Generalised dependency graphs solving a special case of the P conjecture

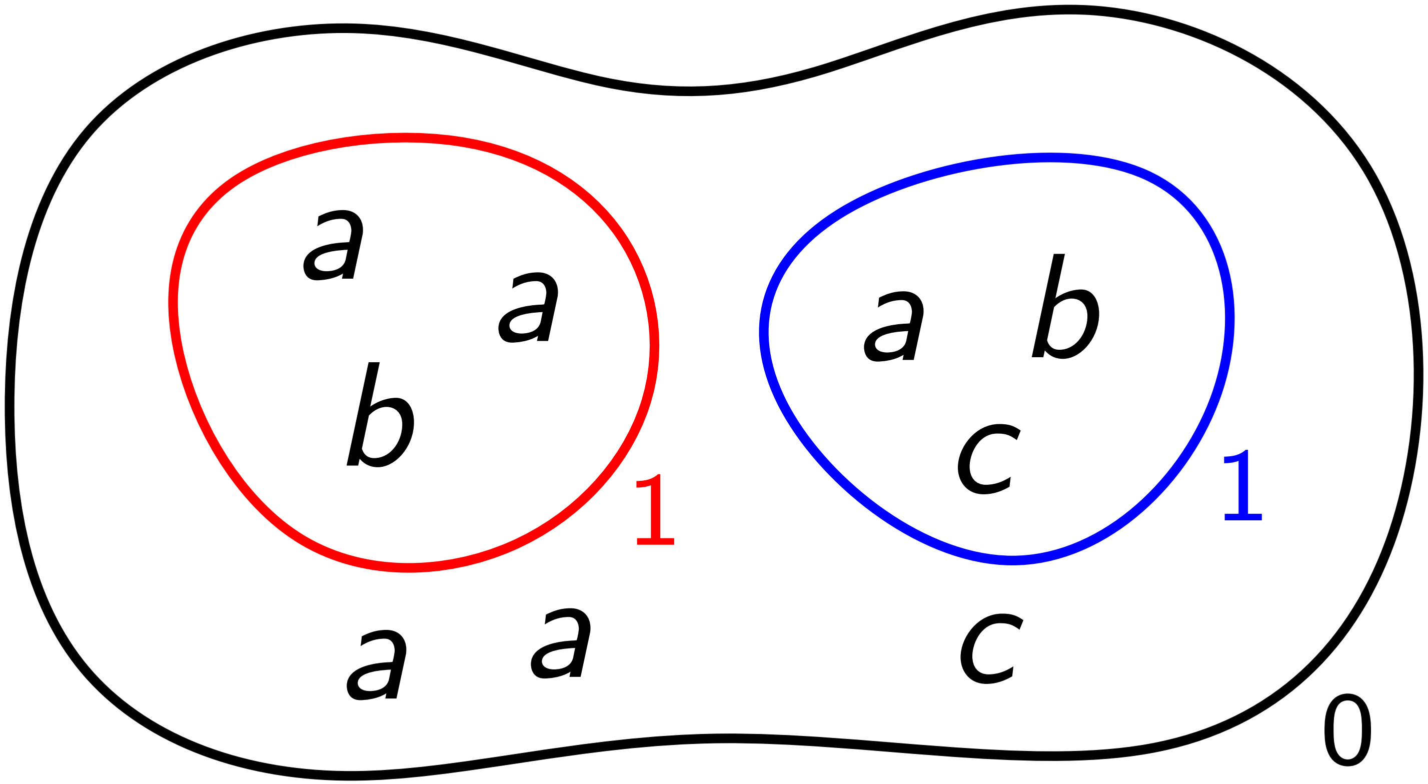
based on work by

Alberto Leporati · Luca Manzoni · Giancarlo Mauri
Antonio E. Porreca · Claudio Zandron

Università degli Studi di Milano-Bicocca

16th Brainstorming Week on Membrane Computing
31 January 2018, Sevilla, Spain

Rules for P systems with active membranes without charges



$[a \to bc]_1$ $\qquad$ $[f]_1 \to [\,]_1\, f$ $\qquad$ $d\,[\,]_1 \to [d]_1$

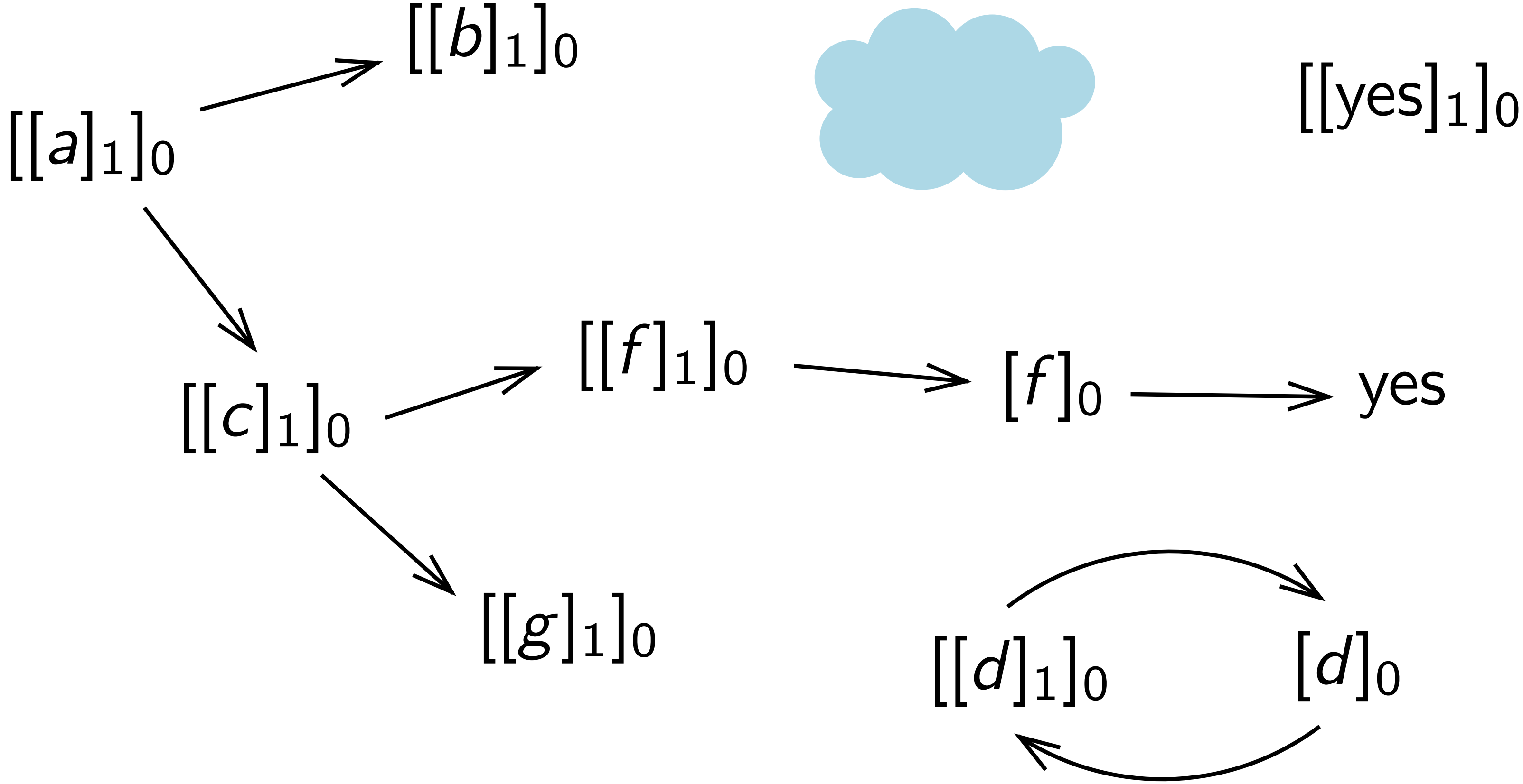
$[c]_1 \to [f]_1\, [g]_1$ $\qquad$ $[f]_0 \to [\,]_0\, \text{yes}$ $\qquad$ $[d]_1 \to [\,]_1\, d$

The P conjecture

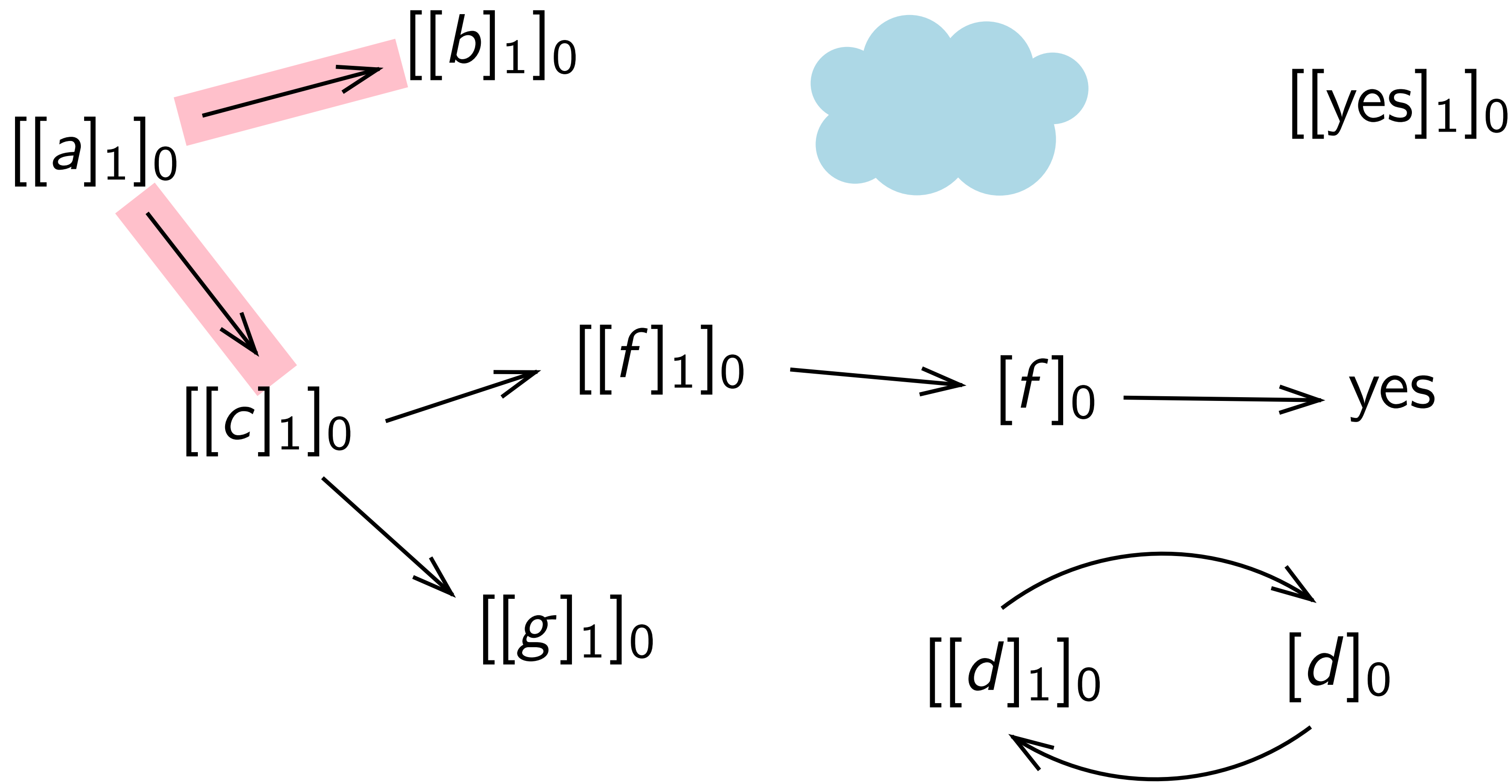
[I]t was shown that [. . .] for solving **NP**-complete problems [in polynomial time] two charges are enough.

Can the polarizations be completely avoided? [. . .] The feeling is that this is not possible

Standard dependency graphs



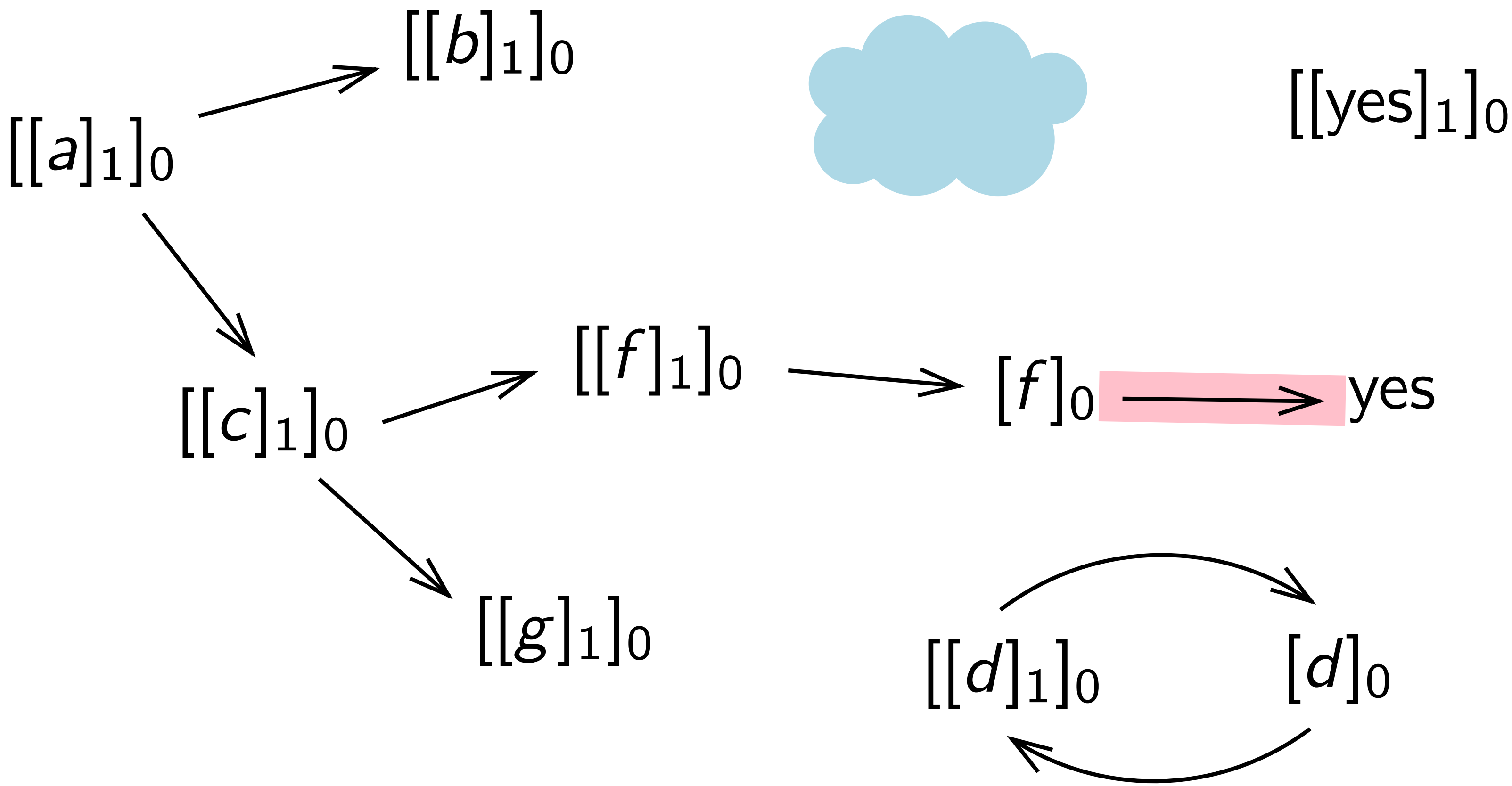
Standard dependency graphs



$[a \to bc]_1$ $\qquad$ $[f]_1 \to [\,]_1\, f$ $\qquad$ $d\,[\,]_1 \to [d]_1$

$[c]_1 \to [f]_1\, [g]_1$ $\qquad$ $[f]_0 \to [\,]_0\, \mathsf{yes}$ $\qquad$ $[d]_1 \to [\,]_1\, d$

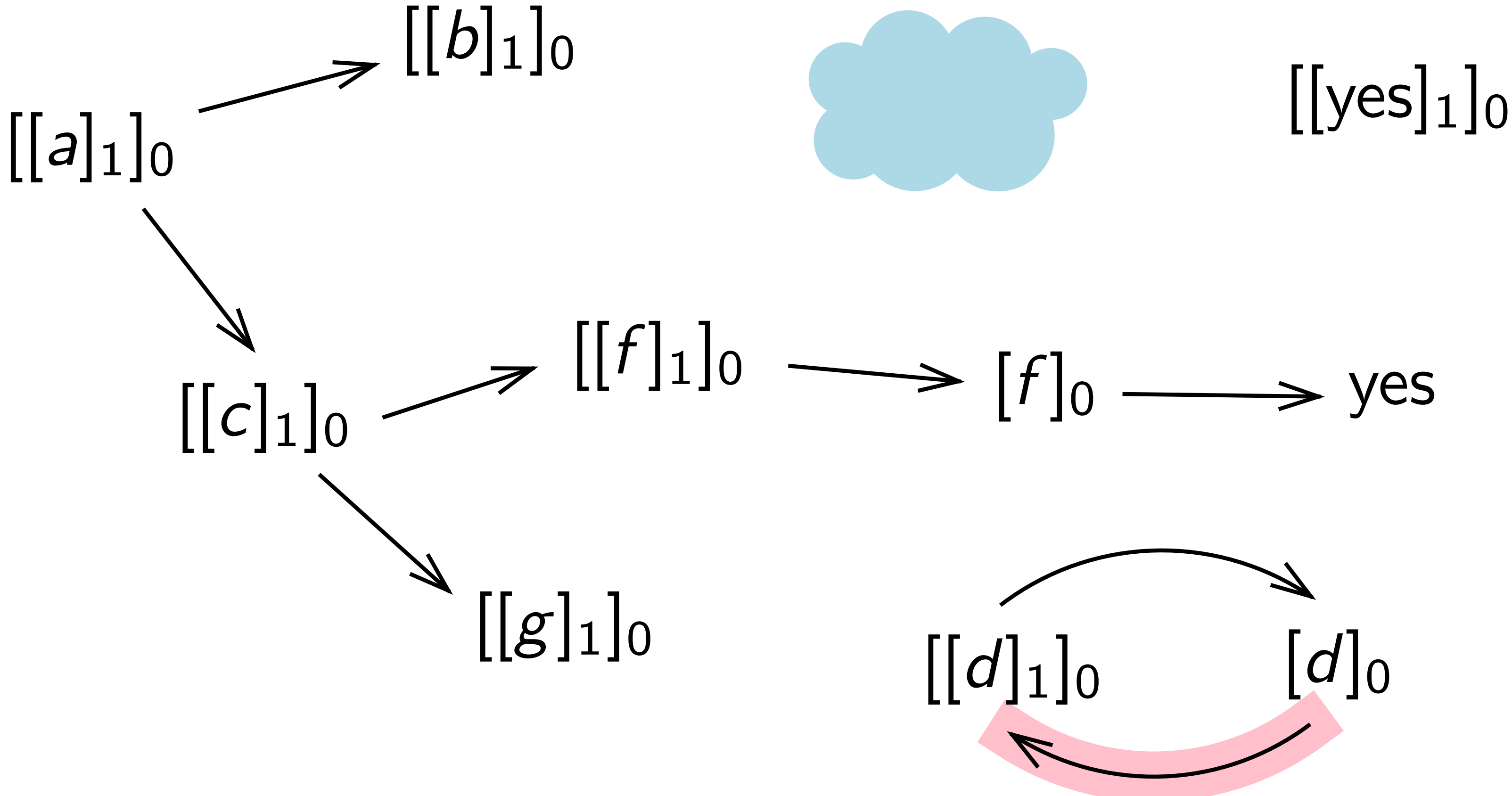
Standard dependency graphs



$[a \to bc]_1$ $\quad$ $[f]_1 \to [\,]_1\, f$ $\quad$ $d\,[\,]_1 \to [d]_1$

$[c]_1 \to [f]_1\,[g]_1$ $\quad$ $[f]_0 \to [\,]_0\ \text{yes}$ $\quad$ $[d]_1 \to [\,]_1\, d$

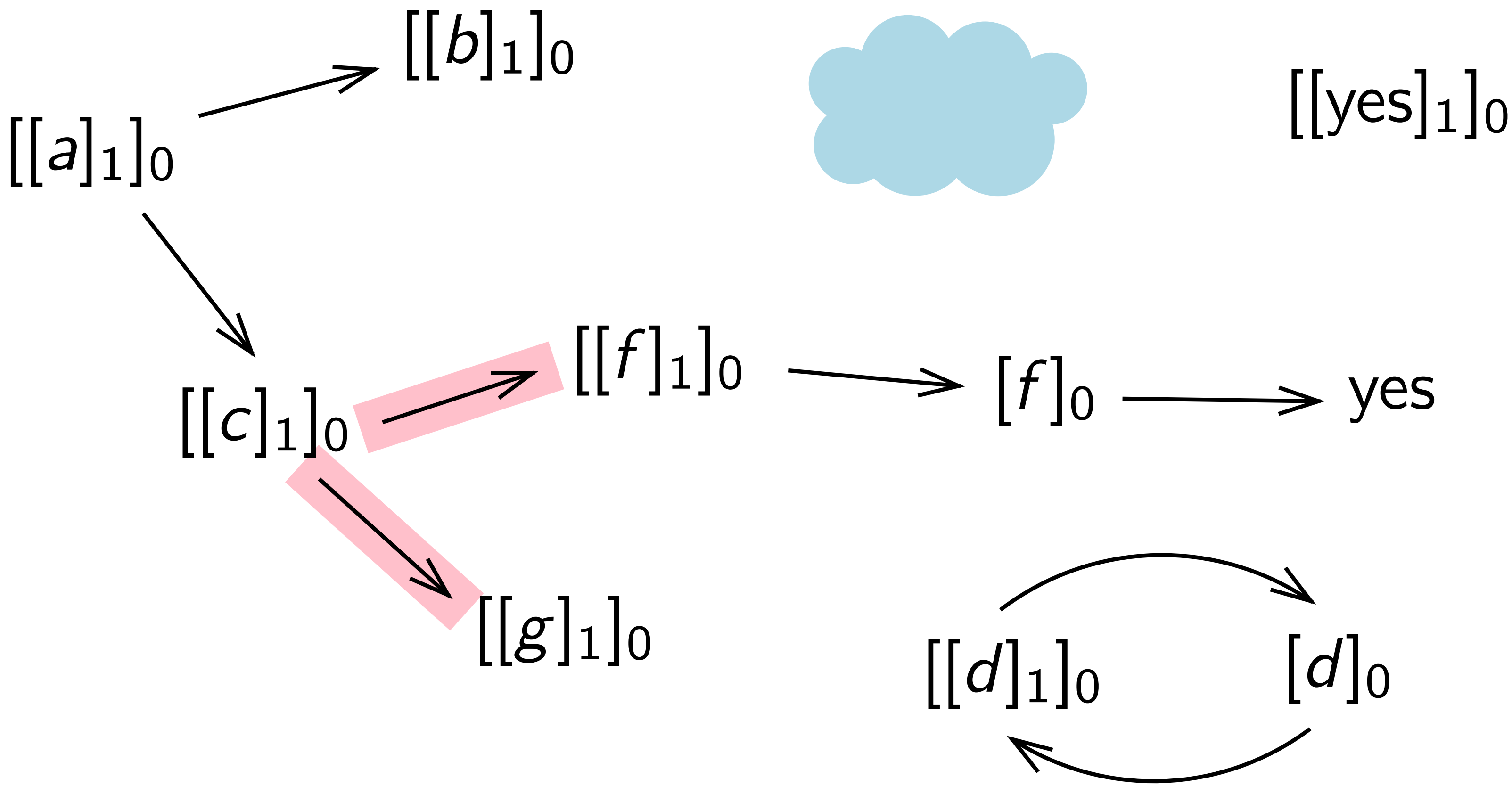
Standard dependency graphs



$[a \to bc]_1$ $\quad$ $[f]_1 \to [\,]_1\, f$ $\quad$ $\boxed{d\,[\,]_1 \to [d]_1}$

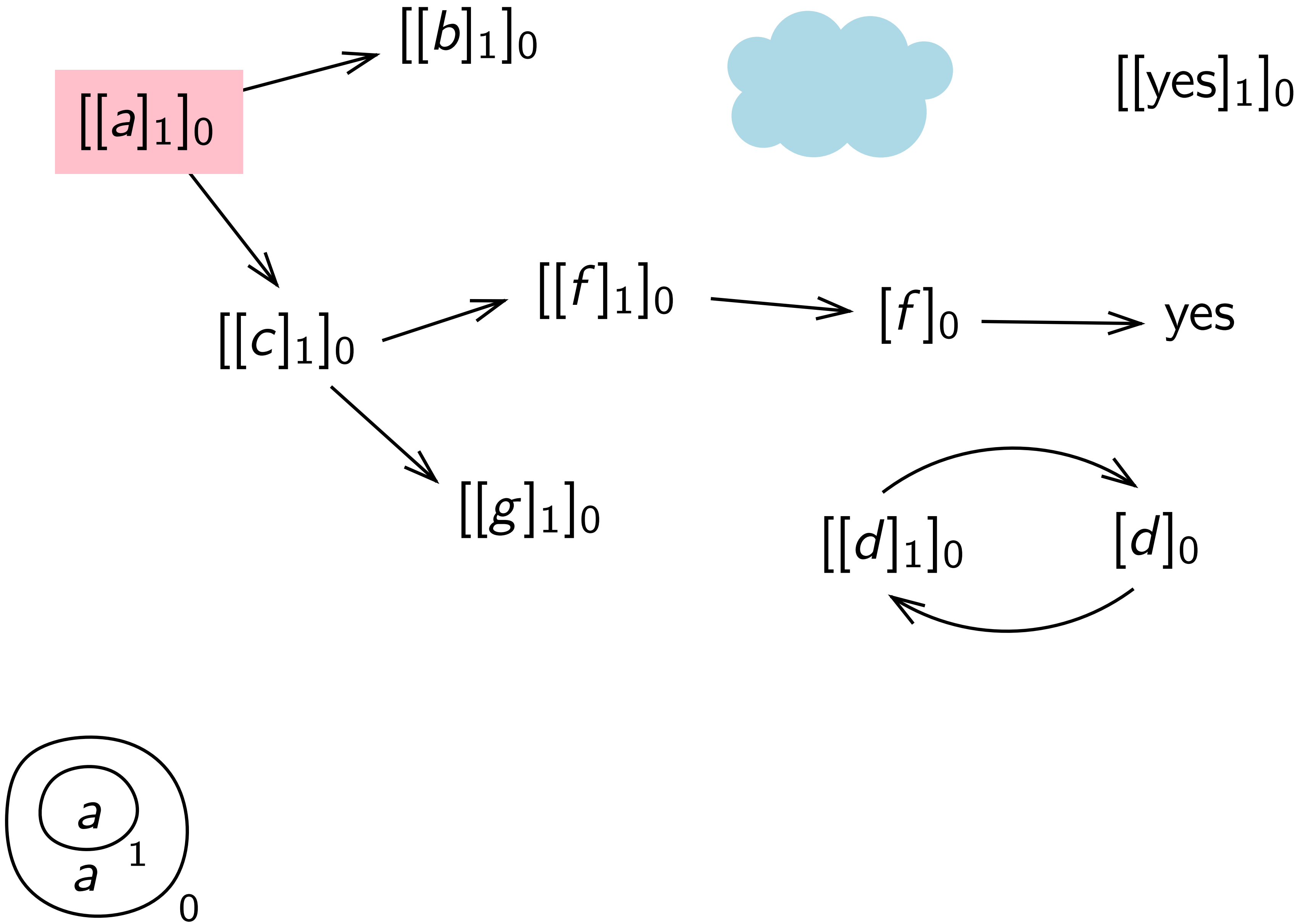
$[c]_1 \to [f]_1\,[g]_1$ $\quad$ $[f]_0 \to [\,]_0$ yes $\quad$ $[d]_1 \to [\,]_1\, d$

Standard dependency graphs

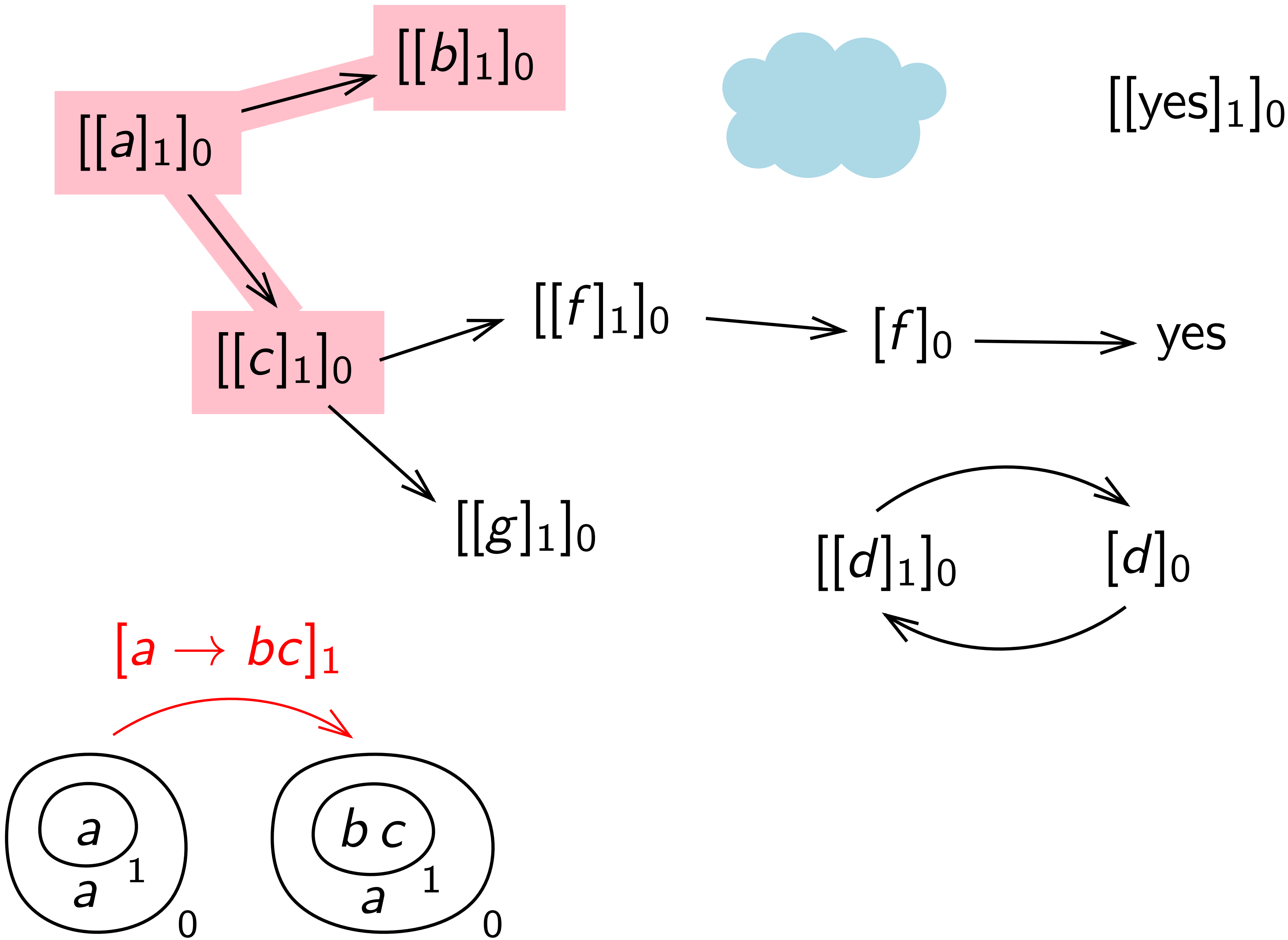


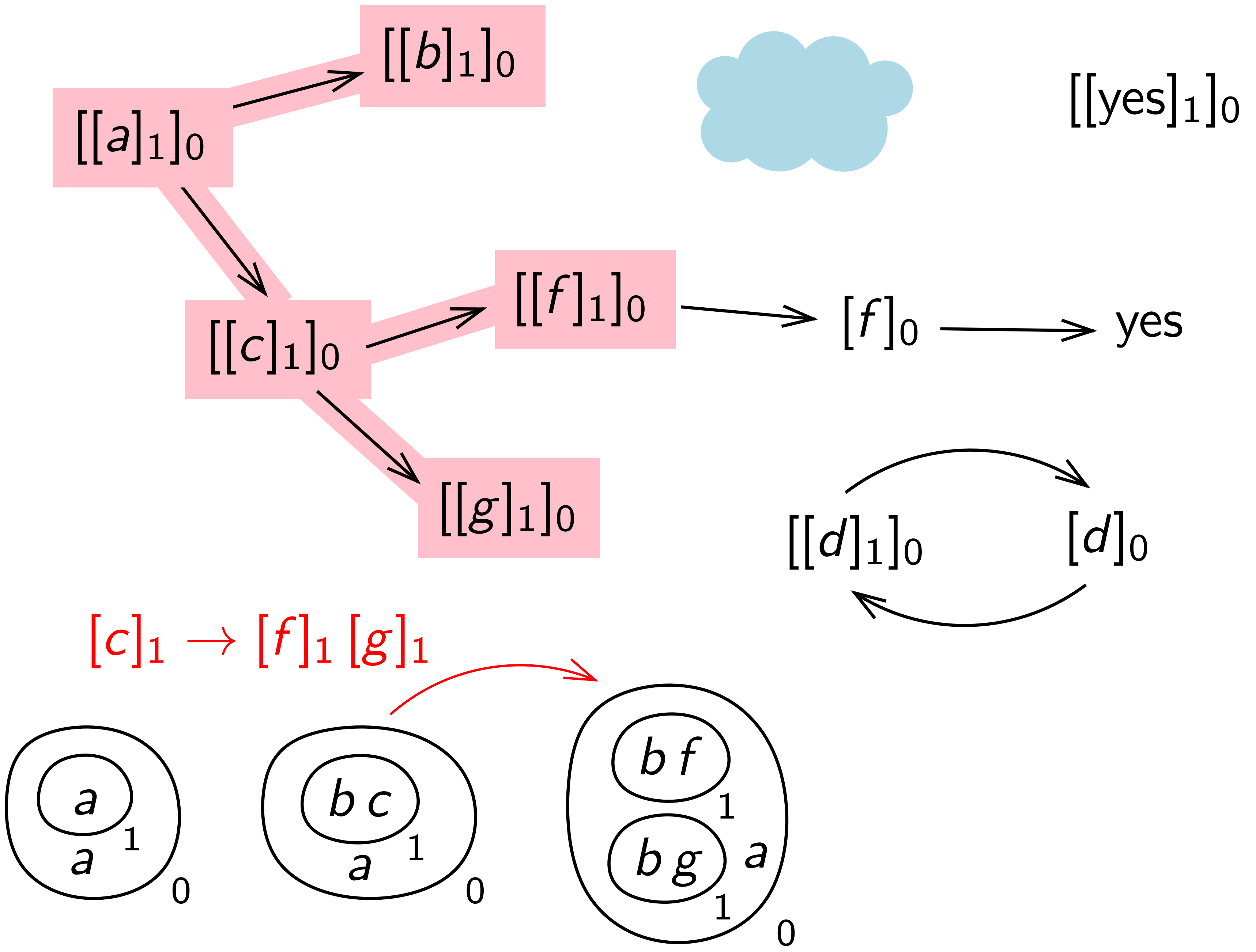
$[a \rightarrow bc]_1$ $\quad$ $[f]_1 \rightarrow [\,]_1\, f$ $\quad$ $d\,[\,]_1 \rightarrow [d]_1$

$[c]_1 \rightarrow [f]_1\,[g]_1$ $\quad$ $[f]_0 \rightarrow [\,]_0\, \mathrm{yes}$ $\quad$ $[d]_1 \rightarrow [\,]_1\, d$

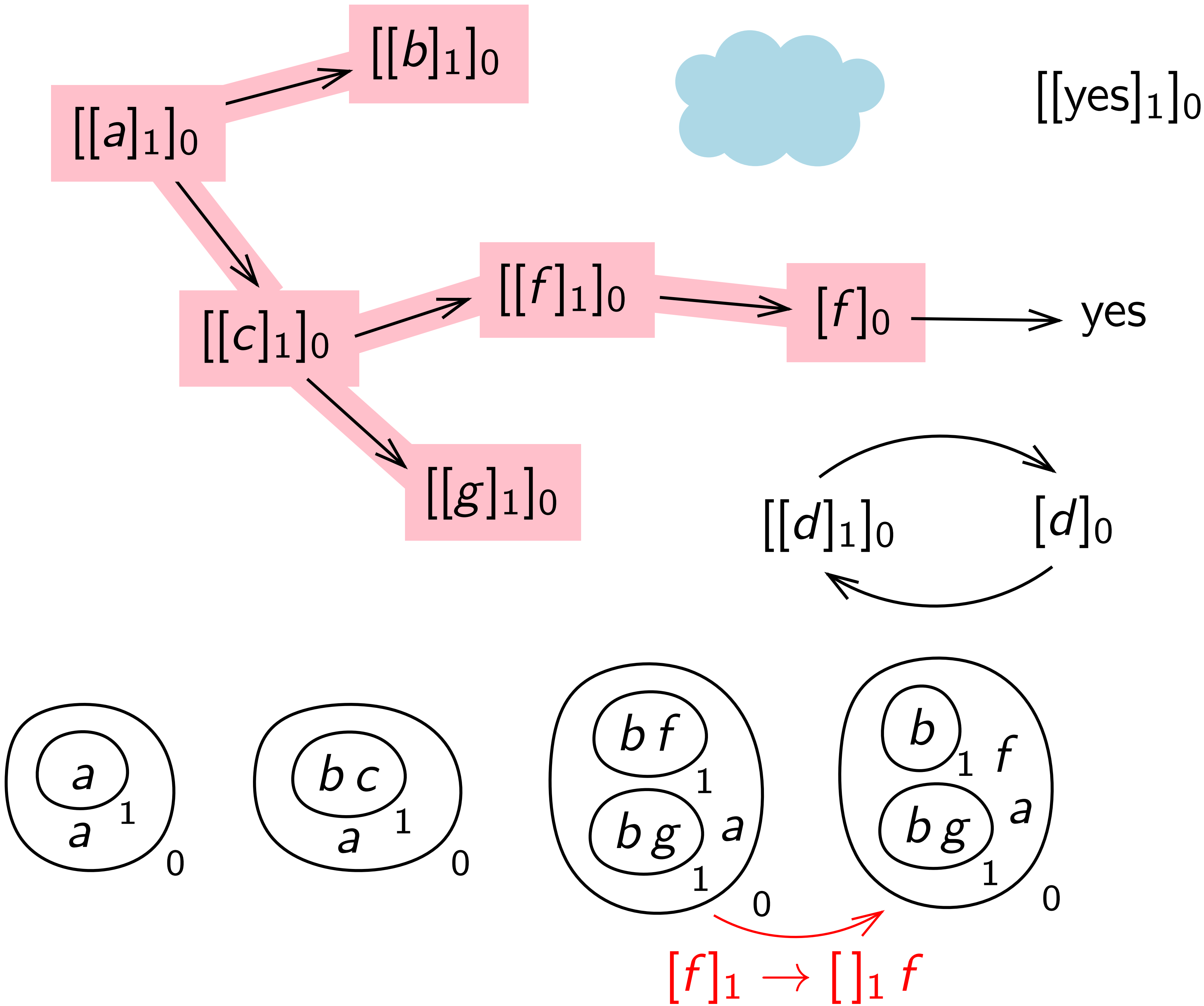


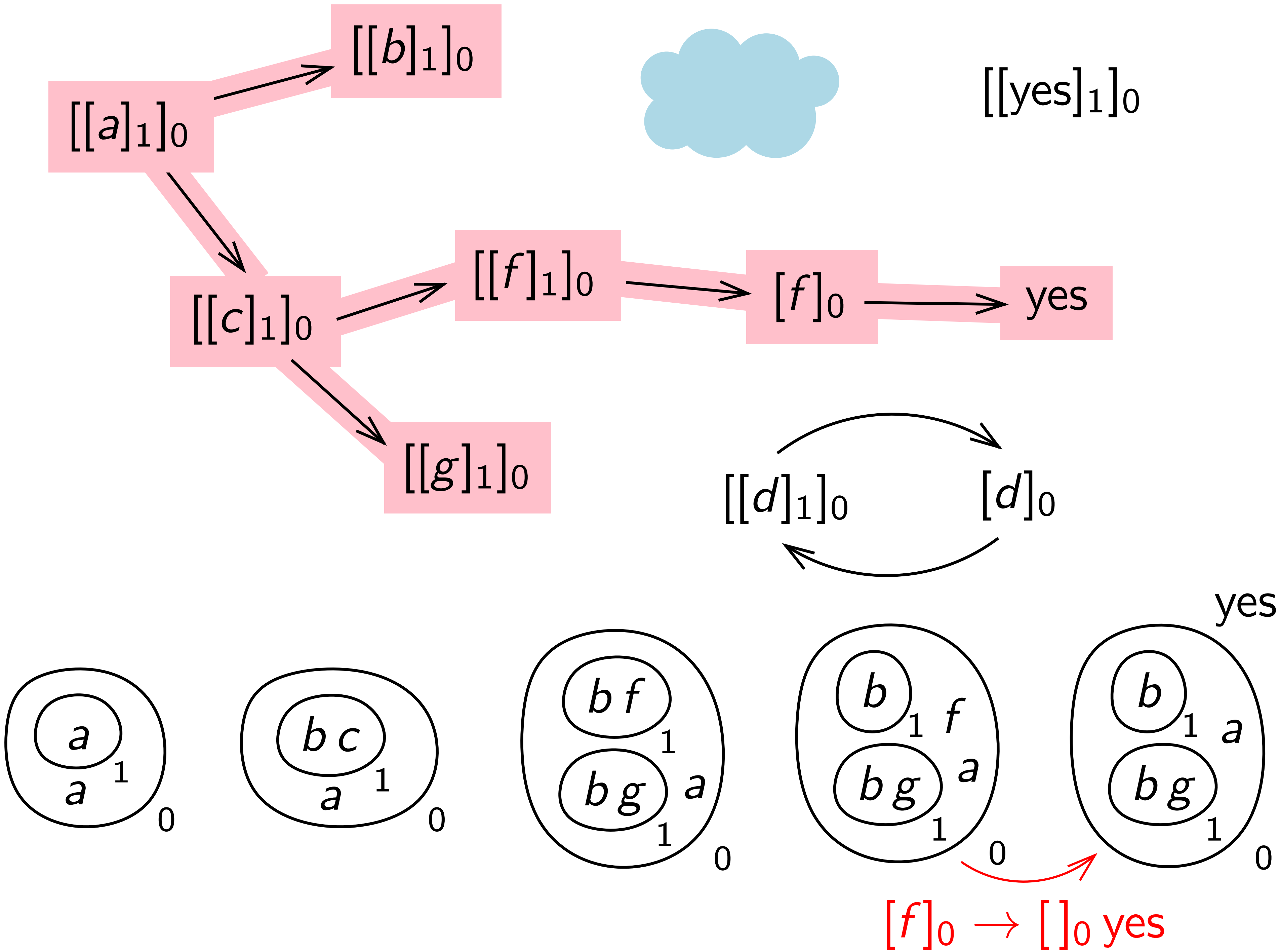
$[[a]_1]_0$
$[[b]_1]_0$
$[[\text{yes}]_1]_0$
$[[c]_1]_0$
$[[f]_1]_0$
$[f]_0$
yes
$[[g]_1]_0$
$[[d]_1]_0$
$[d]_0$
a
a
1
0

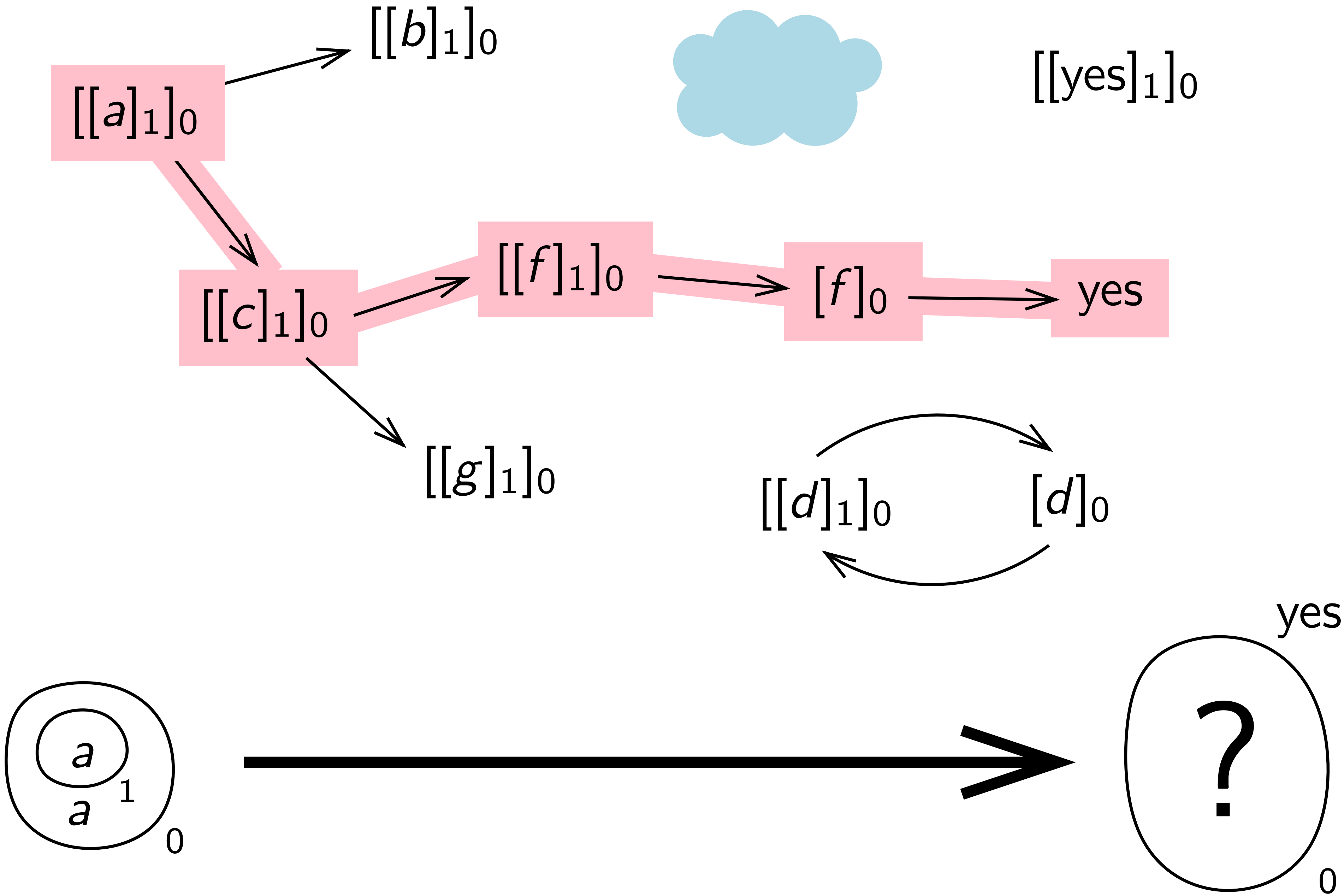

$[[a]_1]_0$
$[[b]_1]_0$
$[[c]_1]_0$
$[[f]_1]_0$
$[f]_0$
yes
$[[yes]_1]_0$
$[[g]_1]_0$
$[[d]_1]_0$
$[d]_0$
$[a \to bc]_1$
a
a
1
0
b c
a
1
0



[[a]1]0
[[b]1]0
[[yes]1]0
[[c]1]0
[[f]1]0
[f]0
yes
[[g]1]0
[[d]1]0
[d]0
[c]1 → [f]1 [g]1
a
a
1
0
b c
a
1
0
b f
1
b g
a
1
0


[[b]1]0
[[a]1]0
[[yes]1]0
[[f]1]0
[f]0
yes
[[c]1]0
[[g]1]0
[[d]1]0
[d]0
a
a 1
0
b c
a 1
0
b f
1
b g
a
1
0
b
1 f
b g
a
1
0
[f]1 → [ ]1 f


[[b]1]0
[[a]1]0
[[yes]1]0
[[f]1]0
[f]0
yes
[[c]1]0
[[g]1]0
[[d]1]0
[d]0
yes
a
1
a
0
b c
1
a
0
b f
1
b g
1
a
0
b
1
f
b g
a
1
0
b
1
a
b g
1
0
[f]0 → [ ]0 yes



$[[b]_1]_0$
$[[a]_1]_0$
$[[yes]_1]_0$
$[[c]_1]_0$
$[[f]_1]_0$
$[f]_0$
yes
$[[g]_1]_0$
$[[d]_1]_0$
$[d]_0$
a
a
1
0
yes
?
0

Dependency graphs can be constructed and explored in polynomial time

Dependency graphs can be constructed and explored in polynomial time

The P conjecture is true for P systems without dissolution rules

Dependency graphs can be constructed and explored in polynomial time

The P conjecture is true for P systems without dissolution rules

And false for those with both non-elementary division and dissolution (they reach **PSPACE**)

In P systems without dissolution the result actually depends on one object

In P systems without dissolution the result actually depends on one object

Consider a restricted version of P systems with dissolution:

- monodirectional
- shallow
- deterministic

The result of the computation depends on at most two objects

$$\mathcal{C}_0 \longrightarrow \mathcal{C}_1 \longrightarrow \mathcal{C}_2 \dashrightarrow \mathcal{C}_{t-1} \longrightarrow \mathcal{C}_t$$

The result of the computation depends on at most two objects

for each computation

$$\mathcal{C}_0 \longrightarrow \mathcal{C}_1 \longrightarrow \mathcal{C}_2 \dashrightarrow \mathcal{C}_{t-1} \longrightarrow \mathcal{C}_t$$

The result of the computation depends on at most two objects

$$
\begin{tikzcd}
\mathcal{C}_0 \arrow[r] \arrow[d] & \mathcal{C}_1 \arrow[r] \arrow[d, phantom, sloped, "\sqsupseteq"] & \mathcal{C}_2 \arrow[r, dashed] \arrow[d, phantom, sloped, "\sqsupseteq"] & \mathcal{C}_{t-1} \arrow[r] \arrow[d, phantom, sloped, "\sqsupseteq"] & \mathcal{C}_t \arrow[d, phantom, sloped, "\sqsupseteq"] \\
\mathcal{D}_0 & \mathcal{D}_1 & \mathcal{D}_2 & \mathcal{D}_{t-1} & \mathcal{D}_t
\end{tikzcd}
$$

there exists a sequence
of small configurations

$[[a\ b]_k]_h$ $[[a]_k]_h$

$[a]_h$ yes no

The result of the computation depends on at most two objects

$$
\begin{tikzcd}
\mathcal{C}_0 \arrow[r] \arrow[d, red] & \mathcal{C}_1 \arrow[r] \arrow[d, phantom, sloped, "\sqsupseteq"] & \mathcal{C}_2 \arrow[rr, dashed] \arrow[d, phantom, sloped, "\sqsupseteq"] & & \mathcal{C}_{t-1} \arrow[r] \arrow[d, phantom, sloped, "\sqsupseteq"] & \mathcal{C}_t \arrow[d, phantom, sloped, "\sqsupseteq"] \\
\mathcal{D}_0 \arrow[dr] & \mathcal{D}_1 \arrow[dr] & \mathcal{D}_2 \arrow[drr, dashed] & & \mathcal{D}_{t-1} \arrow[dr] & \mathcal{D}_t \\
& \mathcal{E}_1 \arrow[u, red] & \mathcal{E}_2 \arrow[u, red] & & \mathcal{E}_{t-1} \arrow[u, red] & \mathcal{E}_t \arrow[u, red]
\end{tikzcd}
$$

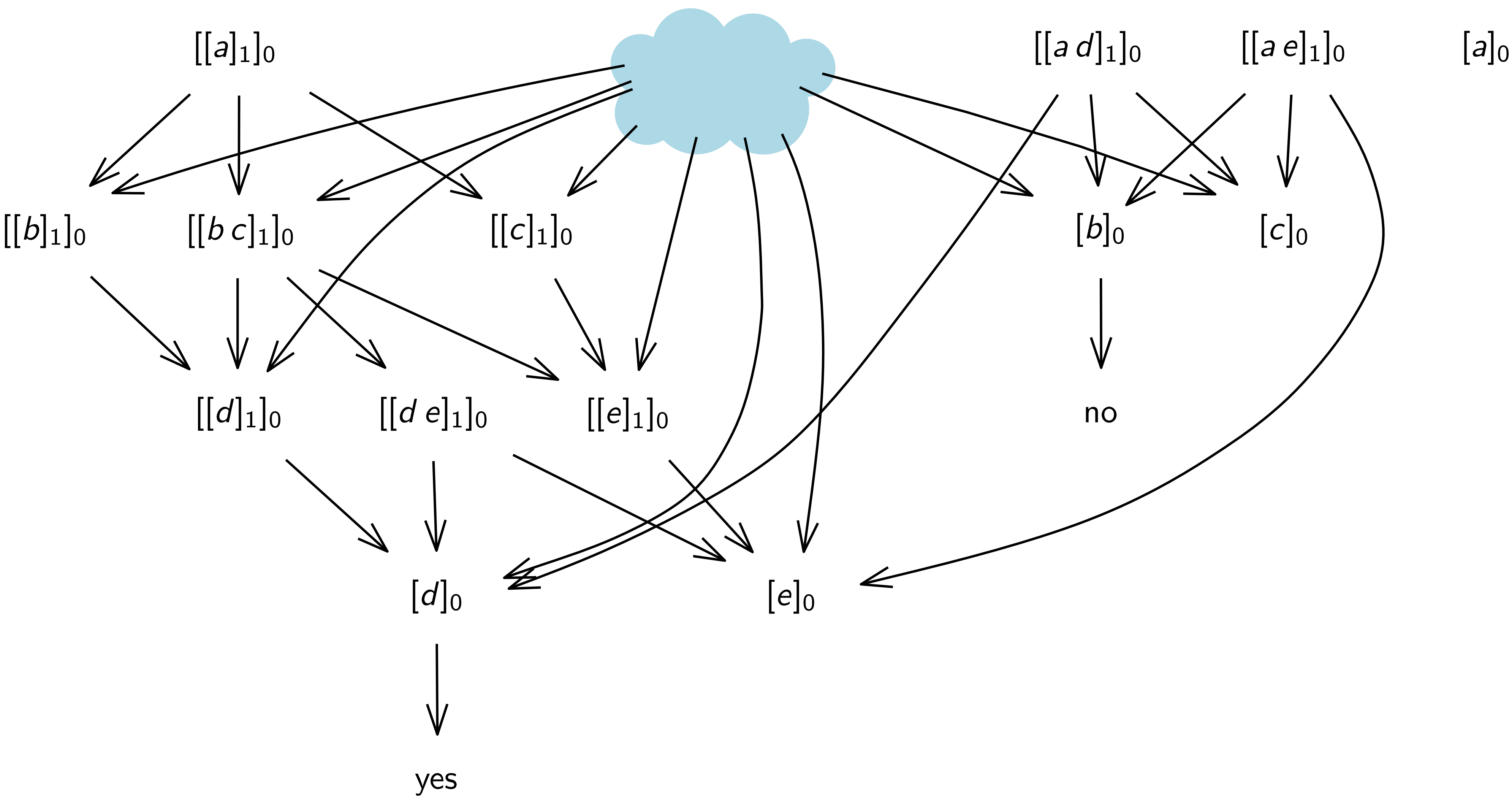
and there exists another
sequence of configurations

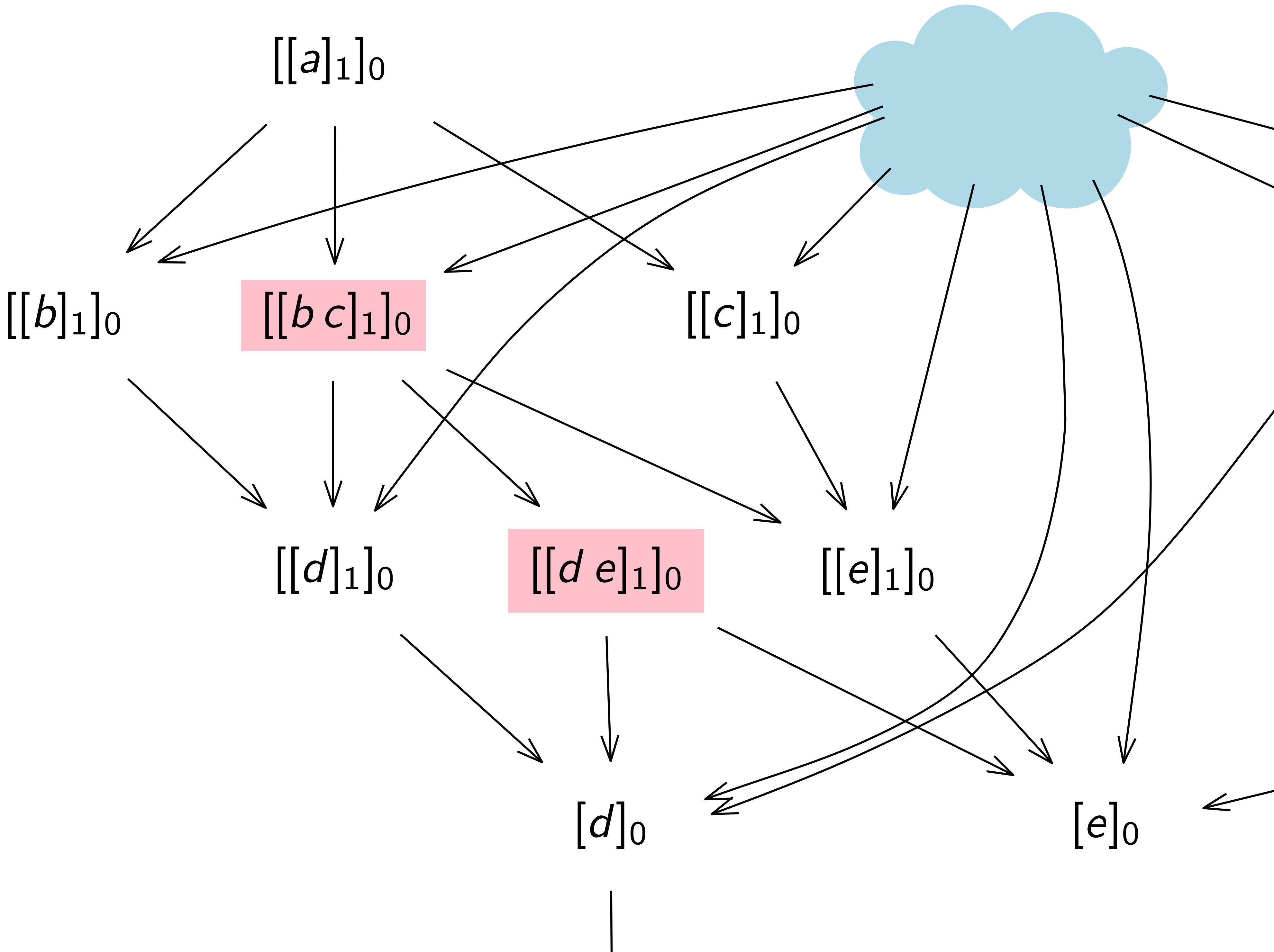
The result of the computation depends on at most two objects

$$
\begin{tikzcd}
\mathcal{C}_0 \arrow[r] \arrow[d, red] & \mathcal{C}_1 \arrow[r] \arrow[d, phantom, "\rotatebox{90}{$\sqsubseteq$}"] & \mathcal{C}_2 \arrow[r, dashed] \arrow[d, phantom, "\rotatebox{90}{$\sqsubseteq$}"] & \mathcal{C}_{t-1} \arrow[r] \arrow[d, phantom, "\rotatebox{90}{$\sqsubseteq$}"] & \mathcal{C}_t \arrow[d, phantom, "\rotatebox{90}{$\sqsubseteq$}"] \\
\mathcal{D}_0 \arrow[dr] & \mathcal{D}_1 \arrow[dr] & \mathcal{D}_2 \arrow[dr, dashed] & \mathcal{D}_{t-1} \arrow[dr] & \mathcal{D}_t \\
 & \mathcal{E}_1 \arrow[u, red] & \mathcal{E}_2 \arrow[u, red] & \mathcal{E}_{t-1} \arrow[u, red] & \mathcal{E}_t \arrow[u, red]
\end{tikzcd}
$$

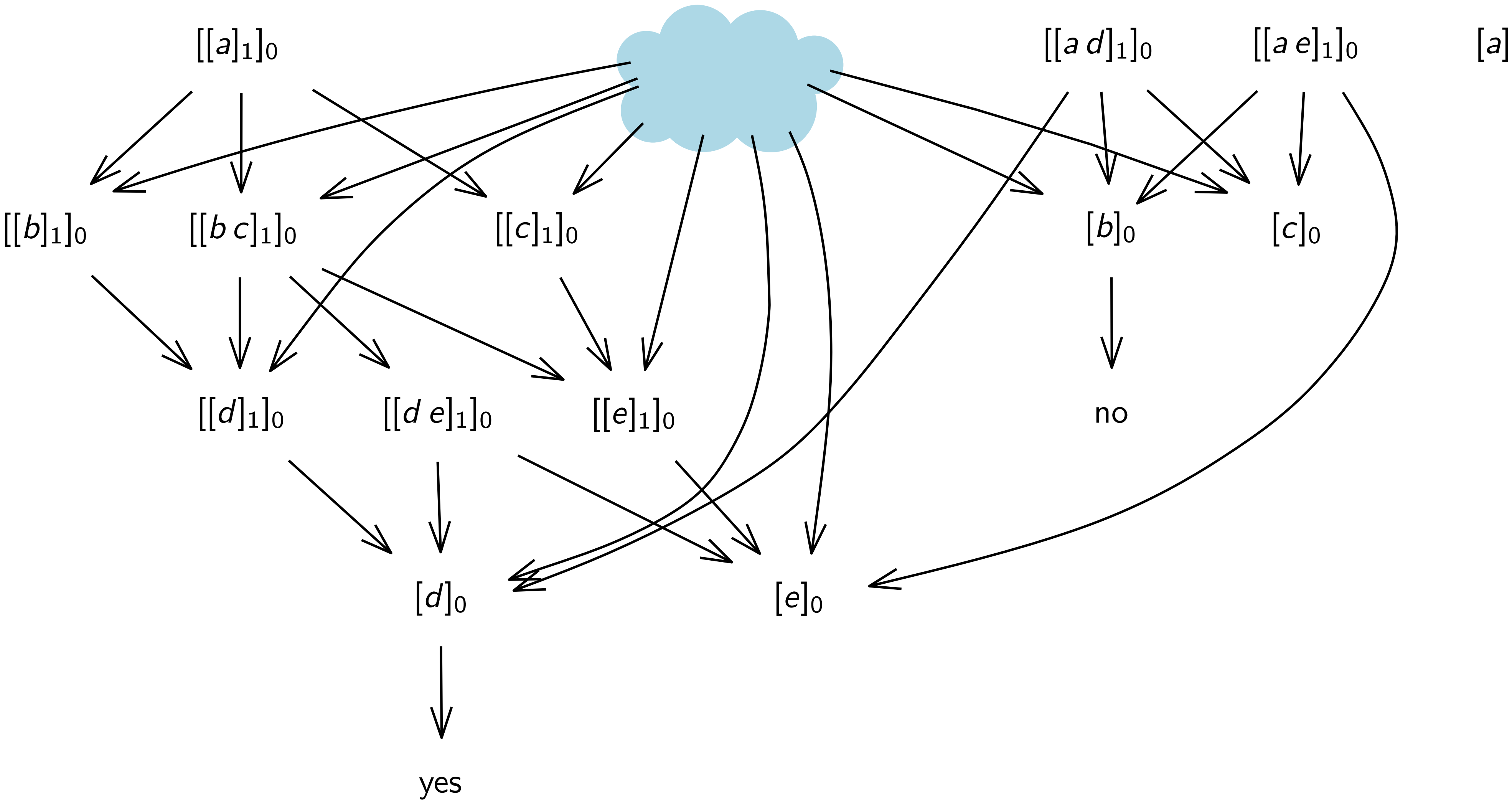
such that the diagram
“commutes” (same result)

Generalised dependency graphs





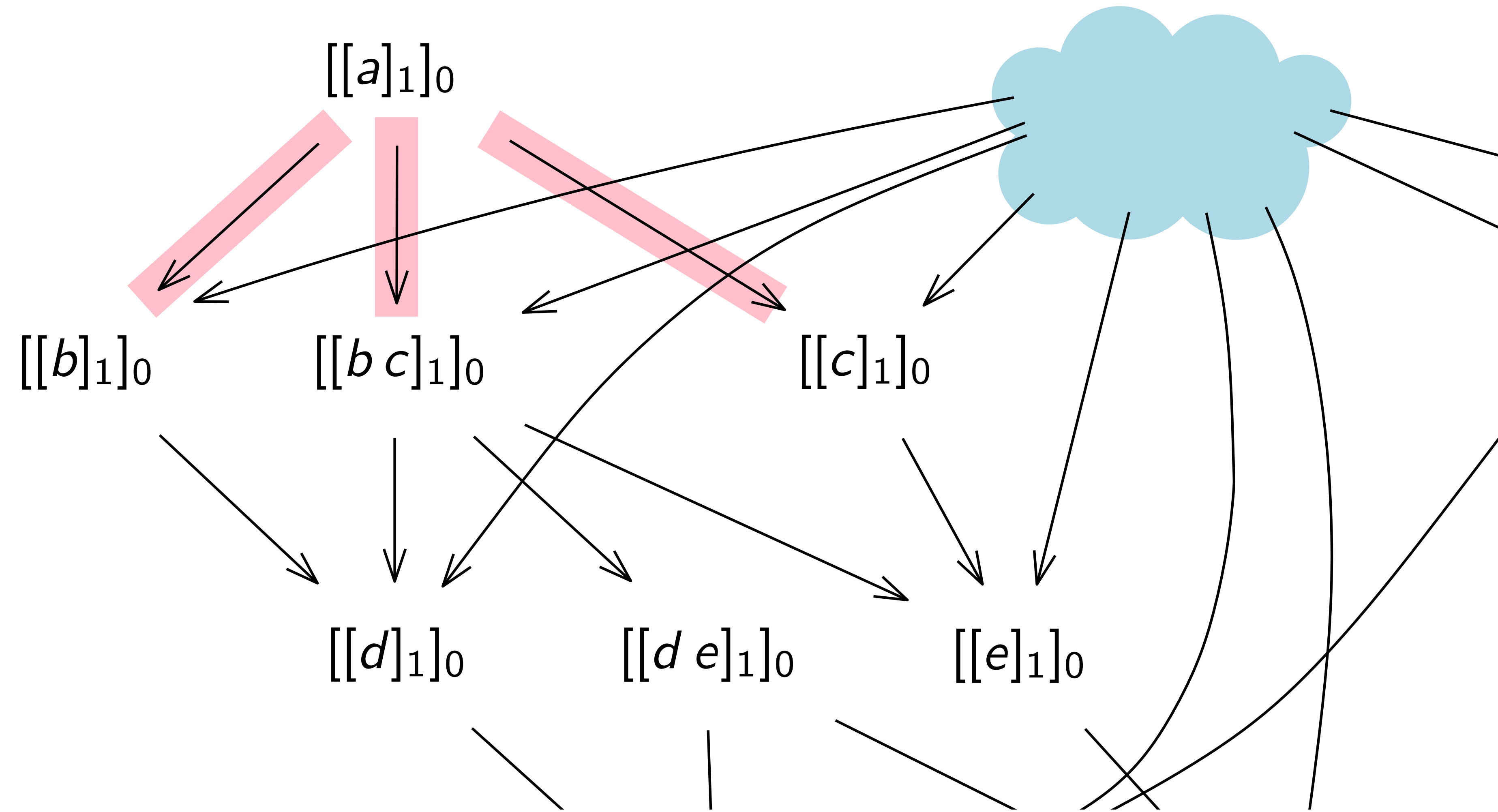
$[[a]_1]_0$
$[[b]_1]_0$
$[[b\ c]_1]_0$
$[[c]_1]_0$
$[[d]_1]_0$
$[[d\ e]_1]_0$
$[[e]_1]_0$
$[d]_0$
$[e]_0$



$[a \to bc]_1$ $\quad$ $[d]_0 \to [\,]_0$ yes $\quad$ $[e]_1 \to e$ $\quad$ $[f]_1 \to [g]_1\,[h]_1$

$[b \to d]_1$ $\quad$ $[b]_0 \to [\,]_0$ no $\quad$ $[g]_1 \to g$

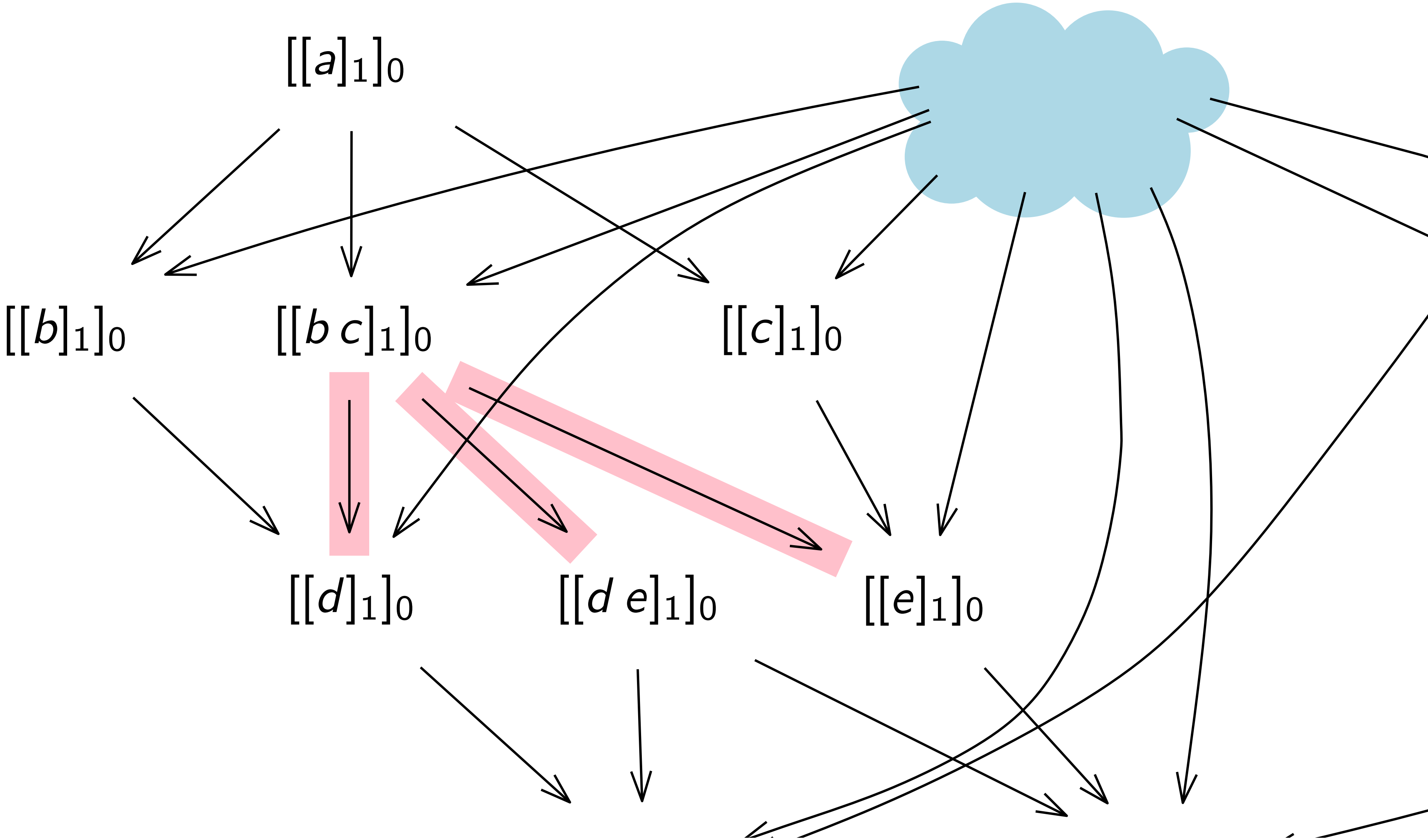
$[c \to e]_1$ $\quad$ $[d]_1 \to d$ $\quad$ $[h]_1 \to h$



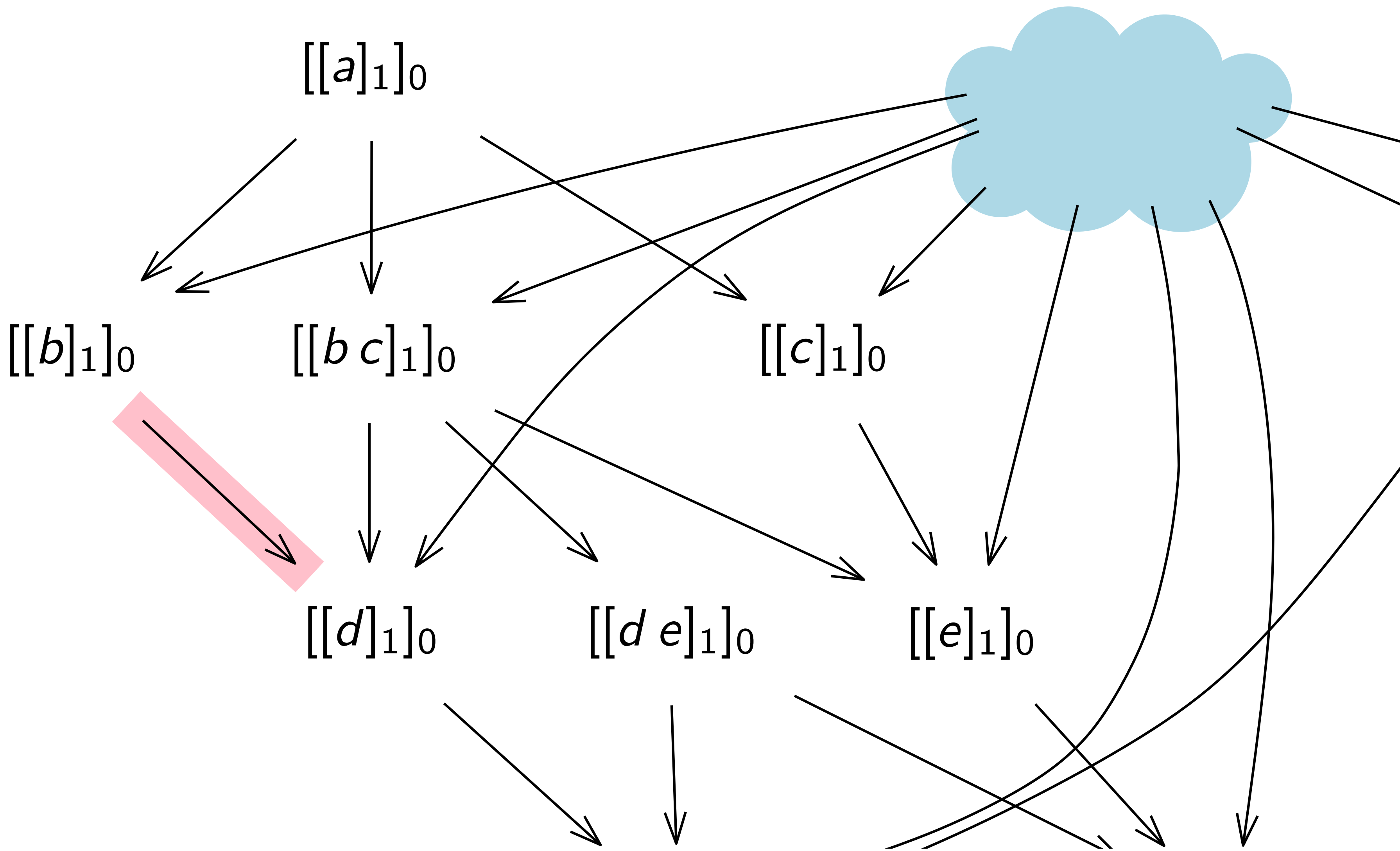
$[a \rightarrow bc]_1$ $\quad$ $[d]_0 \rightarrow [\ ]_0$ yes $\quad$ $[e]_1 \rightarrow e$ $\quad$ $[f]_1 \rightarrow [g]_1\,[h]_1$

$[b \rightarrow d]_1$ $\quad$ $[b]_0 \rightarrow [\ ]_0$ no $\quad$ $[g]_1 \rightarrow g$

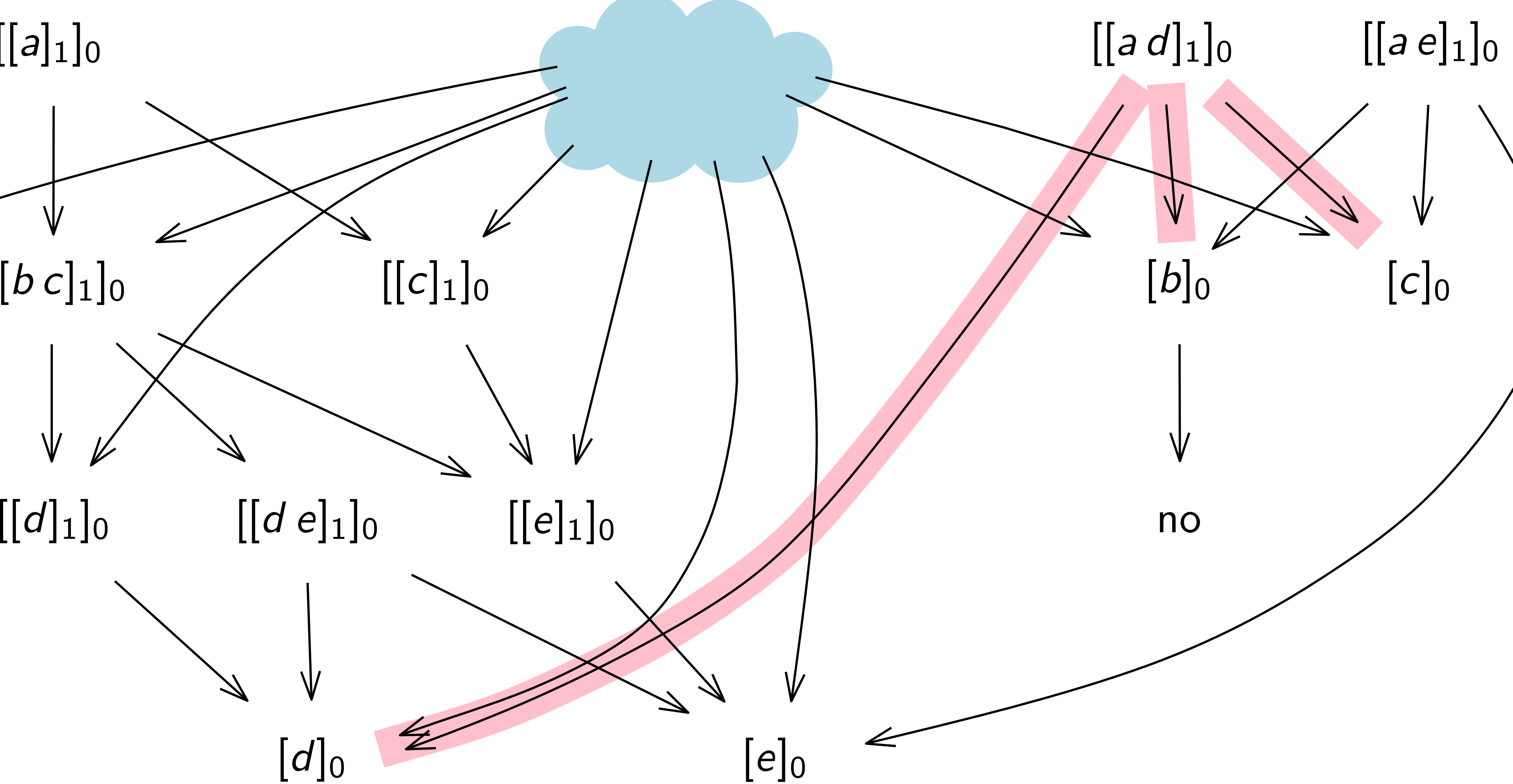
$[c \rightarrow e]_1$ $\quad$ $[d]_1 \rightarrow d$ $\quad$ $[h]_1 \rightarrow h$



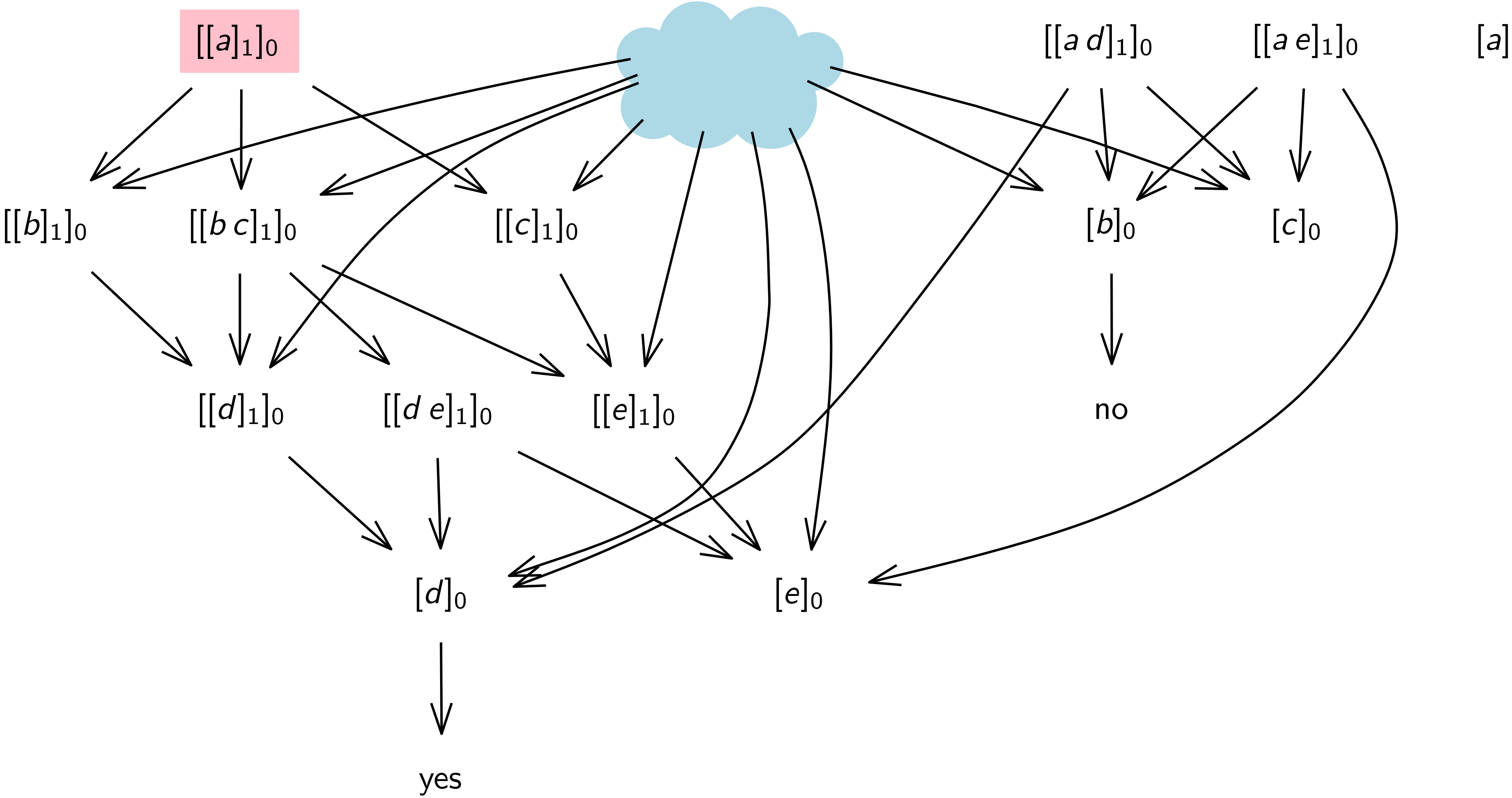
$[a \to bc]_1$	$[d]_0 \to [\,]_0$ yes	$[e]_1 \to e$	$[f]_1 \to [g]_1\,[h]_1$
$[b \to d]_1$	$[b]_0 \to [\,]_0$ no	$[g]_1 \to g$	
$[c \to e]_1$	$[d]_1 \to d$	$[h]_1 \to h$	



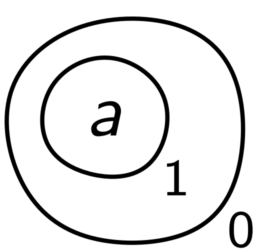
$[a \to bc]_1$	$[d]_0 \to [\,]_0$ yes	$[e]_1 \to e$	$[f]_1 \to [g]_1\,[h]_1$
$[b \to d]_1$	$[b]_0 \to [\,]_0$ no	$[g]_1 \to g$	
$[c \to e]_1$	$[d]_1 \to d$	$[h]_1 \to h$	



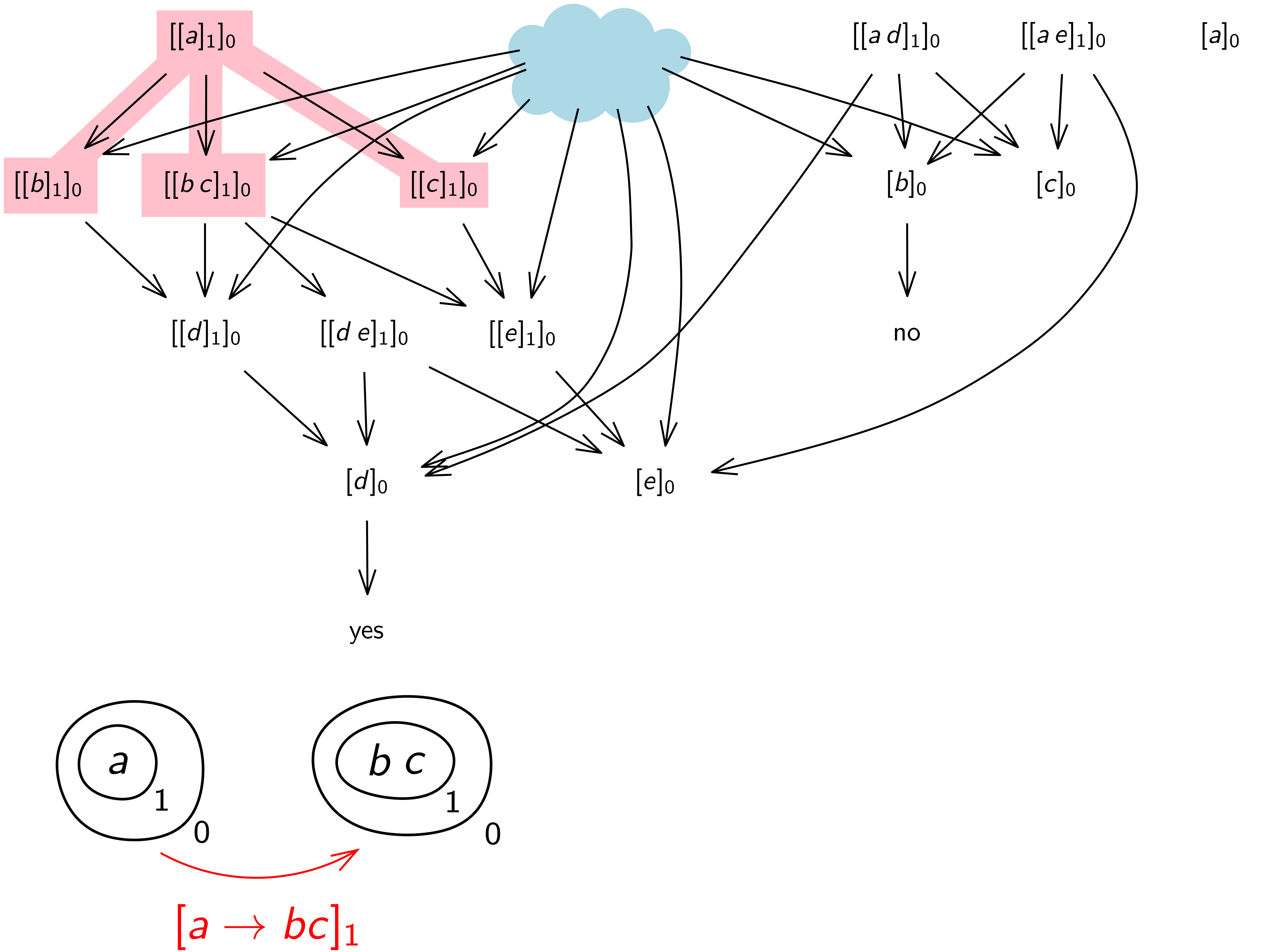
$[a \to bc]_1$	$[d]_0 \to [\,]_0$ yes	$[e]_1 \to e$	$[f]_1 \to [g]_1\,[h]_1$
$[b \to d]_1$	$[b]_0 \to [\,]_0$ no	$[g]_1 \to g$	
$[c \to e]_1$	$[d]_1 \to d$	$[h]_1 \to h$	

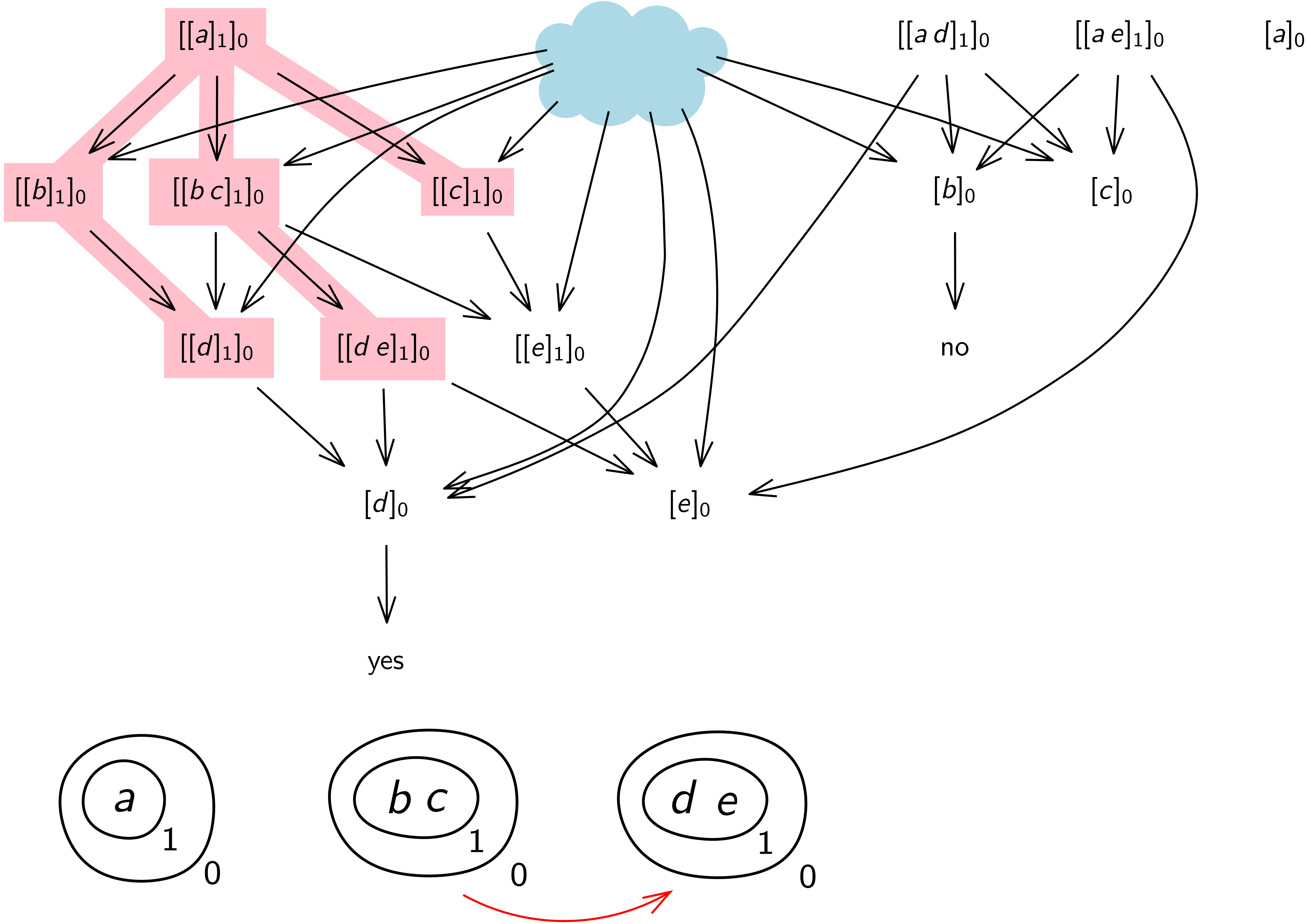


$[[a]_1]_0$
$[[a\ d]_1]_0$
$[[a\ e]_1]_0$
$[a]_0$
$[[b]_1]_0$
$[[b\ c]_1]_0$
$[[c]_1]_0$
$[b]_0$
$[c]_0$
$[[d]_1]_0$
$[[d\ e]_1]_0$
$[[e]_1]_0$
no
$[d]_0$
$[e]_0$
yes

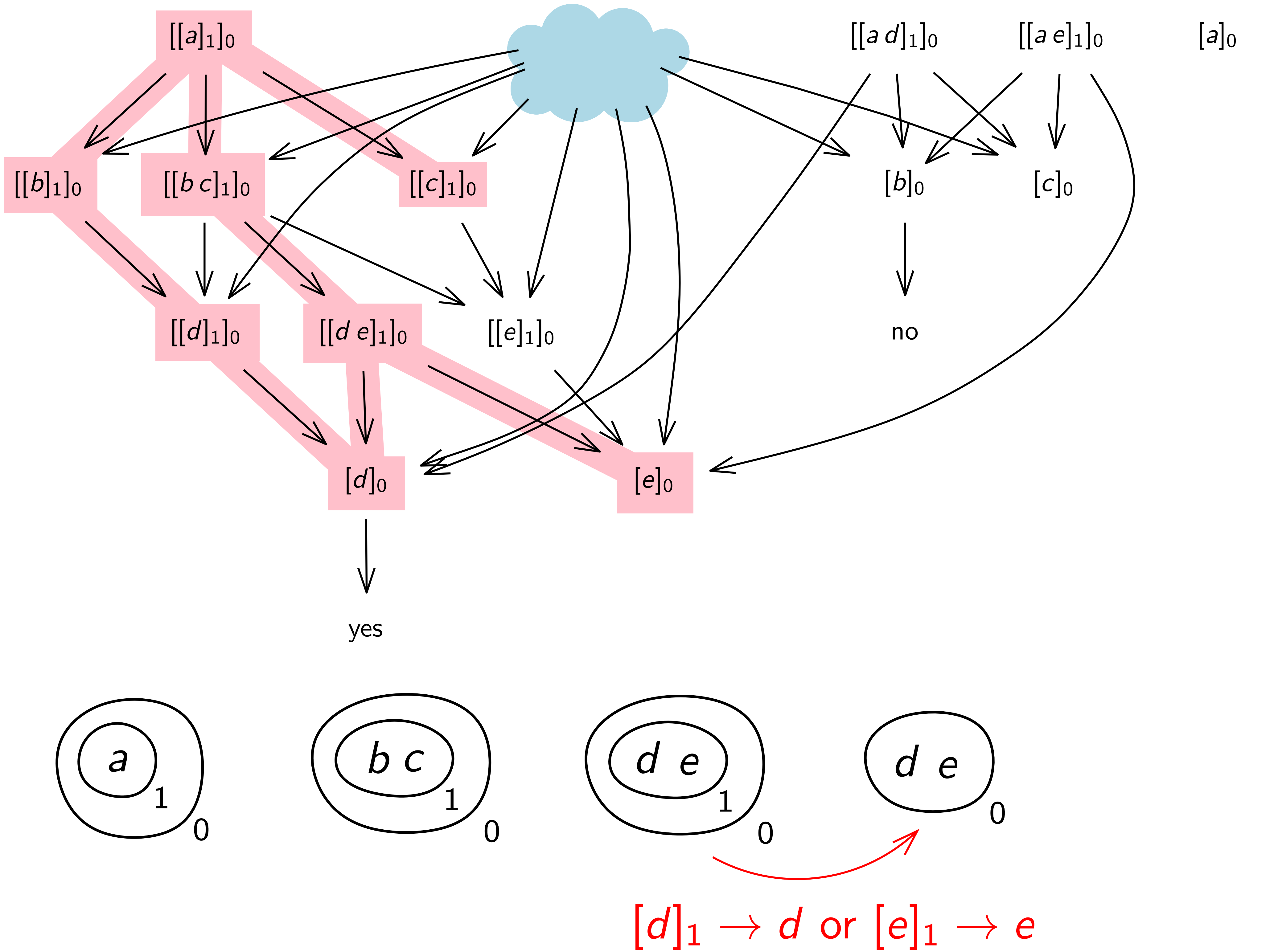


a
1
0

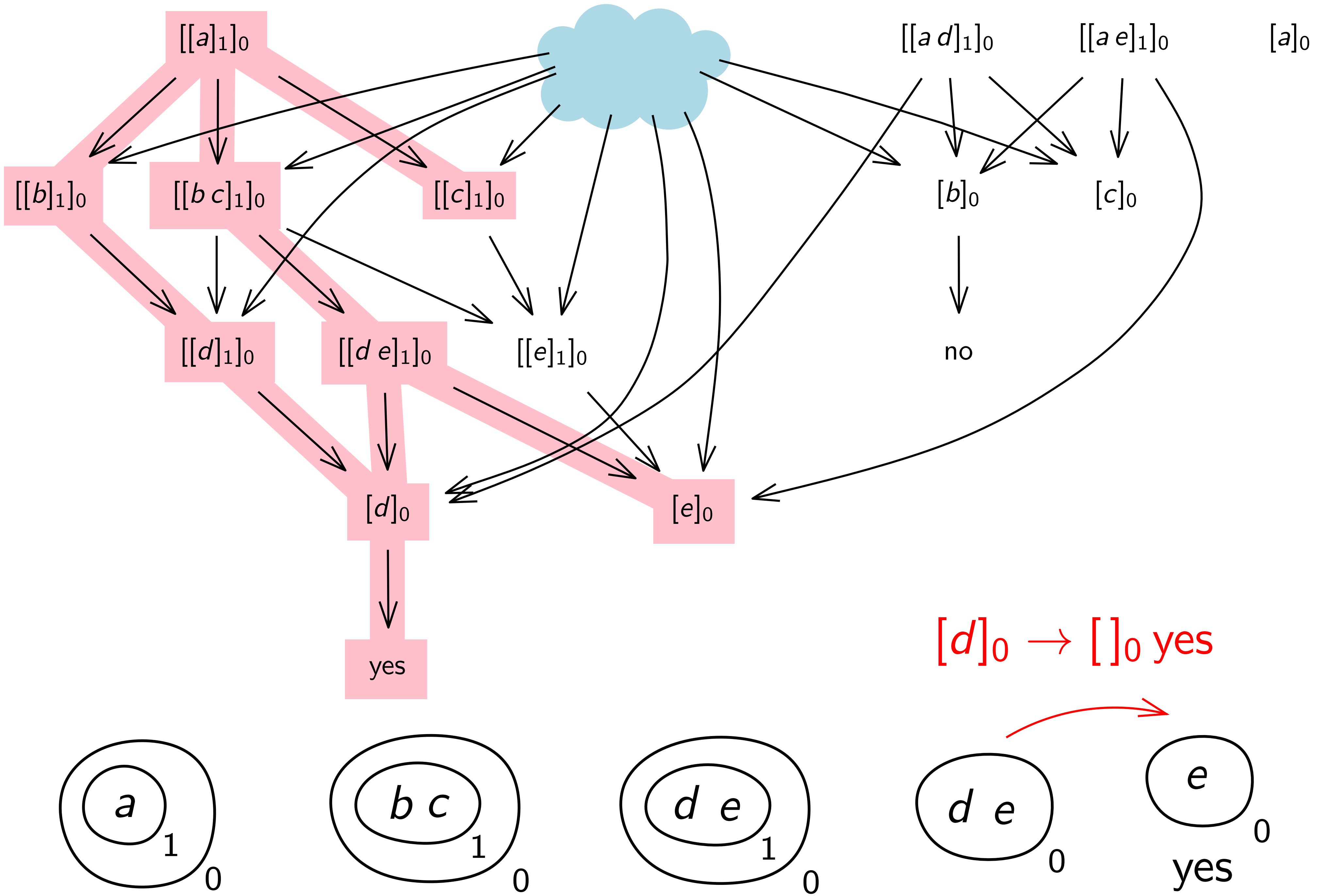

[[a]1]0
[[a d]1]0
[[a e]1]0
[a]0
[[b]1]0
[[b c]1]0
[[c]1]0
[b]0
[c]0
[[d]1]0
[[d e]1]0
[[e]1]0
no
[d]0
[e]0
yes
a
1
0
b c
1
0
[a → bc]1

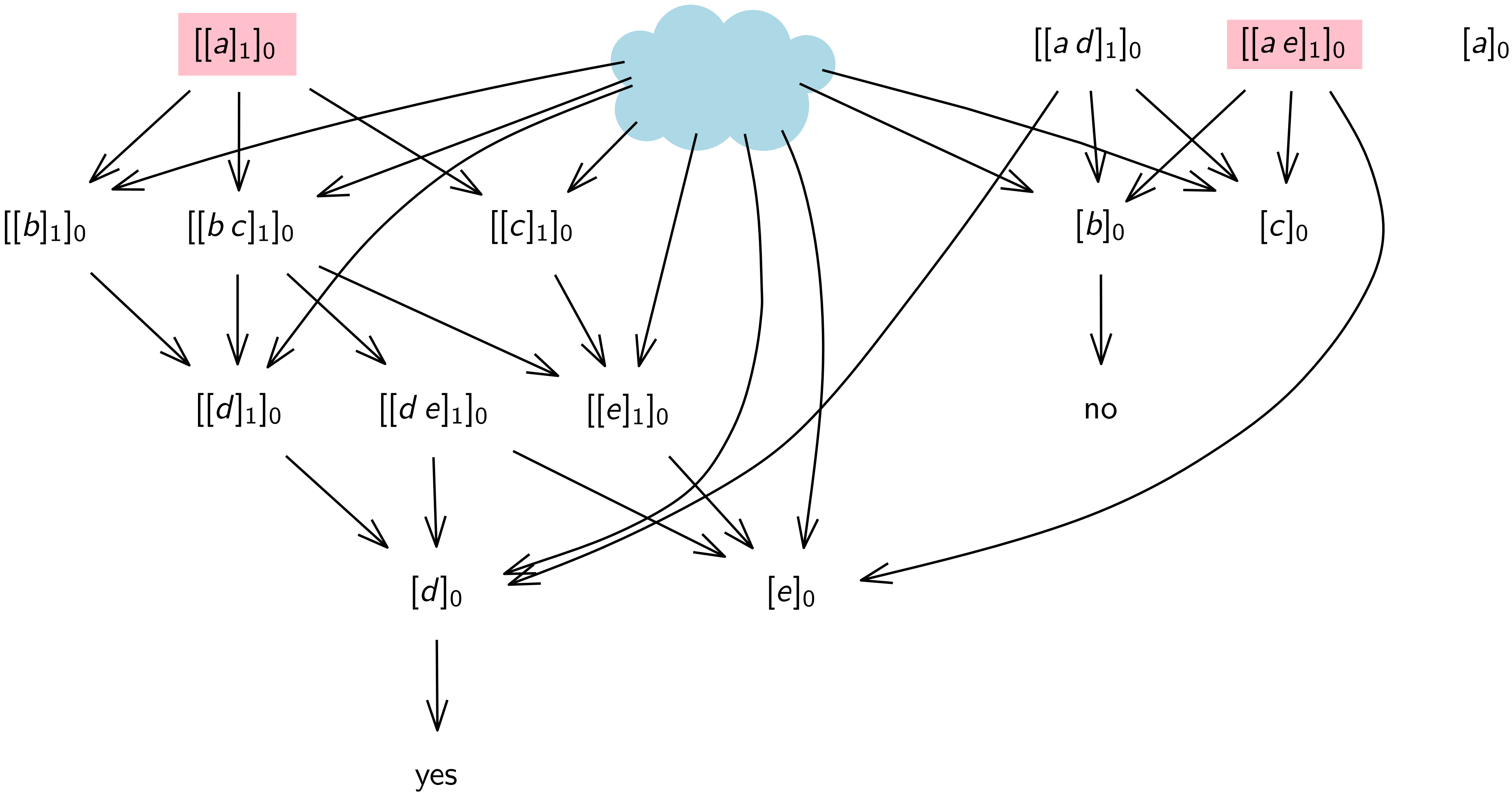


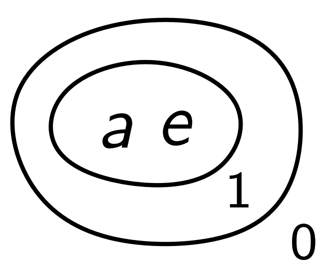
$[b \to d]_1$ and $[c \to e]_1$

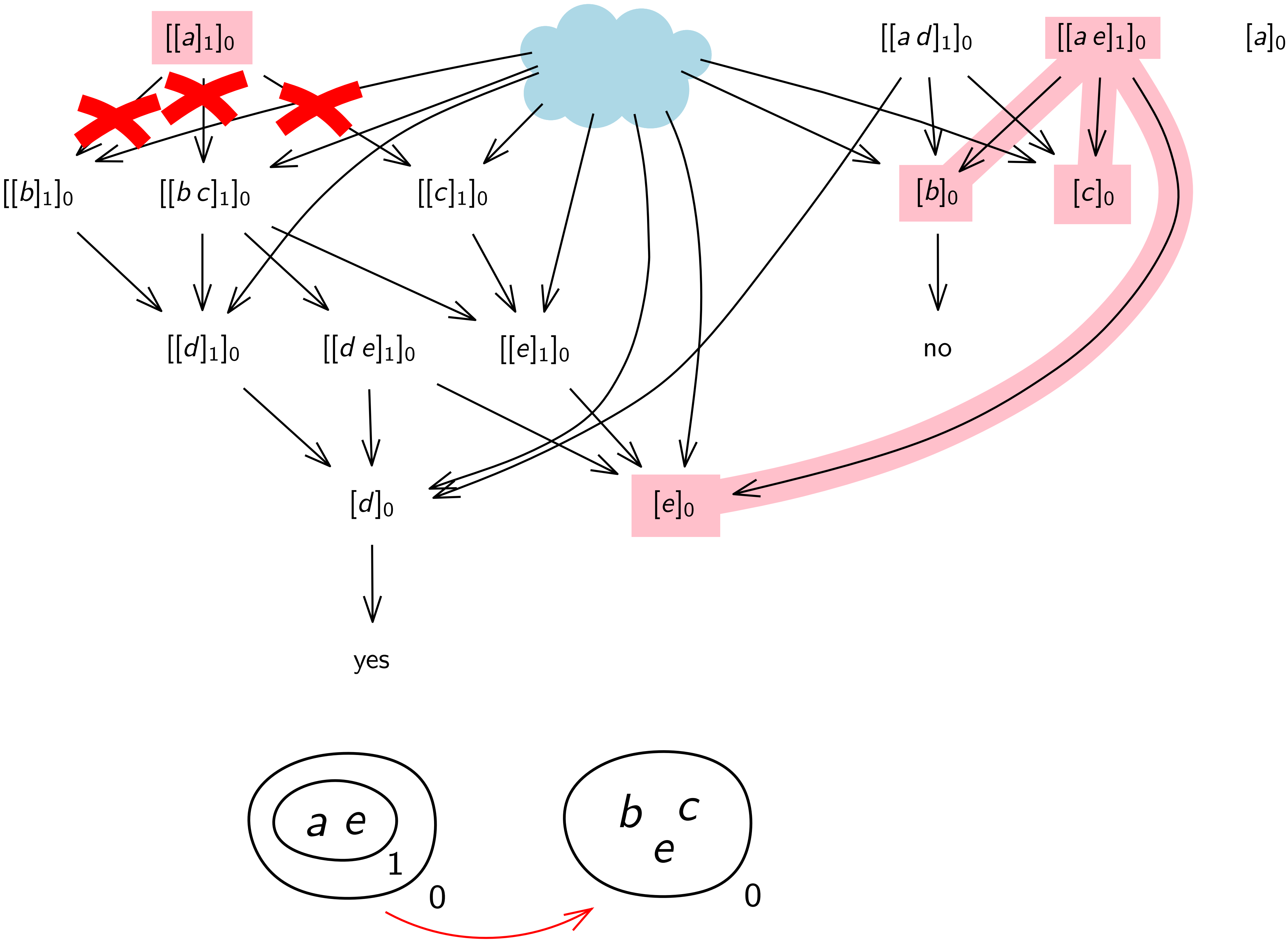


[[a]1]0
[[a d]1]0
[[a e]1]0
[a]0
[[b]1]0
[[b c]1]0
[[c]1]0
[b]0
[c]0
[[d]1]0
[[d e]1]0
[[e]1]0
no
[d]0
[e]0
yes
a 1 0
b c 1 0
d e 1 0
d e 0
[d]1 → d or [e]1 → e

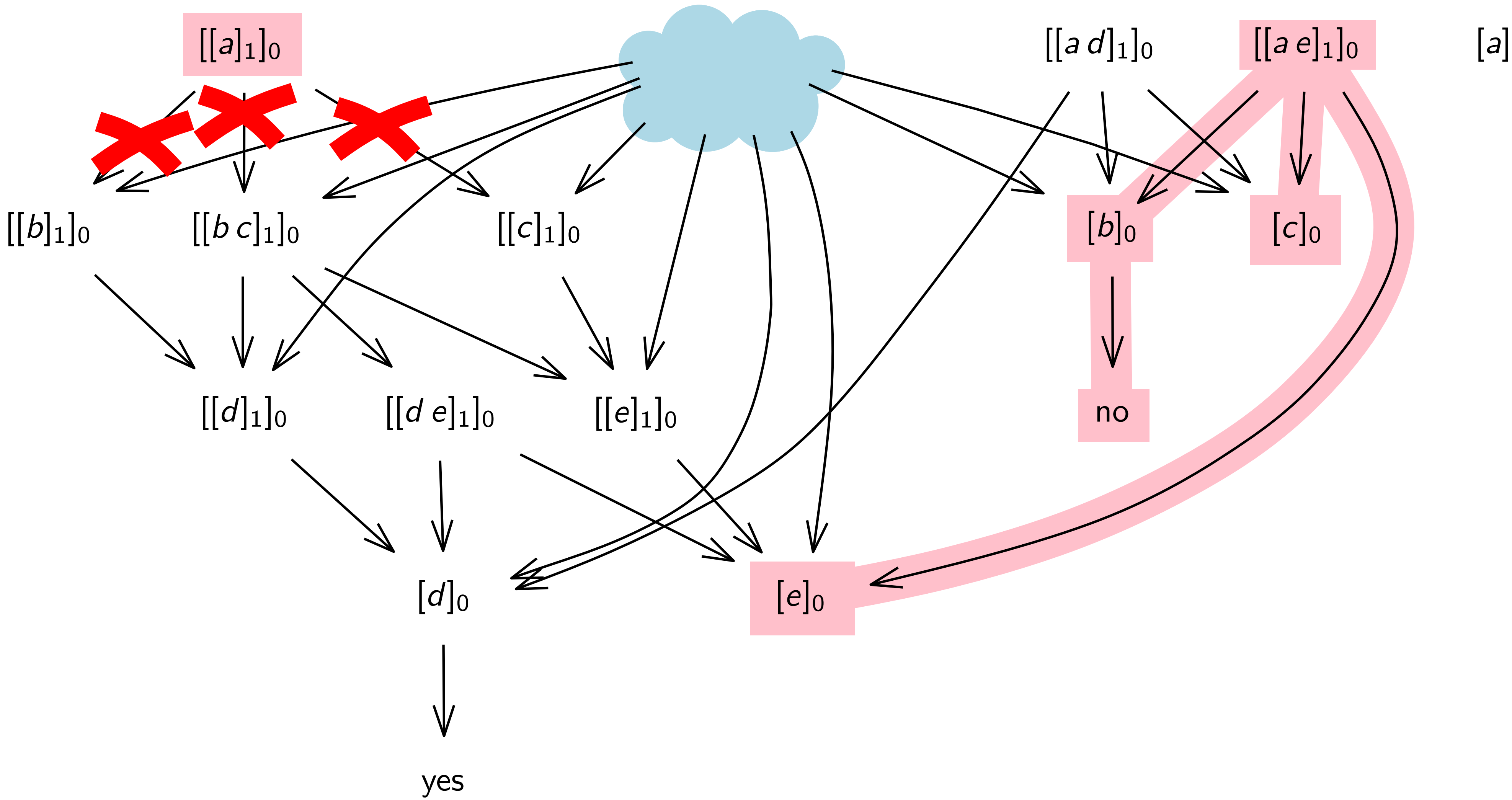

[[a]1]0
[[a d]1]0
[[a e]1]0
[a]0
[[b]1]0
[[b c]1]0
[[c]1]0
[b]0
[c]0
[[d]1]0
[[d e]1]0
[[e]1]0
no
[d]0
[e]0
yes
[d]0 → [ ]0 yes
a 1 0
b c 1 0
d e 1 0
d e 0
e 0
yes

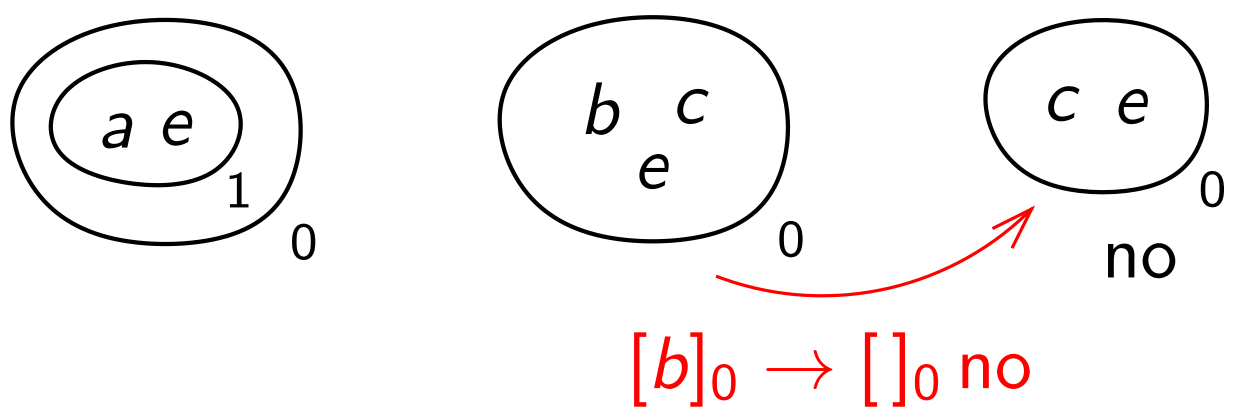

[[a]1]0
[[a d]1]0
[[a e]1]0
[a]0
[[b]1]0
[[b c]1]0
[[c]1]0
[b]0
[c]0
[[d]1]0
[[d e]1]0
[[e]1]0
no
[d]0
[e]0
yes

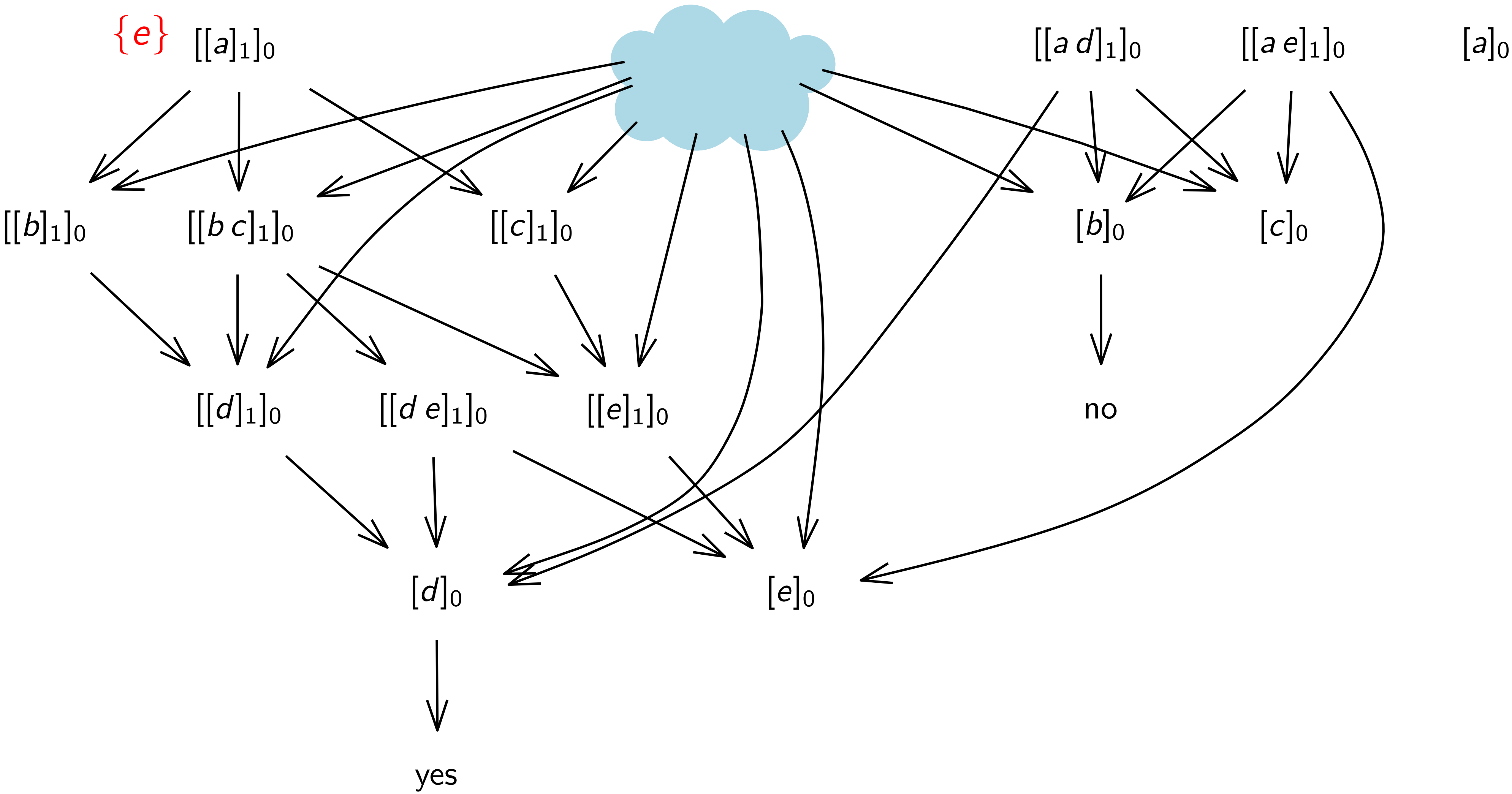

a e
1
0



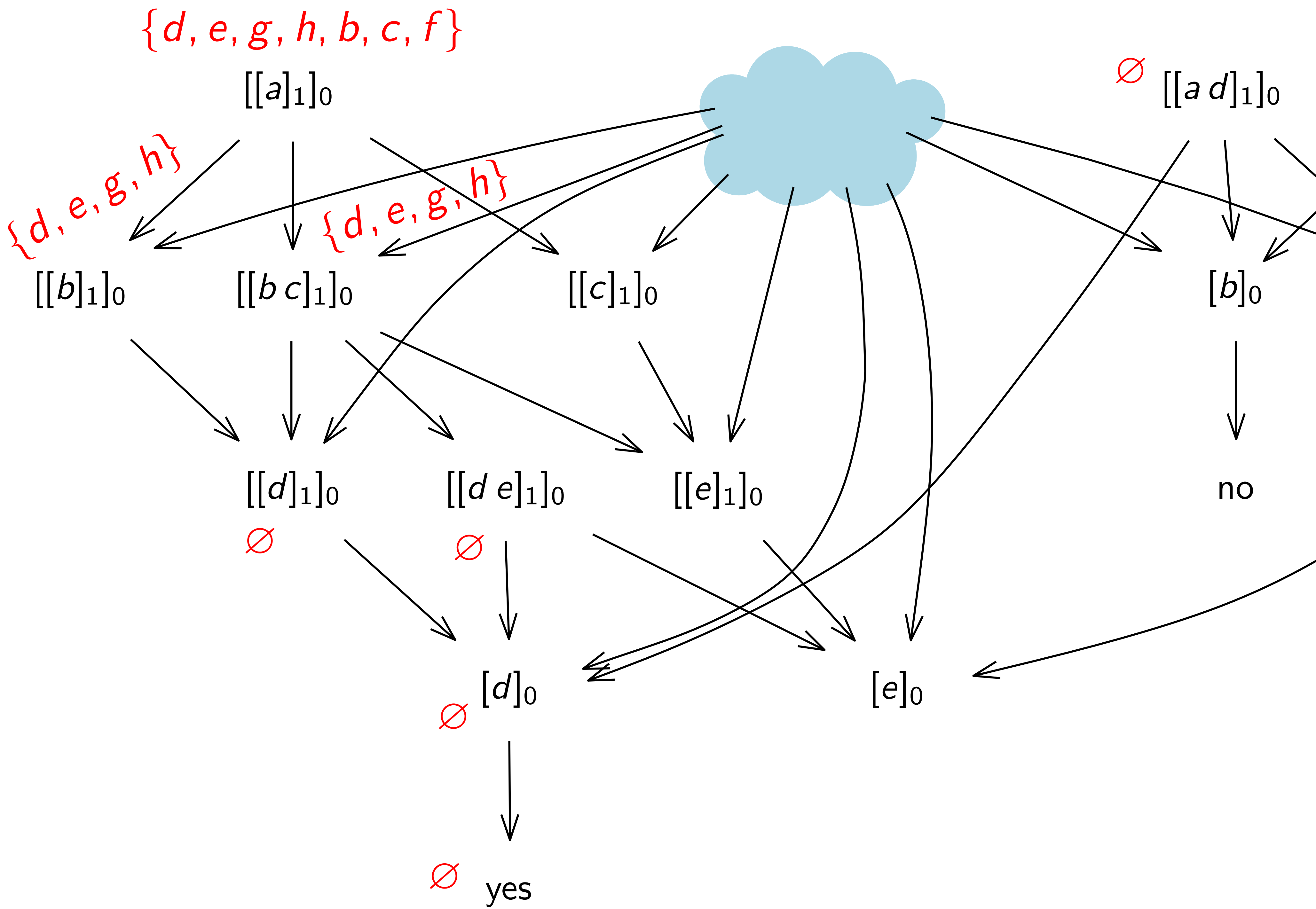
$[a \to bc]_1$ and $[e]_1 \to e$


[[a]1]0
[[a d]1]0
[[a e]1]0
[a]0
[[b]1]0
[[b c]1]0
[[c]1]0
[b]0
[c]0
[[d]1]0
[[d e]1]0
[[e]1]0
no
[d]0
[e]0
yes

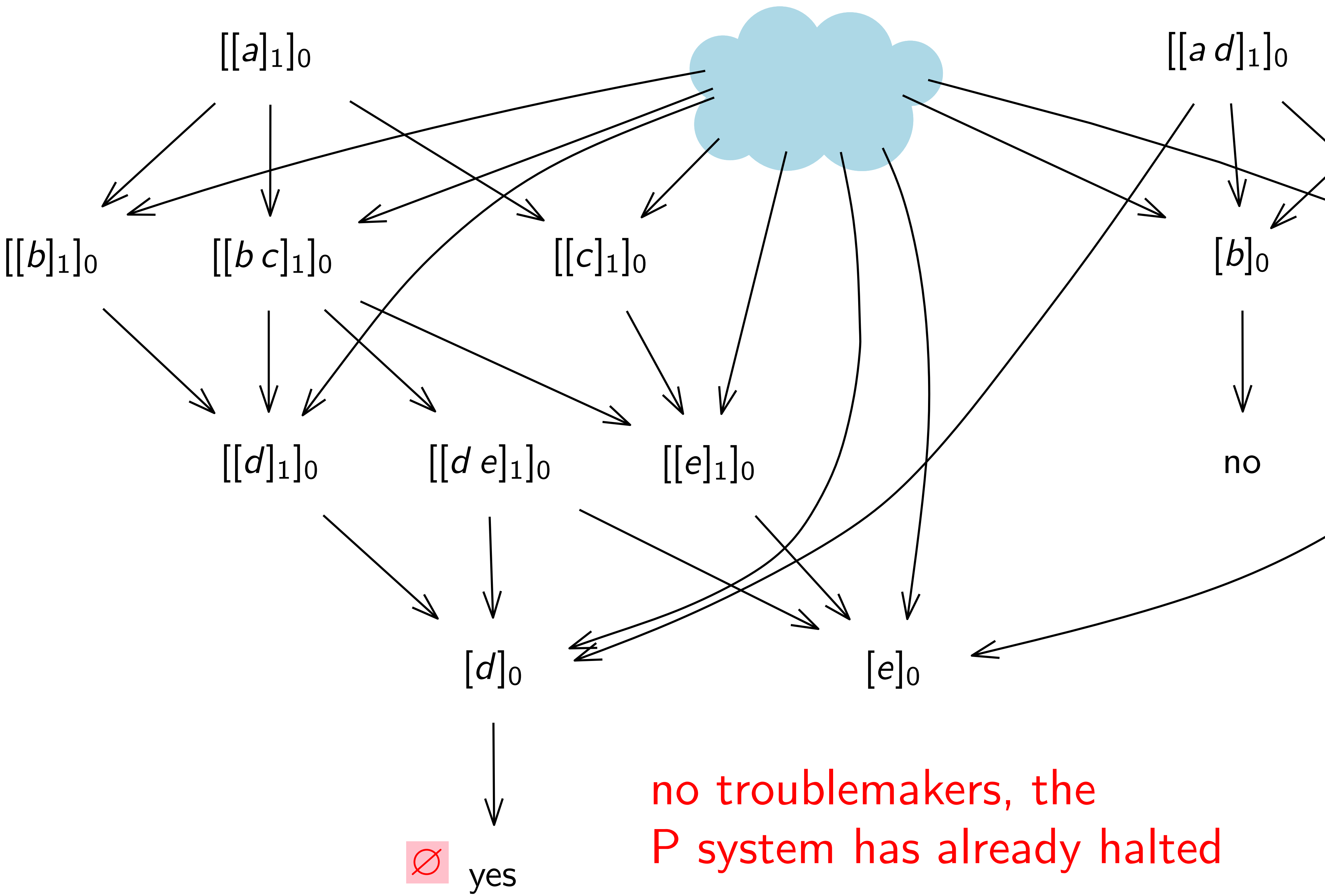

a e
1
0
b c
e
0
c e
0
no
[b]0 → [ ]0 no



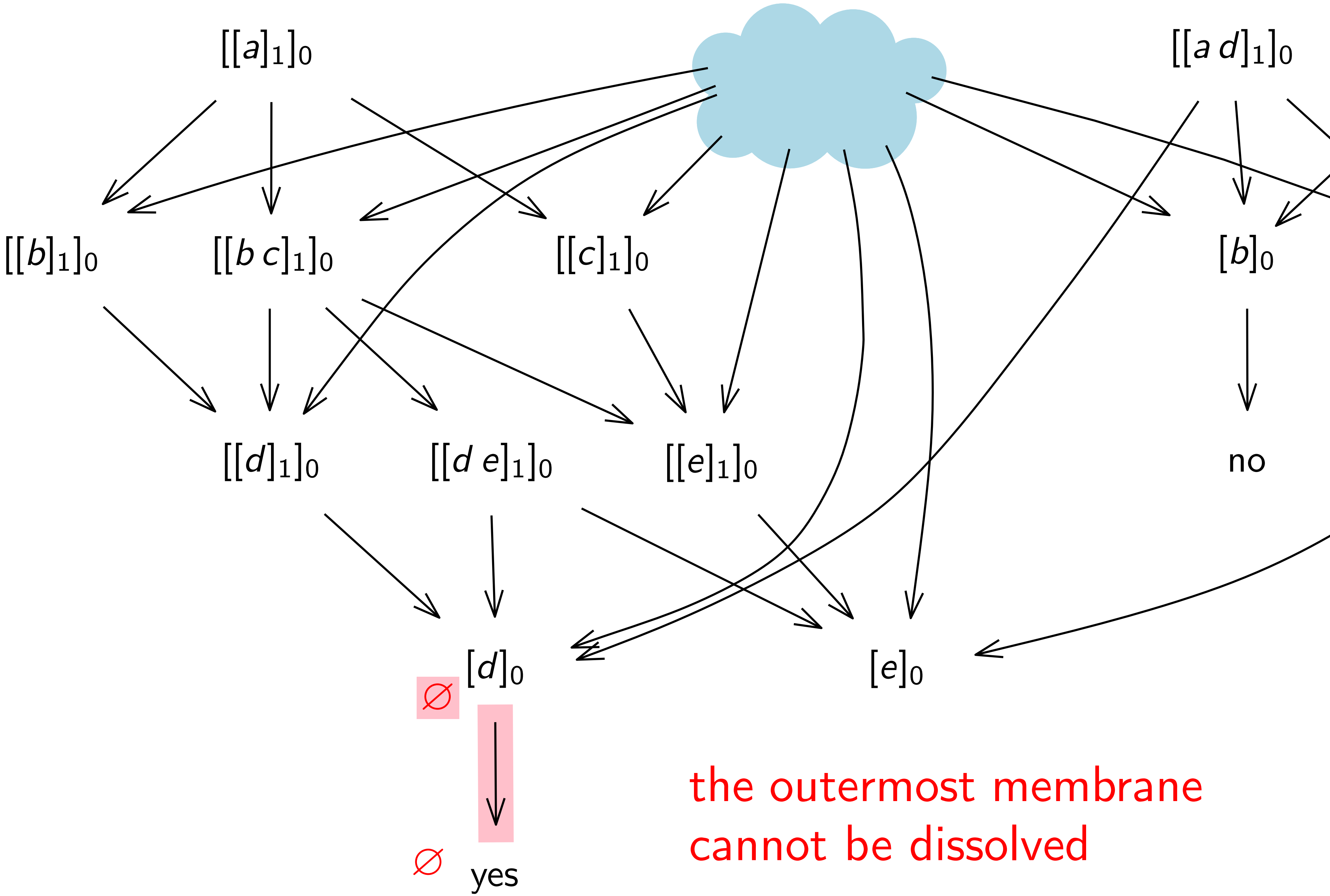
e is a troublemaker for $[[a]_1]_0$



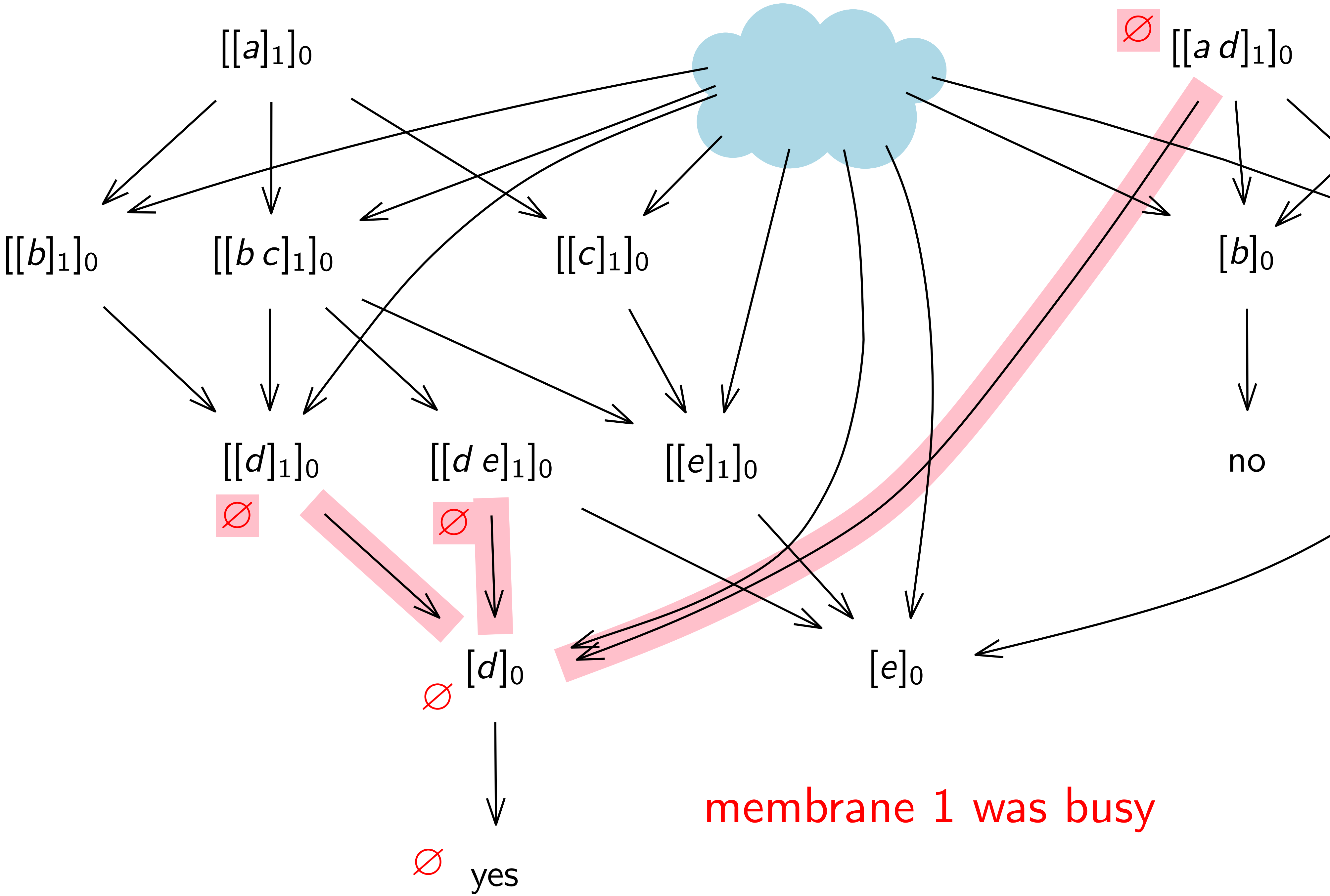
$\{d, e, g, h, b, c, f\}$
$[[a]_1]_0$
$\varnothing$
$[[a\ d]_1]_0$
$\{d, e, g, h\}$
$\{d, e, g, h\}$
$[[b]_1]_0$
$[[b\ c]_1]_0$
$[[c]_1]_0$
$[b]_0$
$[[d]_1]_0$
$[[d\ e]_1]_0$
$[[e]_1]_0$
no
$\varnothing$
$\varnothing$
$[d]_0$
$[e]_0$
$\varnothing$
$\varnothing$
yes



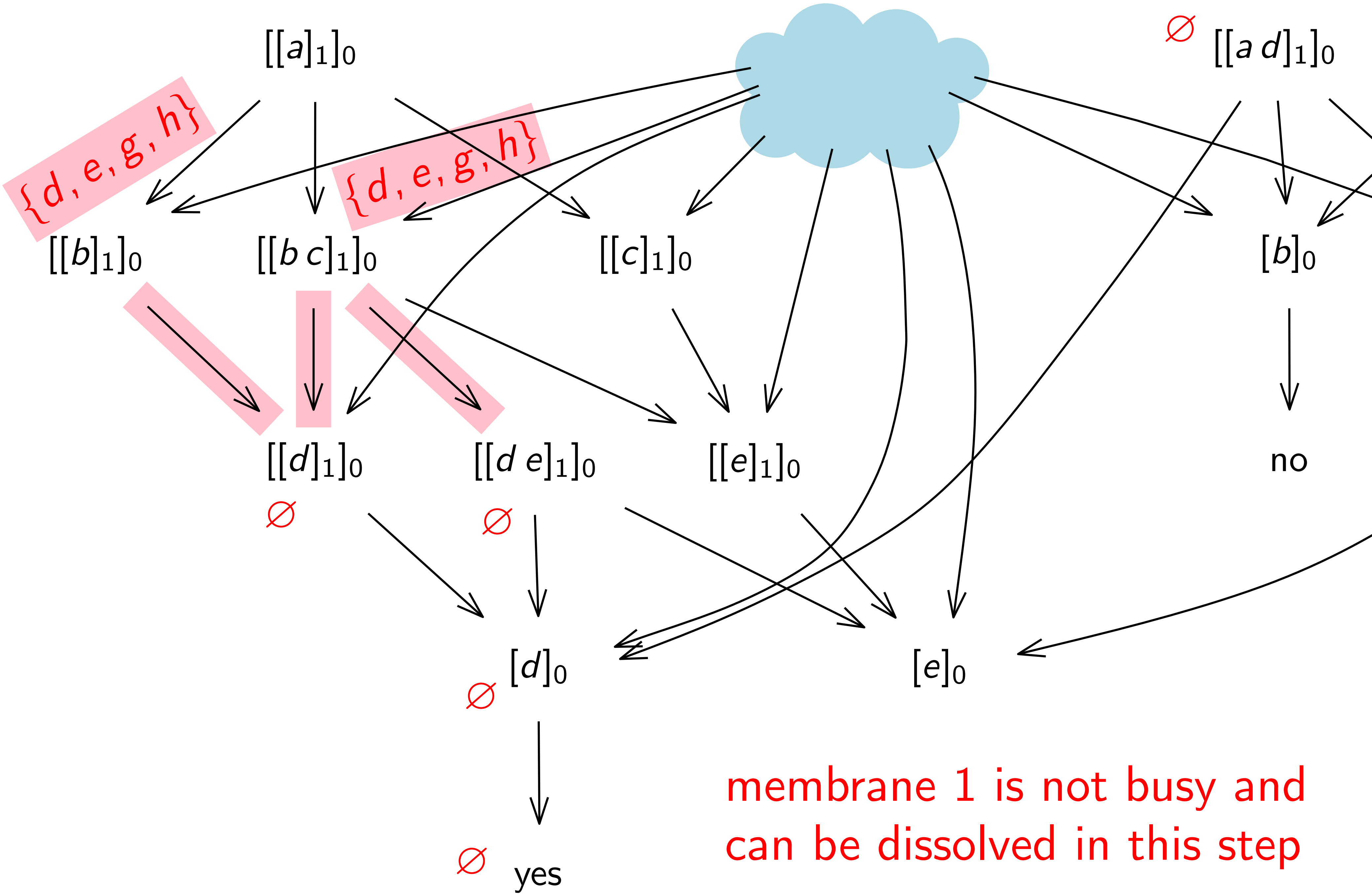
no troublemakers, the
P system has already halted



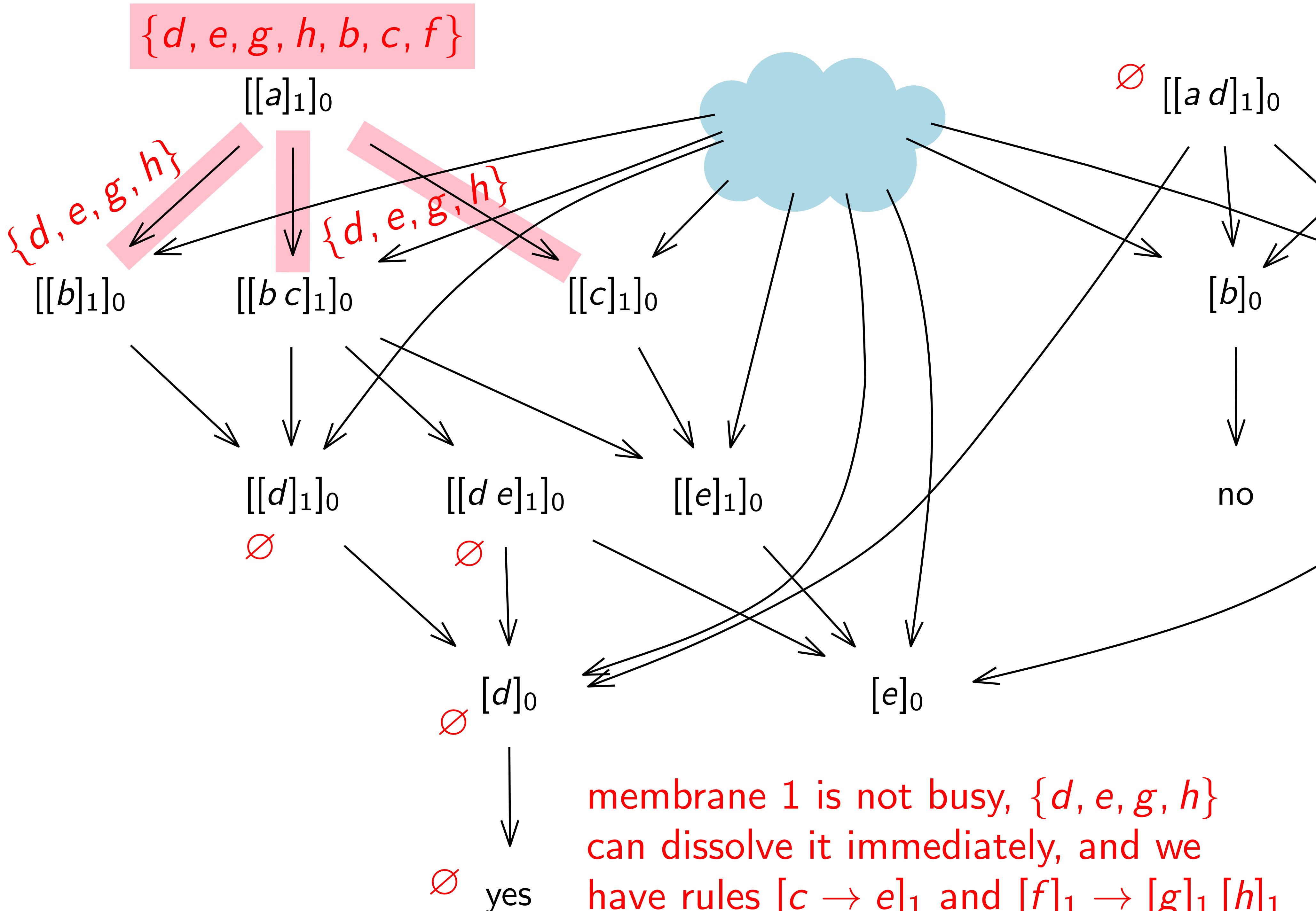
the outermost membrane
cannot be dissolved



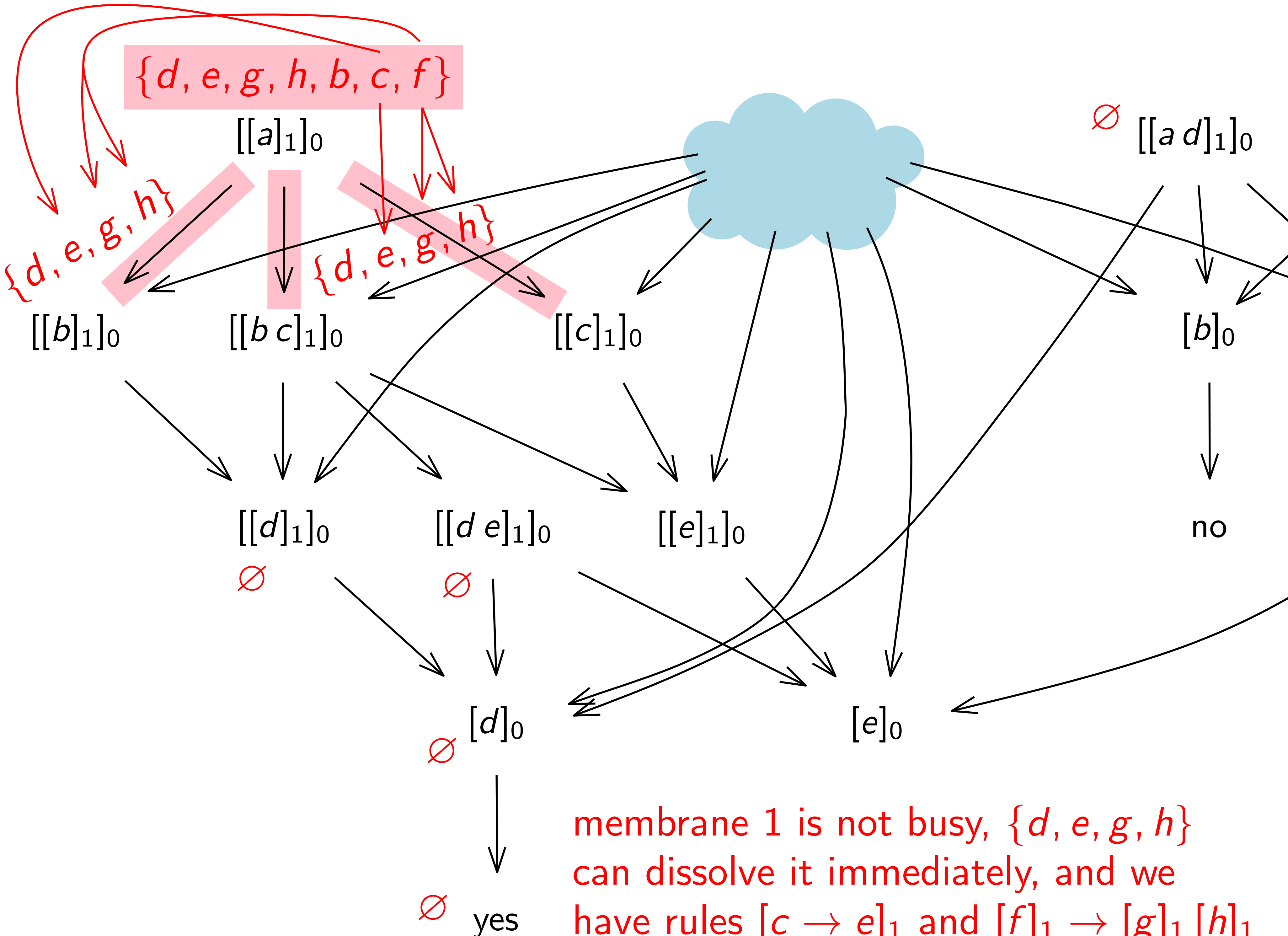
[[a]1]0
[[a d]1]0
[[b]1]0
[[b c]1]0
[[c]1]0
[b]0
[[d]1]0
[[d e]1]0
[[e]1]0
no
[d]0
[e]0
yes
membrane 1 was busy



membrane 1 is not busy and
can be dissolved in this step



membrane 1 is not busy, $\{d, e, g, h\}$ can dissolve it immediately, and we have rules $[c \to e]_1$ and $[f]_1 \to [g]_1\ [h]_1$



membrane 1 is not busy, $\{d, e, g, h\}$ can dissolve it immediately, and we have rules $[c \to e]_1$ and $[f]_1 \to [g]_1\ [h]_1$

A configuration $\mathcal{C}$ of the P system is untroubled if it contains a vertex connected to yes but none of its troublemakers

A configuration $\mathcal{C}$ of the P system is untroubled if it contains a vertex connected to yes but none of its troublemakers

If $\mathcal{C}$ is untroubled and $\mathcal{C} \to \mathcal{D}$, then $\mathcal{D}$ is untroubled

A configuration $\mathcal{C}$ of the P system is untroubled if it contains a vertex connected to yes but none of its troublemakers

If $\mathcal{C}$ is untroubled and $\mathcal{C} \rightarrow \mathcal{D}$, then $\mathcal{D}$ is untroubled

If $\mathcal{D}$ is untroubled and $\mathcal{C} \rightarrow \mathcal{D}$, then $\mathcal{C}$ is untroubled

A configuration $\mathcal{C}$ of the P system is untroubled if it contains a vertex connected to yes but none of its troublemakers

If $\mathcal{C}$ is untroubled and $\mathcal{C} \rightarrow \mathcal{D}$, then $\mathcal{D}$ is untroubled

If $\mathcal{D}$ is untroubled and $\mathcal{C} \rightarrow \mathcal{D}$, then $\mathcal{C}$ is untroubled

Theorem. A P system accepts iff its initial configuration is untroubled

The graph has $O(alphabet^2 \times labels)$ vertices and is constructed by iterating over the rules

The graph has $O(\mathit{alphabet}^2 \times \mathit{labels})$ vertices and is constructed by iterating over the rules

The troublemakers are computed by depth-first search of the (transposed) dependency graph

The graph has $O(alphabet^2 \times labels)$ vertices and is constructed by iterating over the rules

The troublemakers are computed by depth-first search of the (transposed) dependency graph

Untroubledness of the initial configuration of the P system is checked by looking at the vertices it contains

The graph has $O(alphabet^2 \times labels)$ vertices and is constructed by iterating over the rules

The troublemakers are computed by depth-first search of the (transposed) dependency graph

Untroubledness of the initial configuration of the P system is checked by looking at the vertices it contains

Theorem. We can check in polynomial time if the P system accepts

The monodirectional, shallow, deterministic P conjecture is true

The monodirectional, shallow, deterministic
P conjecture is true

$$\mathbf{DPMC}_{\mathcal{D}}^{[\star]} = \mathbf{P} = \mathbf{DMC}_{\mathcal{D}}^{[\star]}$$

Reference:

Leporati, A., Manzoni, L., Mauri, G., Porreca, A.E., Zandron, C.

Solving a special case of the P conjecture using dependency graphs with dissolution

In: Membrane Computing, 18th International Conference, CMC 2017. pp. 196–213. Springer (2018)

Open problems

Prove the result for confluent
(not just deterministic) P systems

Open problems

Prove the result for confluent
(not just deterministic) P systems

Prove the result for P systems with
deeper membrane structures

Open problems

Prove the result for confluent
(not just deterministic) P systems

Prove the result for P systems with
deeper membrane structures

Prove the result for bidirectional P systems
(no idea if we can do this)

Open problems

Prove the result for confluent
(not just deterministic) P systems

Prove the result for P systems with
deeper membrane structures

Prove the result for bidirectional P systems
(no idea if we can do this)

Use generalised dependency graphs for other
variants of P systems to prove **P** upper bound
or find “borderlines” for efficiency

Thanks for your attention!

¡Gracias por vuestra atención!

¿Any preguntas?