w_h)$ is finite

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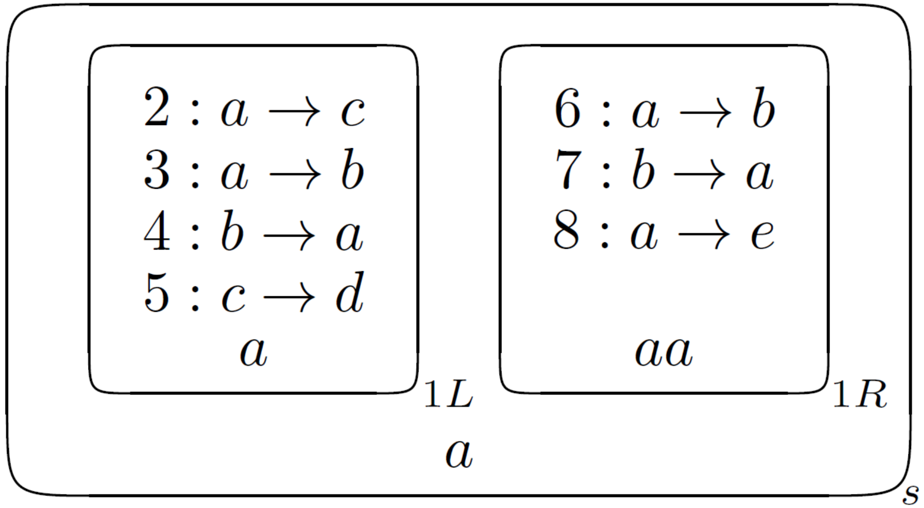
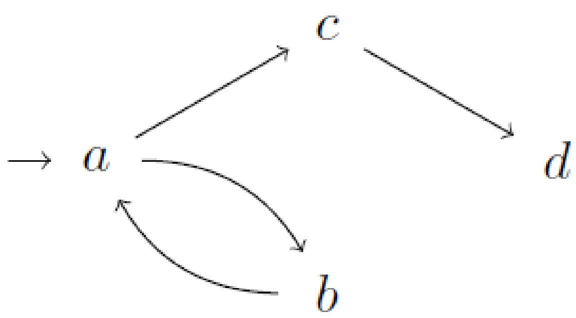
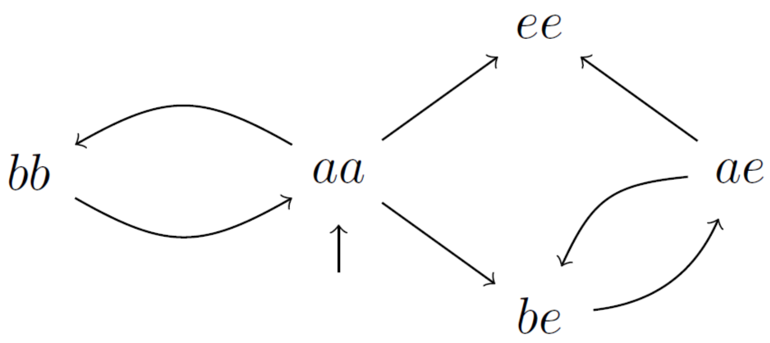
FIN-representability, an example





$$\sigma^*_{0,1L}(a) = \{a, b, c, d\}$$



$$\sigma^*_{0,1R}(aa) = \{aa, bb, ae, be, ee\}$$

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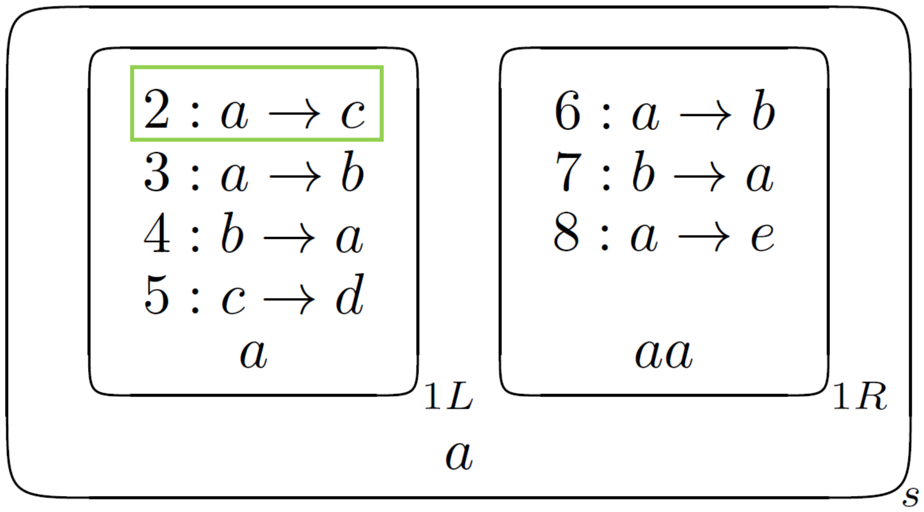
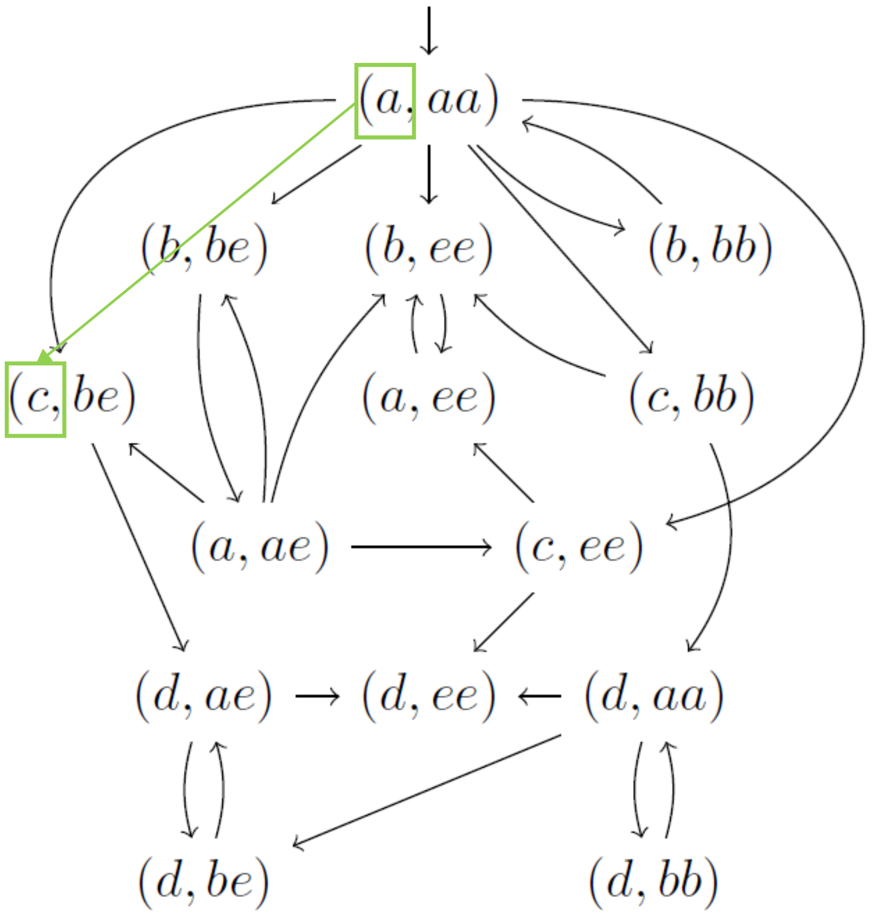
Region *S* is not FIN-representable

FIN-representability, an example





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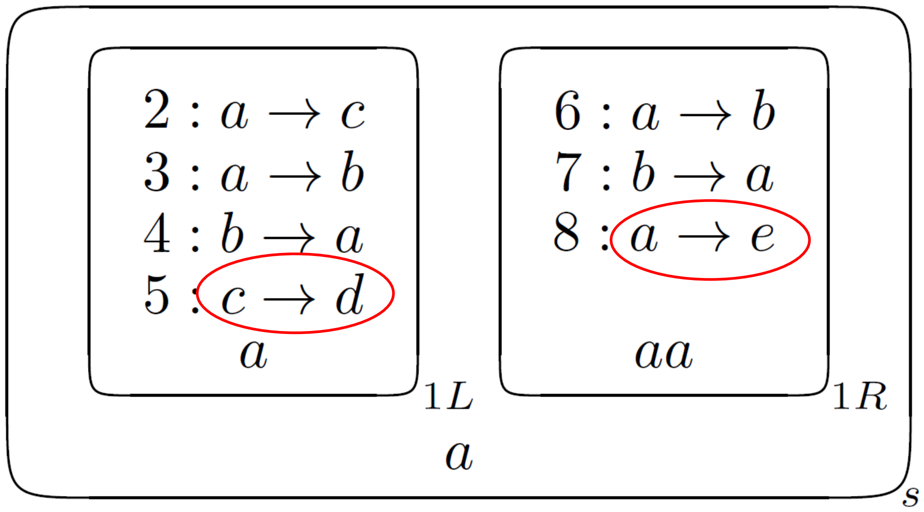
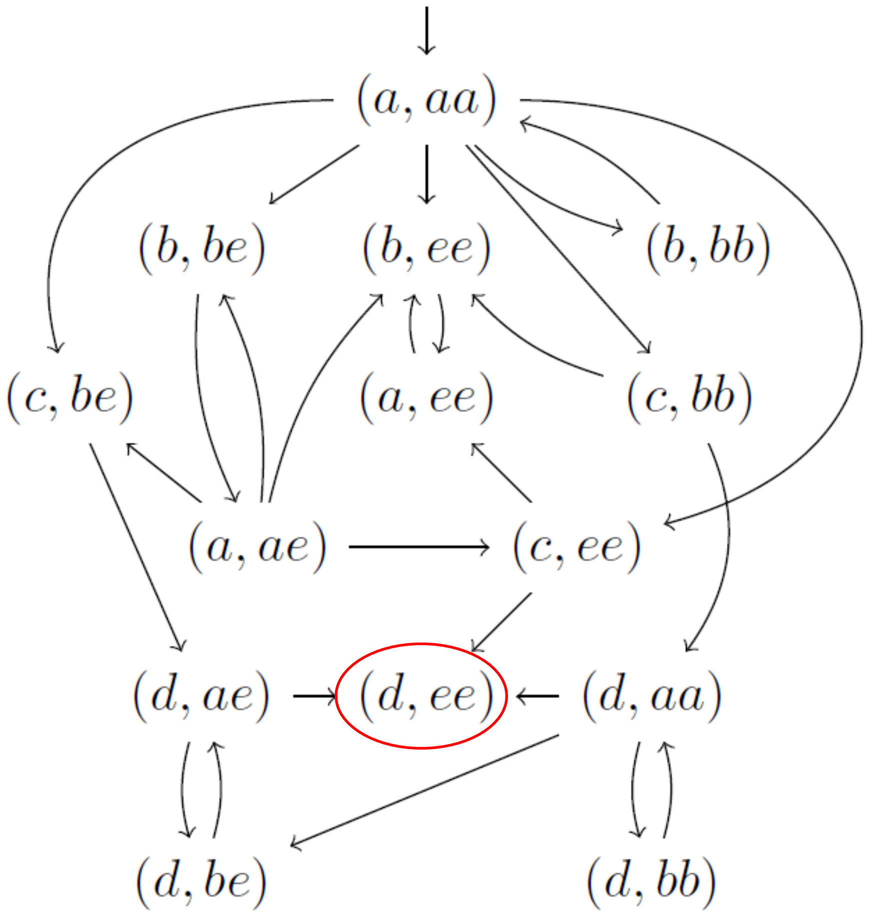
Region S is not FIN-representable

FIN-representability, an example





Region *S* is not FIN-representable

Lemma 2. *For any polymorphic P system* $\Pi \in NOP(polym, ncoo, fin)$*, we can construct a finite transition system* M_Π *which represents the rule configurations of* Π.

Proof

$$\Pi \in NOP(polym, ncoo, fin)$$

$\Pi = (O, T, \mu, w_s, \langle w_{1L}, w_{1R}\rangle, \ldots, \langle w_{nL}, w_{nR}\rangle, s)$, and let $1L, 1R, \ldots, kL, kR$ for some $k \leq n$ be the labels of those regions which are directly enclosed in the skin membrane.

$$\Downarrow$$

$$M_{1\ldots k}$$

Proof

(1) If a membrane labelled by $h \in \{1L, 1R \ldots, nL, nR\}$ is an elementary membrane, then let $M_h = (Q_h, \bar{q}_h, \delta_h)$, where

- $Q_h = \{\bar{q}\} = \{(w_h, \emptyset)\}$, and
- $\delta_h : Q_h \to 2^{Q_h}$ such that $\delta_h(\bar{q}_h) = \emptyset$.

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Proof

(2) If we have already constructed M_{iL}, M_{iR} for the pair of membranes labelled by iL, iR for some $1 \leq i \leq n$, we construct $M_i = (R_i, \bar{r}_i, \delta_i)$ to represent the dynamical rule corresponding to this pair of membranes with

- $R_i = \pi_1(Q_{iL}) \times \pi_1(Q_{iR})$ where π_1 denotes the first projection of the pairs in Q_{iL}, Q_{iR}, (that is, the first components of the states, the components which denote the contents of the corresponding region),
- $\bar{r}_i = (w_{iL}, w_{iR})$, or using the notation with the first projection as above, we might also write that $\bar{r}_i = (\pi_1(\bar{q}_{iL}), \pi_1(\bar{q}_{iR}))$, and
- $\delta_i : R_i \to 2^{R_i}$ such that $(u'_{iL}, u'_{iR}) \in \delta_i(u_{iL}, u_{iR})$, if and only if
  - $u'_{iL} \in \delta_{iL}(u_{iL})$ and $u'_{iR} \in \delta_{iR}(u_{iR})$, or
  - if $\delta_{iL}(u_{iL}) = \emptyset$ and $u'_{iR} \in \delta_{iR}(u_{iR})$ (or if $\delta_{iR}(u_{iR}) = \emptyset$ and $u'_{iL} \in \delta_{iL}(u_{iL})$) then $u'_{iL} = u_{iL}$ (or $u'_{iR} = u_{iR}$, repectively), and

  $\delta_i(u_{iL}, u_{iR}) = \emptyset$, if and only if $\delta_{iL}(u_{iL}) = \delta_{iR}(u_{iR}) = \emptyset$.

Proof

(3) If the membranes that are directly enclosed by the non-elementary membrane (their parent membrane) with label h are labelled by $i_1L, i_1R, \ldots, i_kL, i_kR$, and we have already constructed $M_{i_1}, \ldots, M_{i_k}$ for all the pairs i_jL, i_jR, $1 \leq j \leq k$, then we construct the representation M_h in two steps.

(3.1) We first construct $M_{i_1 \ldots i_k} = (R_{i_1 \ldots i_k}, \bar{r}_{i_1 \ldots i_k}, \delta_{i_1 \ldots i_k})$ with

- $R_{i_1 \ldots i_k} = R_{i_1} \times \ldots \times R_{i_k}$,
- $\bar{r}_{i_1 \ldots i_k} = (\bar{r}_{i_1}, \ldots, \bar{r}_{i_k})$, and
- $\delta_{i_1 \ldots i_k} : R_{i_1 \ldots i_k} \to 2^{R_{i_1 \ldots i_k}}$ such that $(r'_{i_1}, \ldots, r'_{i_k}) \in \delta_{i_1 \ldots i_k}(r_{i_1}, \ldots, r_{i_k})$, if and only if
  - $r'_{i_j} \in \delta_{i_j}(r_{i_j})$ for all i_j, $1 \leq j \leq k$, or
  - if $\delta_{i_j}(r_{i_j}) = \emptyset$, but there is at least one i_l, such that $\delta_{i_l}(r_{i_l}) \neq \emptyset$, $1 \leq j, l \leq k$, then $r'_{i_j} = r_{i_j}$, and

$\delta_{i_1 \ldots i_k}(r_{i_1}, \ldots, r_{i_k}) = \emptyset$, if and only if, $\delta_{i_j}(r_{i_j}) = \emptyset$ for all i_j, $1 \leq j \leq k$.

Proof

(3.2) Given $M_{i_1 \ldots i_k}$ and the initial multiset w_h, we can construct $M_h = (Q_h, \bar{q}_h, \delta_h)$ as

- $Q_h = \sigma^*_{0,h}(w_h) \times R_{i_1 \ldots i_k}$, the direct product of the possible contents of region h and the possible k-tuples of rules by the pairs of regions $i_j L, i_j R$, $1 \leq j \leq k$,
- $\bar{q}_h = (w_h, \bar{r}_{i_1 \ldots i_k})$, the pair of the initial contents and the k-tuple of rules represented by the initial configuration, and
- $\delta_h : Q_h \to 2^{Q_h}$ such that $(u'_h, r'_{i_1 \ldots i_k}) \in \delta_h(u_h, r_{i_1 \ldots i_k})$ if and only if
  - the multiset u'_h can be obtained from u_h by the maximal parallel application of the set of rules of $r_{i_1 \ldots i_k}$, denoted as $u_h \Rightarrow_{\{r_{i_1}, \ldots, r_{i_k}\}} u'_h$ where $r_{i_1 \ldots i_k} = (r_{i_1}, \ldots, r_{i_k})$, and
  - $r'_{i_1 \ldots i_k} \in \delta_{i_1 \ldots i_k}(r_{i_1 \ldots i_k})$, or if $\delta_{i_1 \ldots i_k}(r_{i_1 \ldots i_k}) = \emptyset$, then $r'_{i_1 \ldots i_k} = r_{i_1 \ldots i_k}$, or
  - if $r'_{i_1 \ldots i_k} \in \delta_{i_1 \ldots i_k}(r_{i_1 \ldots i_k})$, but the rules of $r_{i_1 \ldots i_k} = (r_{i_1}, \ldots, r_{i_k})$ are not applicable to u_h, then $u'_h = u_h$.

Proof

If none of the cases above holds, that is, none of the rules of $r_{i_1 \ldots i_k} = (r_{i_1}, \ldots, r_{i_k})$ are applicable to u_h, and $\delta_{i_1 \ldots i_k}(r_{i_1 \ldots i_k}) = \emptyset$, then $\delta_h(u_h, r_{i_1 \ldots i_k}) = \emptyset$, thus, the state $(u_h, r_{i_1 \ldots i_k})$ is a halting state in M_h.

Proof

If $\Pi = (O, T, \mu, w_s, \langle w_{1L}, w_{1R} \rangle, \ldots, \langle w_{nL}, w_{nR} \rangle, s)$ with $1L, 1R, \ldots, kL, kR$, $k \leq n$ being the labels of those regions which are directly enclosed in the skin membrane, and we let $M_\Pi = M_{1\ldots k}$, then the states of M_Π represent the dynamically changing collections of rules applicable in the skin region, which can change as allowed by the possible transitions of M_Π, in short, M_Π represents the rule configurations of Π.

Theorem 3. $\mathcal{L}(NOP(polym, ncoo, fin)) \subseteq PsET0L$.

Proof. Let $\Pi = (O, T, \mu, w_s, \langle w_{1L}, w_{1R}\rangle, \ldots, \langle w_{nL}, w_{nR}\rangle, s)$ be a polymorphic P system, $\Pi \in NOP(polym, ncoo, fin)$, and let us assume (without loss of generality) that the membranes that are directly contained in the skin region are labelled by the labels $1L, 1R, \ldots, kL, kR$, $k \leq n$. Since both the left- and right-hand membranes iL, iR, $1 \leq i \leq k$, are FIN-representable, we can construct the transition system $M_\Pi = (R_\Pi, \bar{r}_\Pi, \delta_\Pi) = M_{1\ldots k} = (R_{1\ldots k}, \bar{r}_{1\ldots k}, \delta_{1\ldots k})$ as described in the proof of Lemma 2.

Now, based on M_Π, we construct an ET0L system $G = (V, T, U, w)$, where V contains the alphabet, T the terminal alphabet with $T \subseteq V$, w is the initial string, and U is a set of tables, $U = (P_1, P_2, \ldots, P_m)$ containing at most three tables for each state of M_Π and one additional table.

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A characterization of *PsET0L*

Theorem: $\mathcal{L}(NOP(polym\ , ncoo, fin)) \subseteq PsET0L.$

Corollary: $\mathcal{L}(NOP(polym\ , ncoo, fin)) = PsET0L.$

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Why does the corollary hold?

Thank you for your attention!

Example 4. Let us construct the representation $M_\Pi = M_1$ for the P system of Example 3. Starting with the elementary membranes, we get $M_{2L} = (\{(a, \emptyset)\}, (a, \emptyset), \delta_{2L})$, $M_{2R} = (\{(c, \emptyset)\}, (c, \emptyset), \delta_{2R})$ such that $\delta_{2L}(a, \emptyset) = \delta_{2R}(c, \emptyset) = \emptyset$, and $M_2 = (R_2, \bar{r}_2, \delta_2)$ with $R_2 = \{a\} \times \{c\} = \{(a, c)\}$, $\bar{r}_2 = (a, c)$, and $\delta_2(a, c) = \emptyset$. Similarly, we can construct M_i for all i, $2 \le i \le 8$, which all have a similar structure.

Now, given the transition systems $M_2, \ldots, M_5$ we construct M_{1L} as follows. We start with the construction of $M_{2\ldots5} = (R_{2\ldots5}, \bar{r}_{2\ldots5}, \delta_{2\ldots5})$ as

- $R_{2\ldots5} = \{((a, c), (a, b), (b, a), (c, d))\}$,
- $\bar{r}_{2\ldots5} = ((a, c), (a, b), (b, a), (c, d))$, and
- $\delta_{2\ldots5}((a, c), (a, b), (b, a), (c, d)) = \emptyset$.

If denote the rules by $r_2 = (a, c)$, $r_3 = (a, b)$, $r_4 = (b, a)$, and $r_5 = (c, d)$, then $M_{1L} = (Q_{1L}, \bar{q}_{1L}, \delta_{1L})$ is as follows.

The set of possible states is $Q_{1L} = \{a, b, c, d, e\} \times \{(r_2, r_3, r_4, r_5)\}$, that is,

$$\begin{aligned} Q_{1L} = \{&(a,\ (r_2, r_3, r_4, r_5)),\ (b,\ (r_2, r_3, r_4, r_5)), (c,\ (r_2, r_3, r_4, r_5)),\\ &(d,\ (r_2, r_3, r_4, r_5),\ (e,\ (r_2, r_3, r_4, r_5)\},\end{aligned}$$

the initial state is $\bar{q}_{1L} = (a,\ (r_2, r_3, r_4, r_5))$, and the transition mapping is defined as

$$\begin{aligned} \delta_{1L}(a,\ (r_2, r_3, r_4, r_5)) &= \{(c,\ (r_2, r_3, r_4, r_5)), (b,\ (r_2, r_3, r_4, r_5)\},\\ \delta_{1L}(c,\ (r_2, r_3, r_4, r_5)) &= \{(d,\ (r_2, r_3, r_4, r_5))\},\\ \delta_{1L}(b,\ (r_2, r_3, r_4, r_5)) &= \{(a,\ (r_2, r_3, r_4, r_5))\},\\ \delta_{1L}(d,\ (r_2, r_3, r_4, r_5)) &= \emptyset.\end{aligned}$$

With a similar construction, we can construct $M_{1R} = (Q_{1R}, \bar{q}_{1R}, \delta_{1R})$ as

$$Q_{1R} = \{(aa,\ (r_6, r_7, r_8)),\ (bb,\ (r_6, r_7, r_8,)),\ (be,\ (r_6, r_7, r_8)),\\ (ae,\ (r_6, r_7, r_8)),\ (ee,\ (r_6, r_7, r_8))\},$$

where $r_6 = (a, b)$, $r_7 = (b, a)$, and $r_8 = (a, e)$. Further, $\bar{q}_{1R} = (aa,\ (r_6, r_7, r_8, r_9))$, and

$$\begin{aligned}
\delta_{1R}(aa,\ (r_6, r_7, r_8)) &= \{(bb,\ (r_6, r_7, r_8)), (be,\ (r_6, r_7, r_8)), (ee,\ (r_6, r_7, r_8))\},\\
\delta_{1R}(bb,\ (r_6, r_7, r_8)) &= \{(aa,\ (r_6, r_7, r_8))\},\\
\delta_{1R}(be,\ (r_6, r_7, r_8)) &= \{(ae,\ (r_6, r_7, r_8))\},\\
\delta_{1R}(ae,\ (r_6, r_7, r_8)) &= \{(be,\ (r_6, r_7, r_8)), (ee,\ (r_6, r_7, r_8))\},\\
\delta_{1R}(ee,\ (r_6, r_7, r_8)) &= \emptyset.
\end{aligned}$$

Now, given M_{1L} and M_{1R} we can construct $M_1 = M_\Pi$ as $M_1 = (R_1, \bar{r}_1, \delta_1)$ where

$$R_1 = \pi_1(Q_{1L}) \times \pi_1(Q_{1R}) = \{a, b, c, d\} \times \{aa, bb, be, ae, ee\},$$

is the set of states, $\bar{r}_1 = (a, aa)$ is the initial state, and

$$\begin{aligned}
\delta_1(a, aa) &= \{(b, bb), (b, be), (b, ee), (c, bb), (c, be), (c, ee)\},\\
\delta_1(b, bb) &= \{(a, aa)\},\\
\delta_1(b, be) &= \{(a, ae)\},\\
\delta_1(b, ee) &= \{(a, ee)\},\\
\delta_1(c, bb) &= \{(d, aa)\},\\
\delta_1(c, be) &= \{(d, ae)\},\\
\delta_1(c, ee) &= \{(d, ee)\},\\
\delta_1(a, ae) &= \{(c, be), (c, ee), (b, be), (b, ee)\}.\\
\delta_1(a, ee) &= \{(c, ee), (b, ee)\},\\
\delta_1(d, aa) &= \{(d, ee), (d, be), (d, bb)\},\\
\delta_1(d, ae) &= \{(d, ee), (d, be)\},\\
\delta_1(d, bb) &= \{(d, aa)\},\\
\delta_1(d, be) &= \{(d, ae)\},\\
\delta_1(d, ee) &= \emptyset.
\end{aligned}$$