Cooperation in communication in cell-like P systems

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Introduction

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$$[\quad]_1$$
$$|$$
$$[\quad]_2$$
$$|$$
$$[\quad]_3 \; n$$

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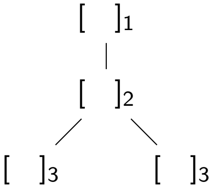

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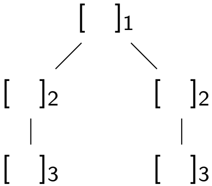

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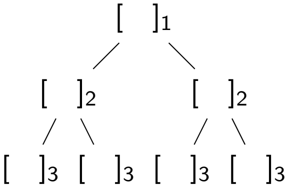

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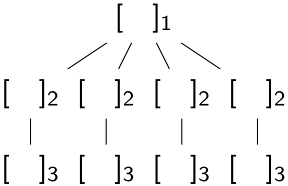

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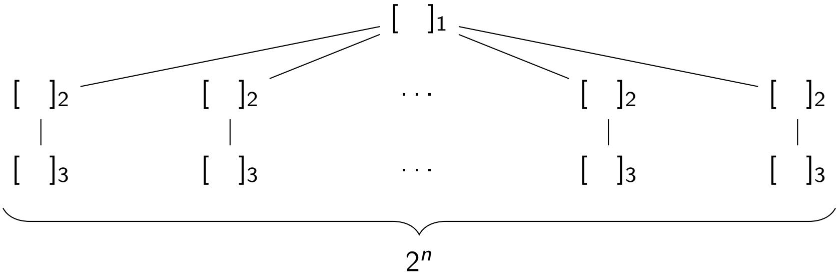

New results

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- Apart from cooperation, when **separation** rules are used instead of **division** rules, generating new objects seems mandatory.

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Division vs. Separation + Evolution

$$[\,a\,]_h \rightarrow [\,b\,]_h\,[\,c\,]_h$$

$$\wr\wr$$

$$[\,a\,]_h \rightarrow [\,\Gamma_0\,]_h\,[\,\Gamma_1\,]_h$$
$$+$$
$$[\,a \rightarrow u\,]_h, u \in M_f(\Gamma)$$

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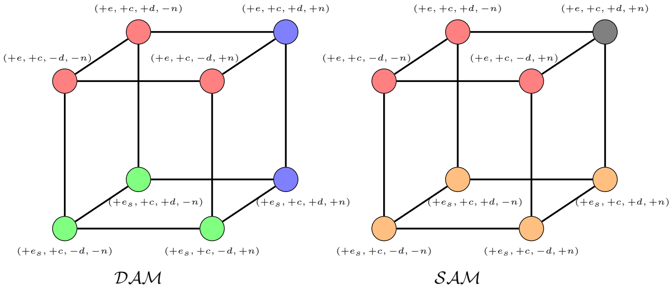
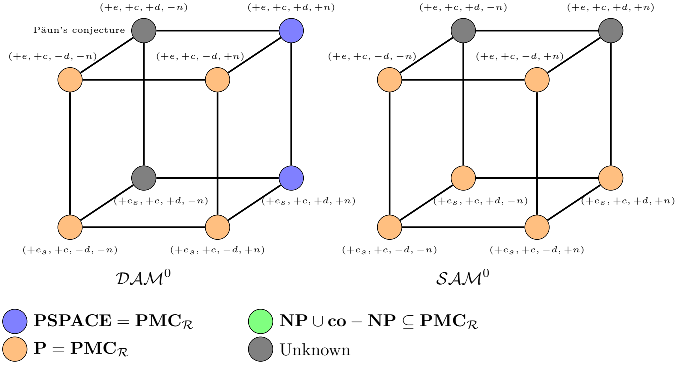
State-of-the-art

- With them, we can increase the landscape of computational complexity theory in Membrane Computing

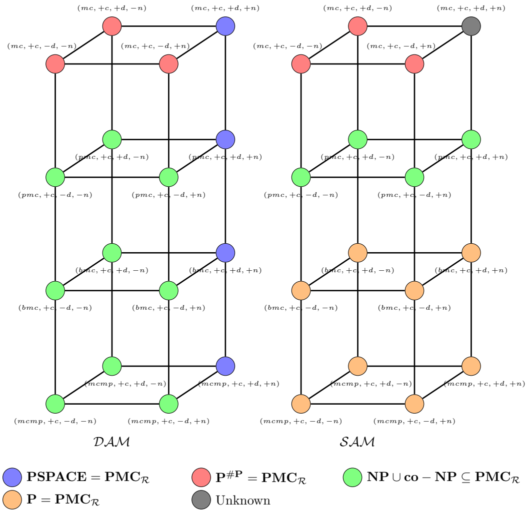


- $\mathbf{PSPACE} = \mathbf{PMC}_{\mathcal{R}}$
- $\mathbf{P}^{\#\mathbf{P}} = \mathbf{PMC}_{\mathcal{R}}$
- $\mathbf{NP} \cup \mathbf{co-NP} \subseteq \mathbf{PMC}_{\mathcal{R}}$
- $\mathbf{P} = \mathbf{PMC}_{\mathcal{R}}$
- Unknown

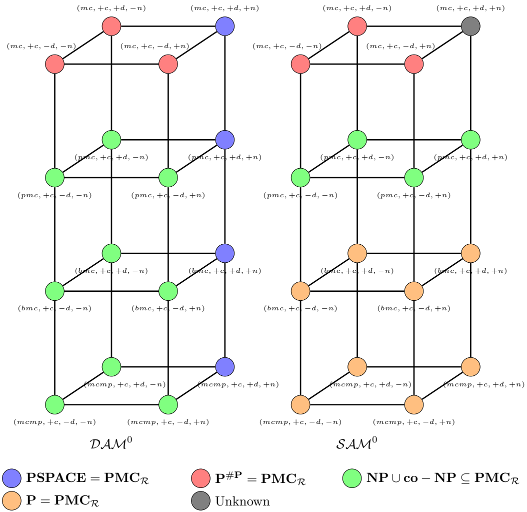
Polarizationless P systems with active membranes



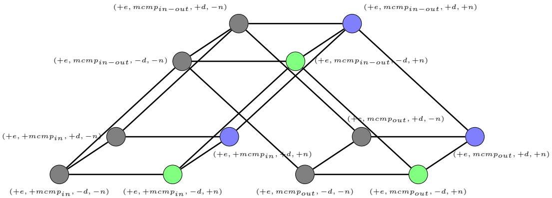
P systems with active membranes with minimal cooperation in ev. rules



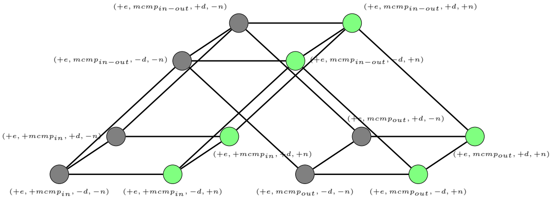
Polarizationless P systems with active membranes and with minimal cooperation in ev. rules



Polarizationless P systems with active membranes with minimal cooperation in comm. rules



$\mathcal{DAM}^0$



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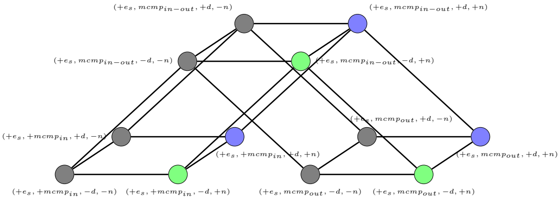
Polarizationless P systems with active membranes with simple ev. rules and minimal cooperation in comm. rules



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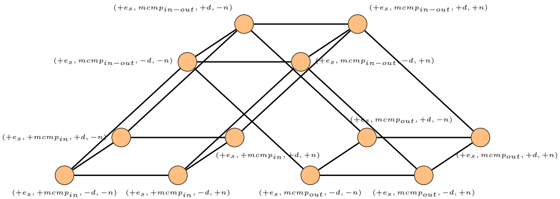

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- (blue) $\mathbf{PSPACE} = \mathbf{PMC}_{\mathcal{R}}$
- (green) $\mathbf{NP} \cup \mathbf{co-NP} \subseteq \mathbf{PMC}_{\mathcal{R}}$
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Future work

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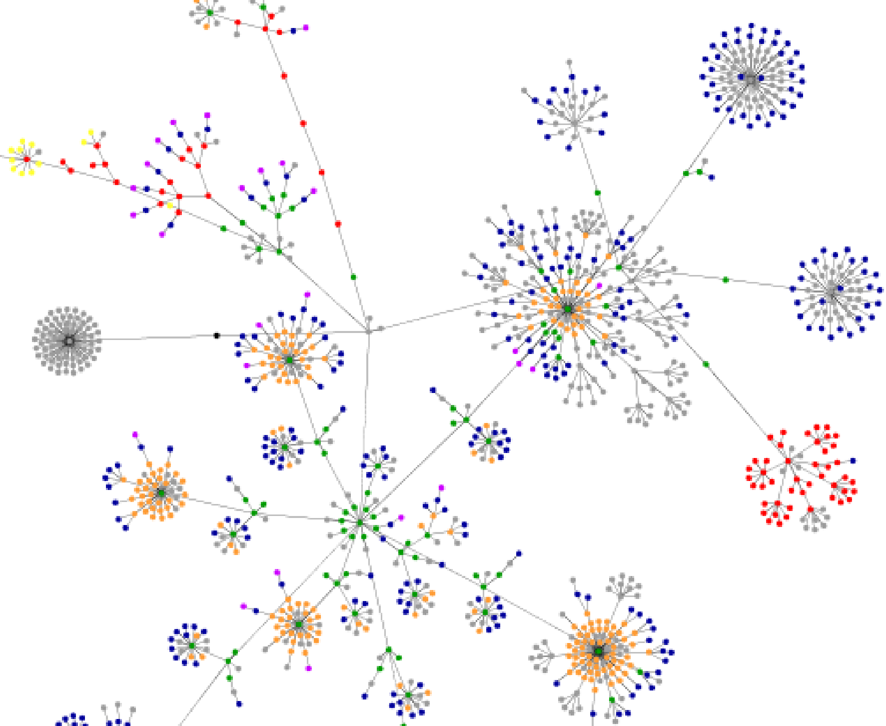
Keeping in mind that...

$\mathcal{F}_1$

$\mathcal{F}_2$




P=NP
$e^{\pi i} = -1$

Thanks for your attention!