Some questions from information theory

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The research topic I would like to propose is COUNTING. I do not mean #**P**, I mean enumerating membrane structures or configurations of a given "size", modulo isomorphism or not. After finishing this email I reformulated this question as: "how much information is stored in a configuration of a membrane system with symbol objects?"

Modulo isomorphism: as "size" it is enough to specify the number m of membranes and t of objects. In total: since alphabets, e.g., membrane alphabet (labels) and object alphabet (symbols) are not bounded, the number of configurations, even with a specified number of membranes and objects would be infinite unless we somehow bound all alphabets. One of alternative approaches: fix the alphabets, in particular —O—=n kinds of objects, the number m of membranes and maximal number k of objects of each kind in each region. Then the total number of objects is not fixed, but bounded by m*n*k.

Some preliminary results.

I One membrane, modulo isomorphism. $t = 0 \to 1$ (empty multiset), $t = 1 \to 1$, $t = 2 \to 2(aa, ab)$, $t = 3 \to (aaa, aab, abc)$, $t = 4 \to 5$, $t = 5 \to 7$, $t = 6 \to 11$. Can be specified by a recurrent two-argument function. Seems to correspond to number sequence `https://oeis.org/A000041`.

II Membrane structures without objects, without considering labels/polarizations, modulo isomorphism. $m = 1 \to 1$ (only skin), $m = 2 \to 1$ (two nested membranes), $m = 3 \to 2$ ([[ ][ ]] and [[[ ]]]), $m = 4 \to 4$, $m = 5 \to 9$. Seems to correspond to number sequence `https://oeis.org/A000081`.

Note: with labels, already $m = 2$ gives 2 different configurations, $[\,[\;\;]_2\,]_1$ and $[\,[\;\;]_1\,]_1$.

III One membrane, one label, one polarization, no isomorphism. Fixing the alphabet size $|O| = n$, the number of different multisets of cardinality t is the number of t-combinations with repetitions of set O, equal to $C(n + t - 1, t)$, where $C(n, k) = n!/(k! * (n - k)!)$. `https://en.wikipedia.org/wiki/Combination\#Number_of_combinations_with_repetition`

IV m membranes, $|O| = n$, at most k objects of any kind in any membrane, no isomorphism. Then the number of different configurations would seem to be equal to the number of different membrane structures with m membranes, multiplied by $(mn)^{(k+1)}$. Correction: but even in this case it is not so simple, because $[\,[\,a\,]\,[\quad]\,]$ and $[\,[\quad]\,[\,a\,]\,]$ are the same configuration; the expression above only holds under the assumption that there are no indistinguishable membranes, e.g., all membranes have different labels.

Goal: to have a general formula for each typical set of parameters specifying "size", of all configurations of this "size".

If counting tree structures is difficult, start with tissue.

Why? to understand how much information is indeed stored in a configuration of a P system, because the general impression that, with m membranes and t objects, there are approximately exponentially many different configurations, is too inaccurate and in some settings incorrect.