A semantic frontier of the efficiency in membrane systems

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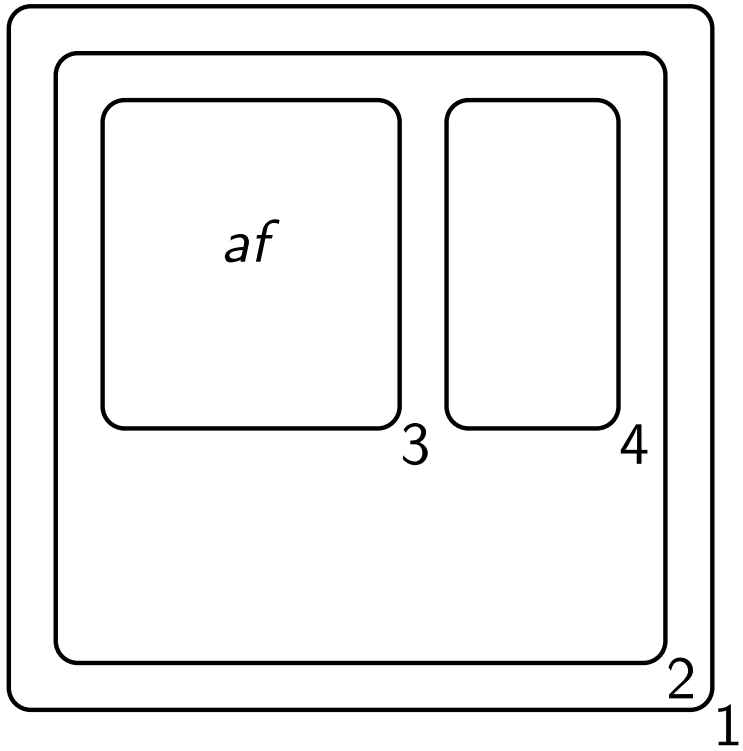
1. Introduction
2. Previously, on The Computational Complexity Theory
3. Teaching an old dog old tricks
4. Complexity results
5. Future work

Introduction

- A long time ago...



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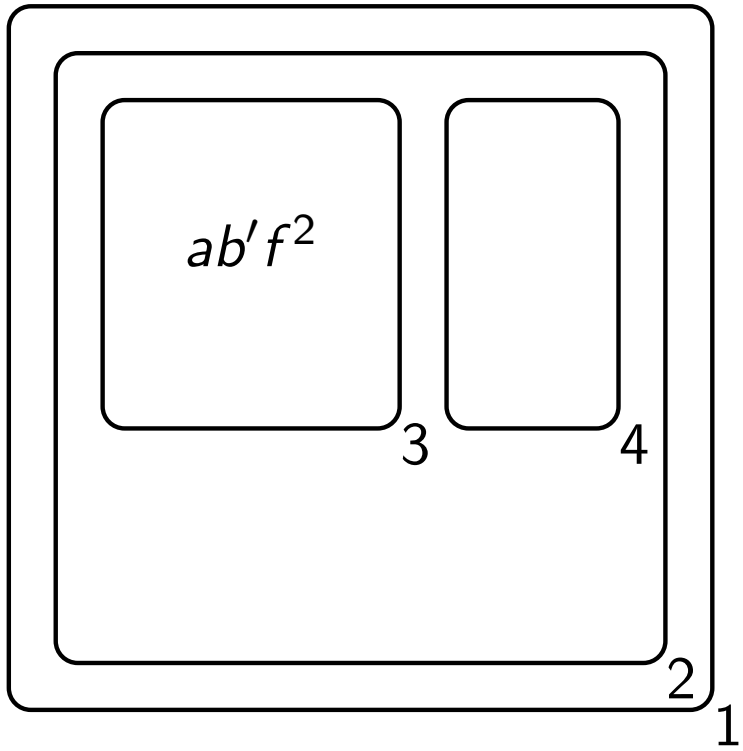

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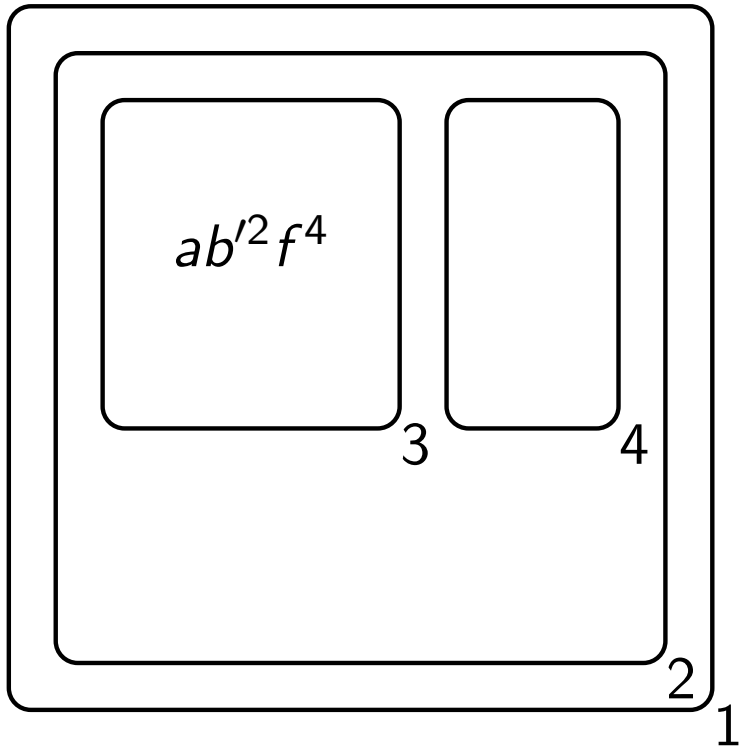

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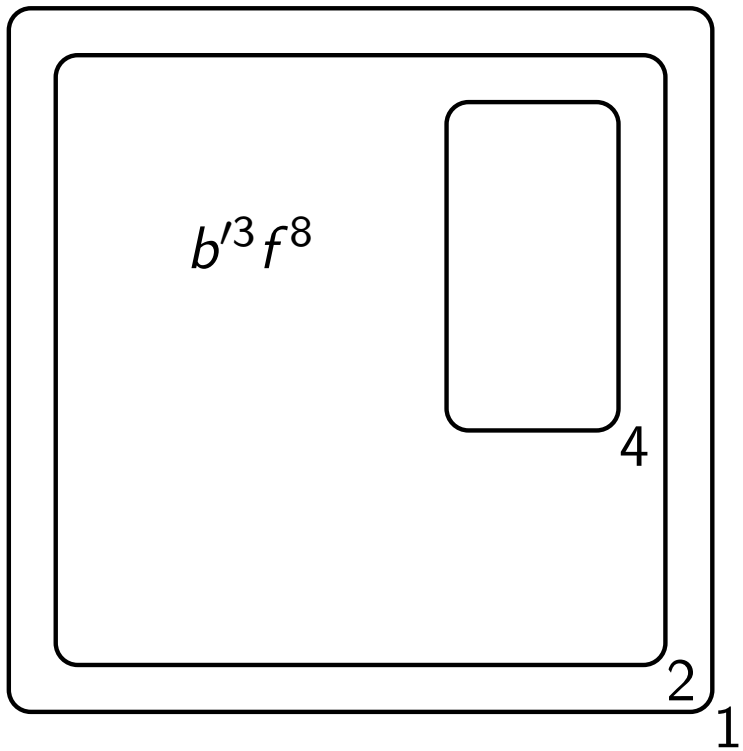

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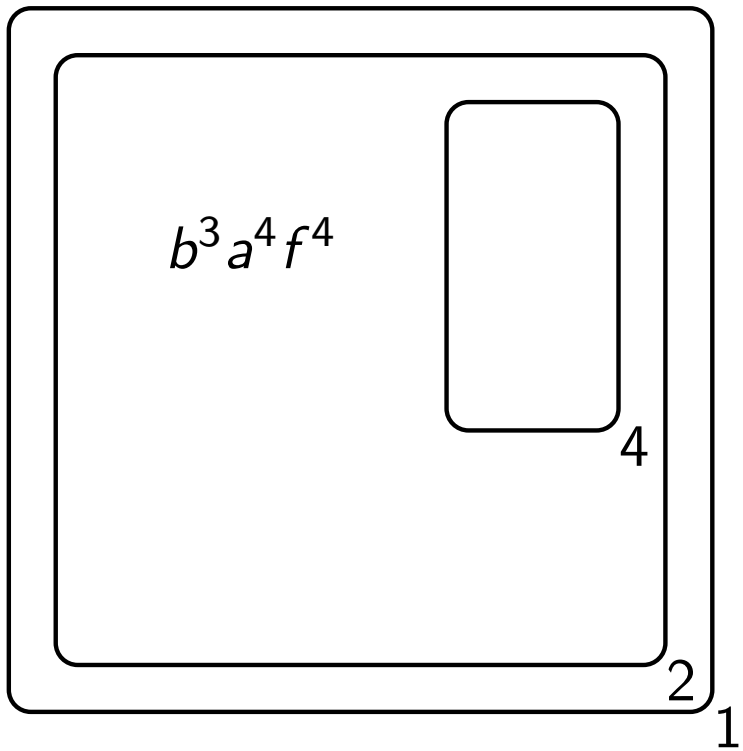

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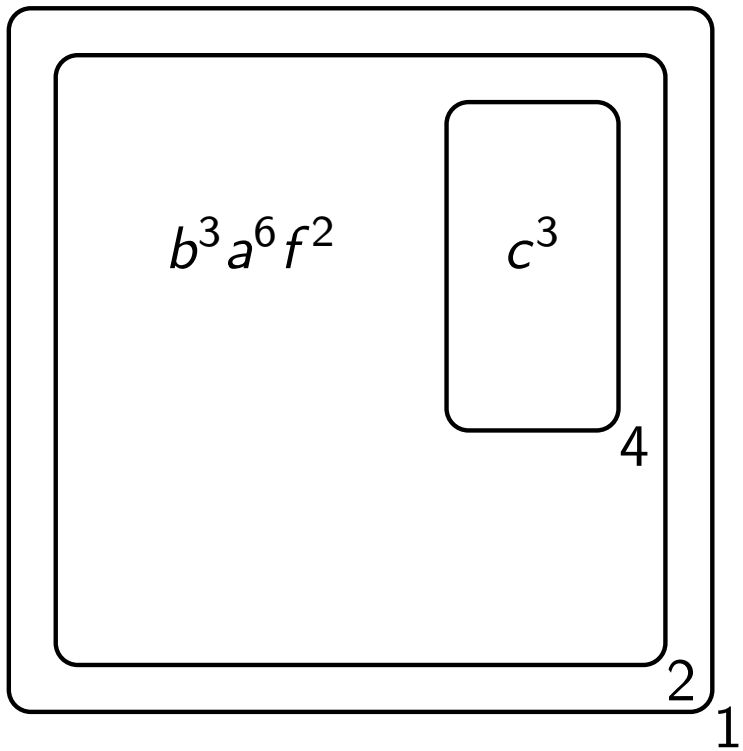

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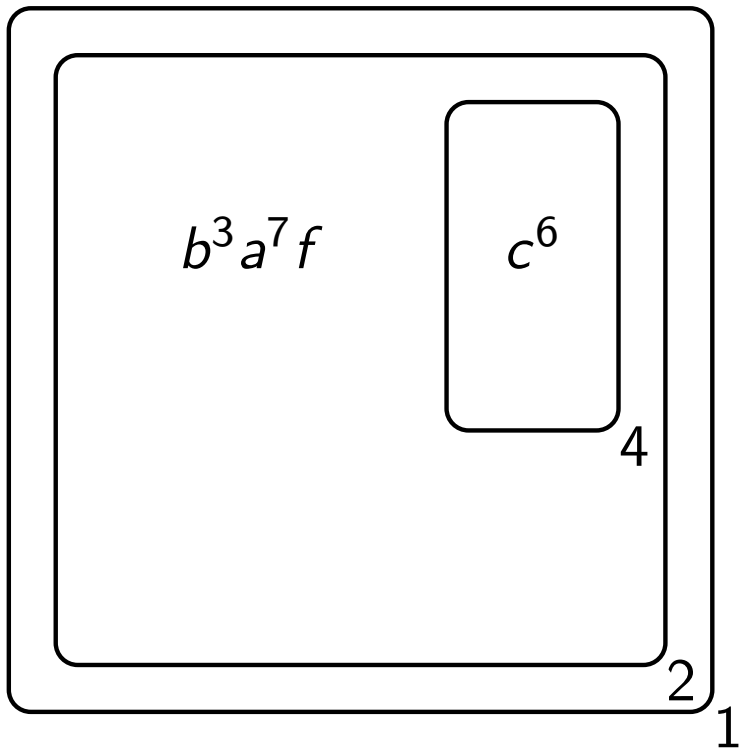

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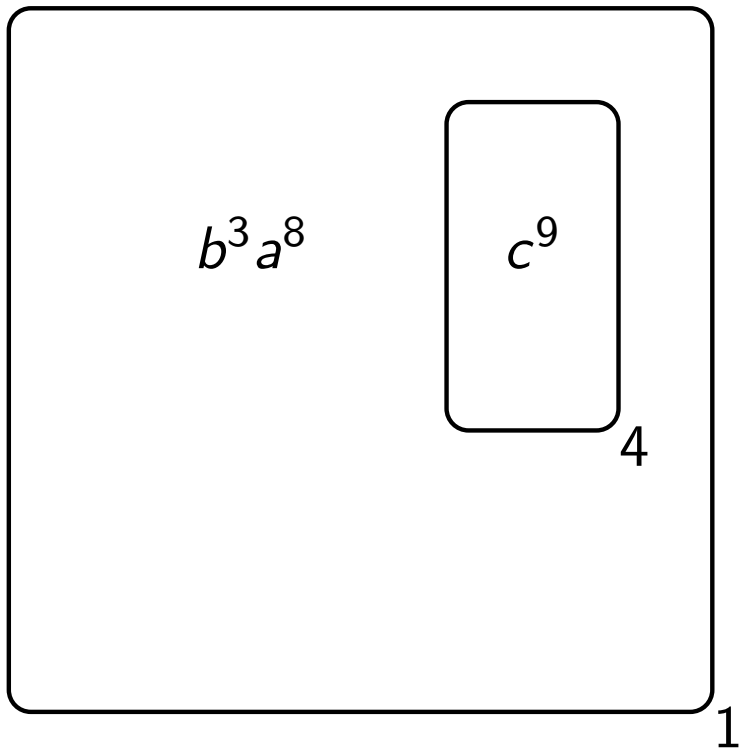

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- And... What about Membrane Creation?

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Previously, on The Computational Complexity Theory

- Subset Sum $\in \mathbf{PMC}_{\mathcal{MC}}$

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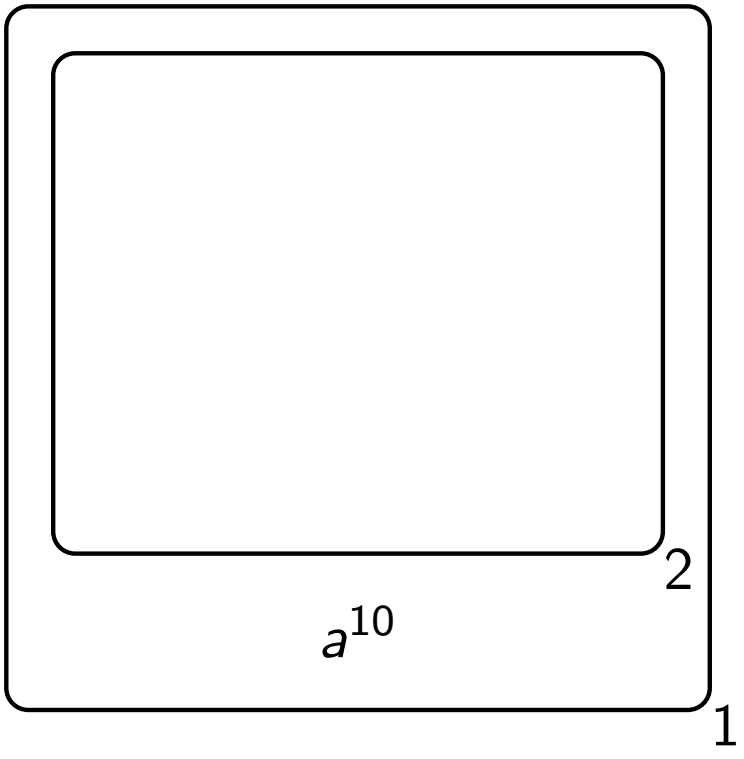

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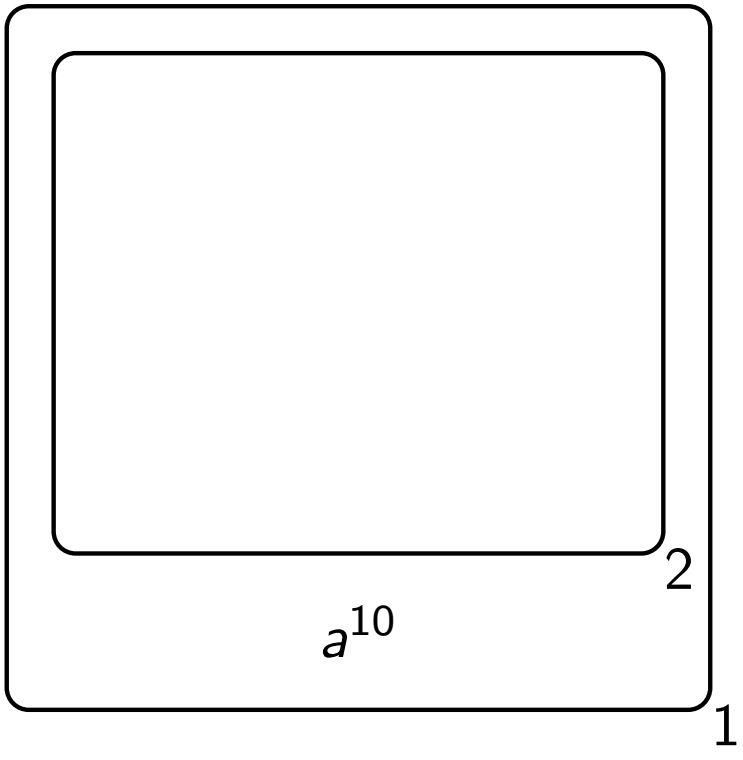

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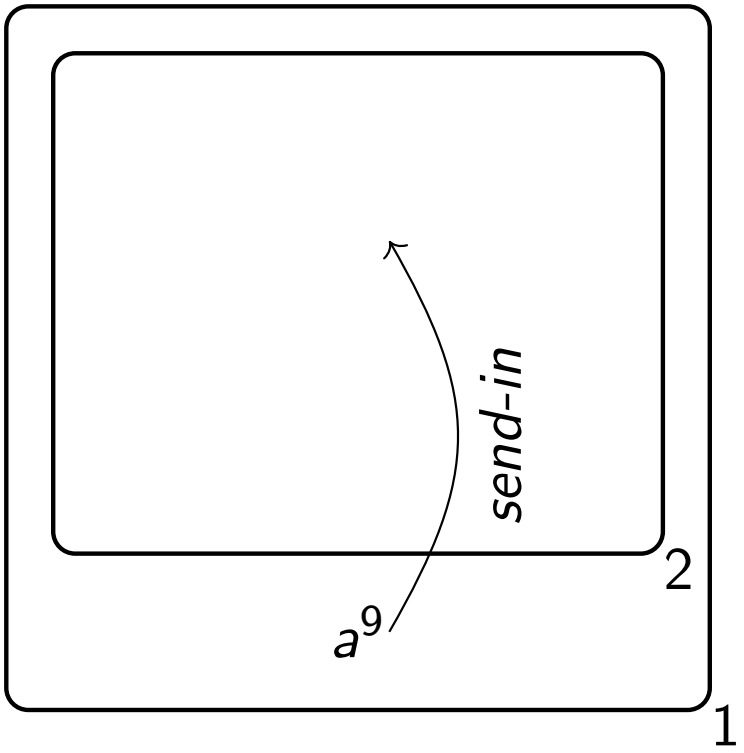

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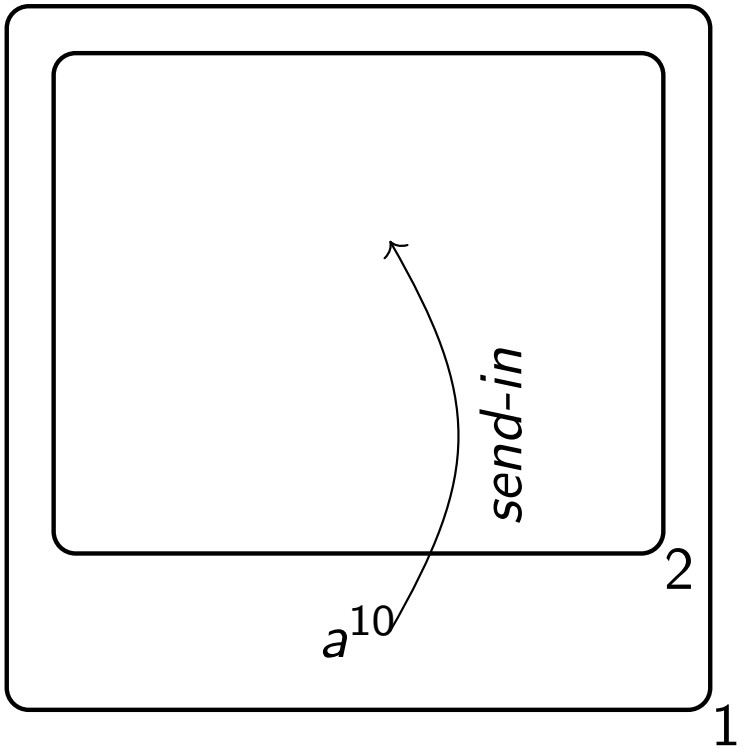

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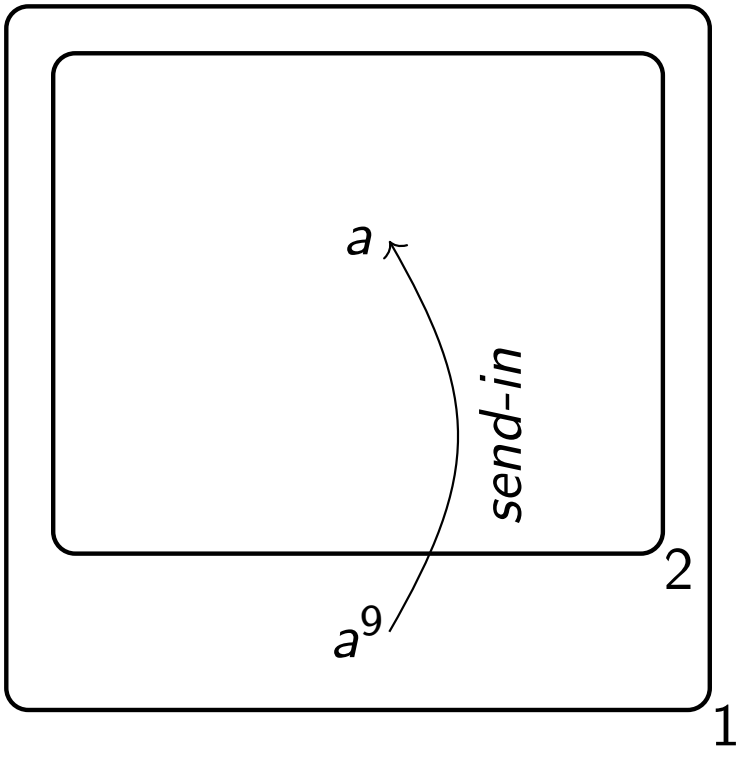

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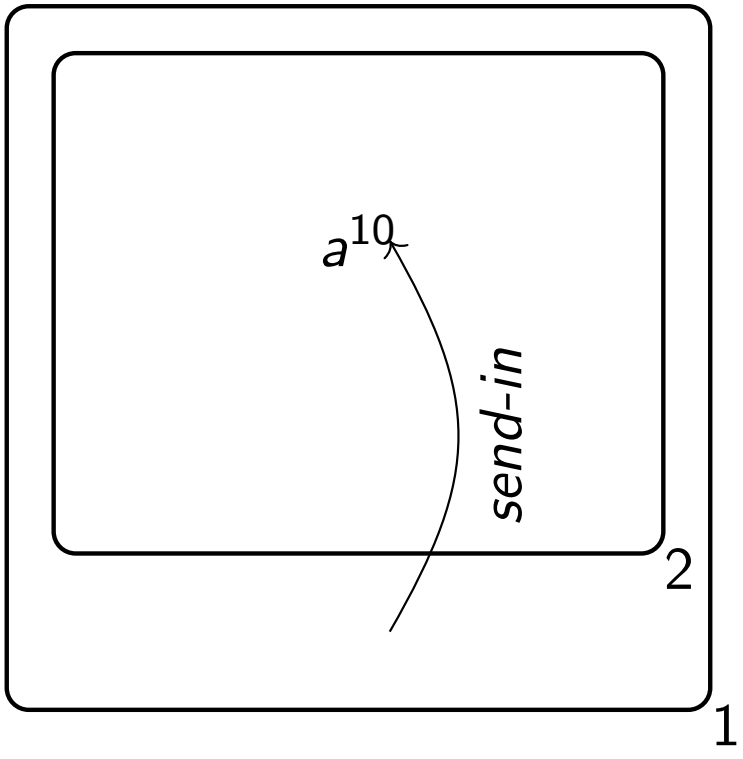

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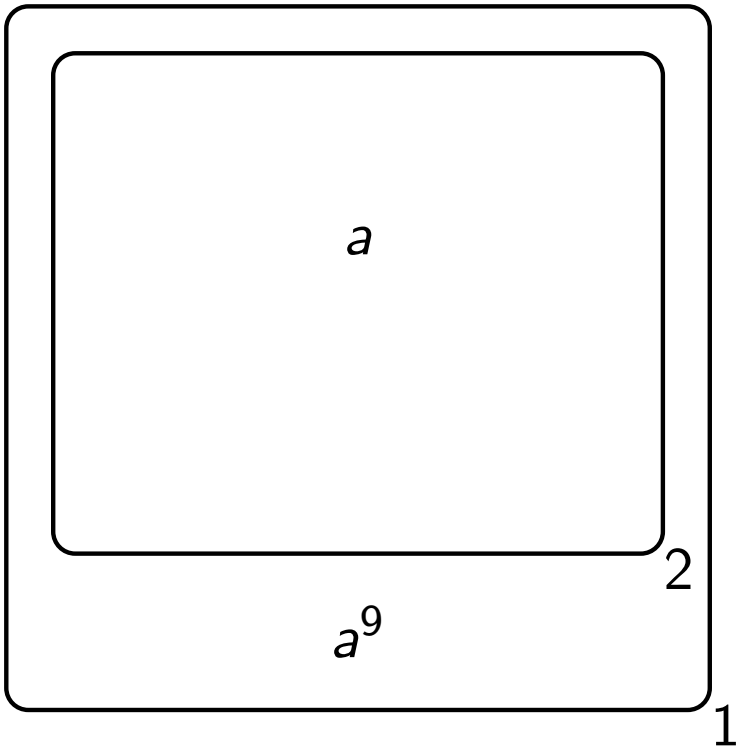

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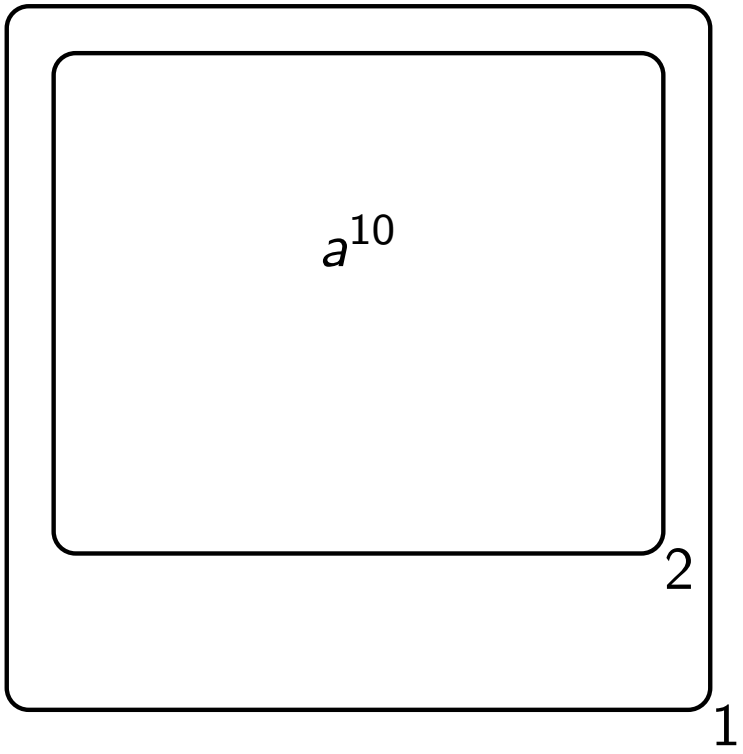

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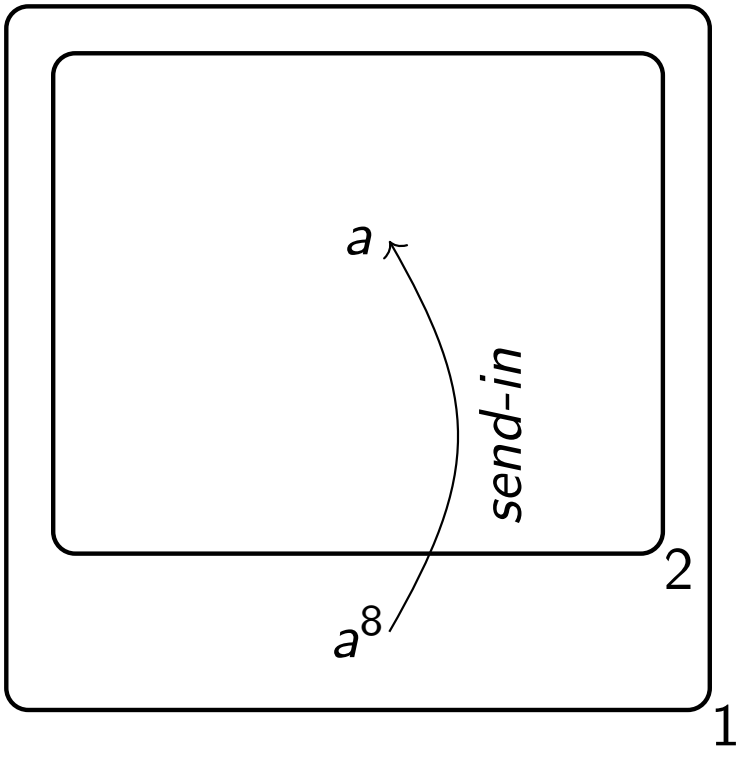

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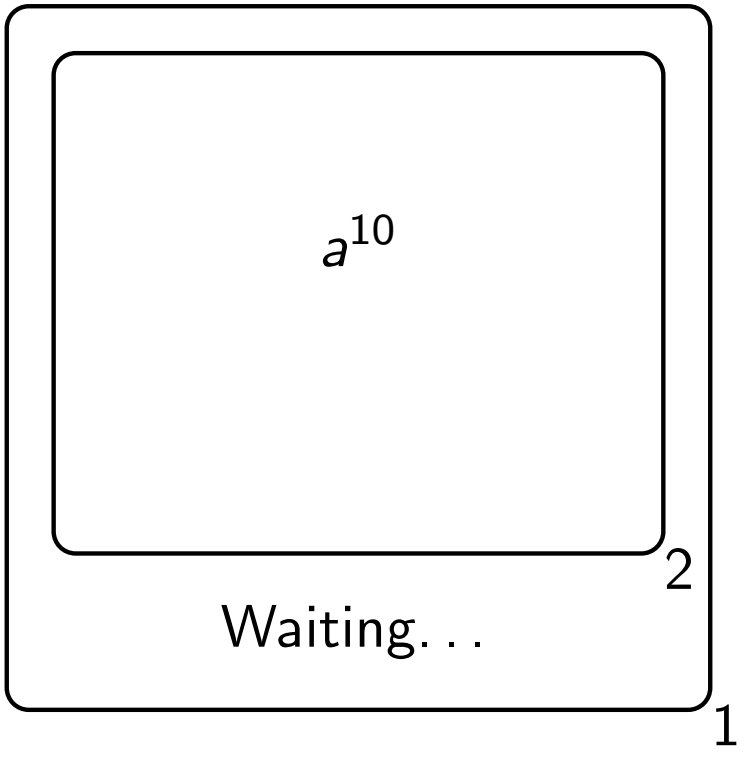

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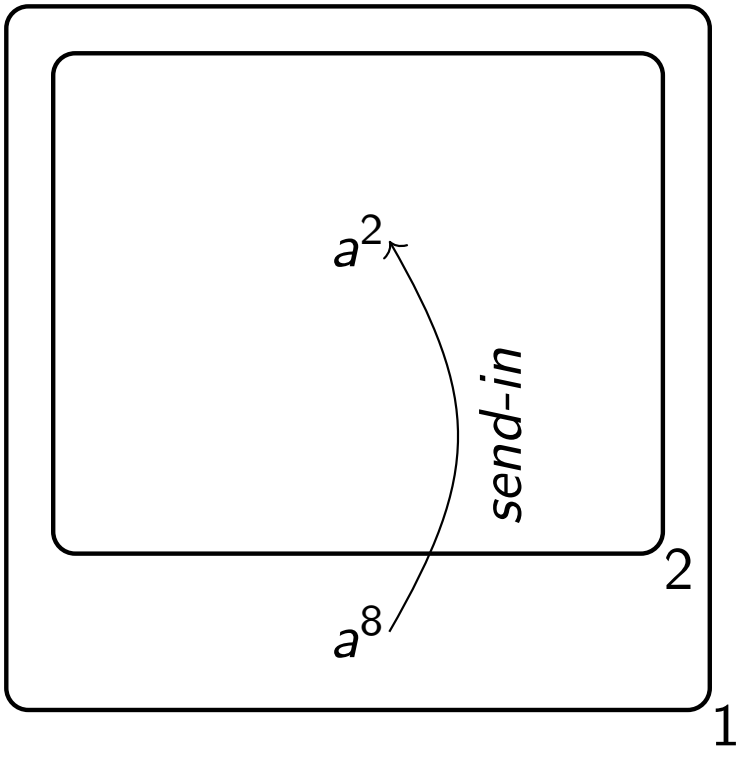

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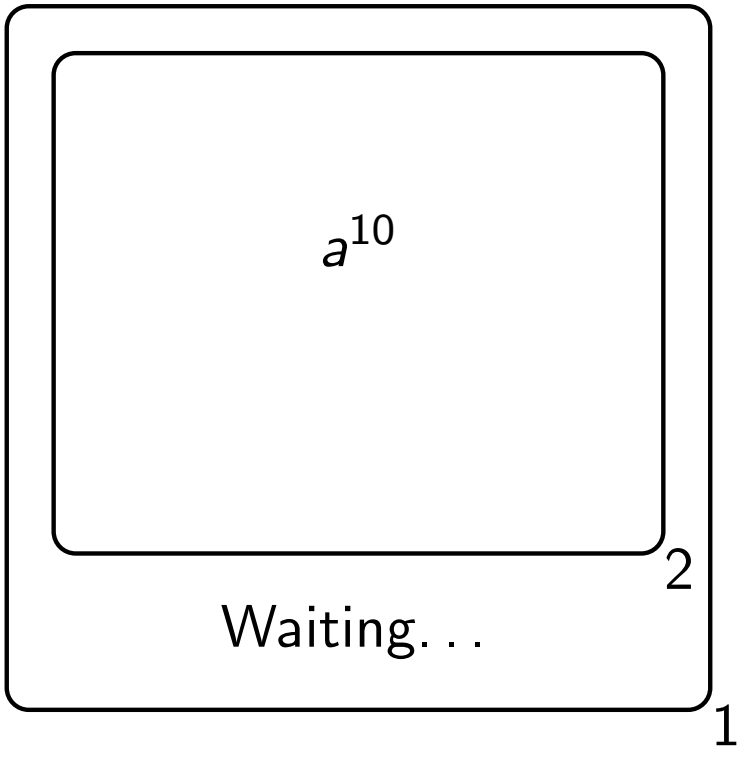

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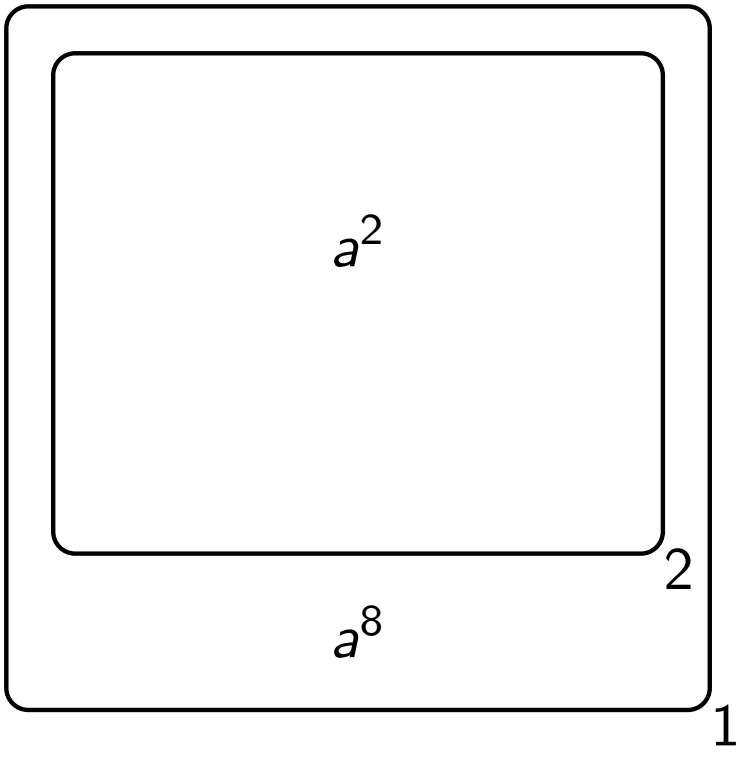

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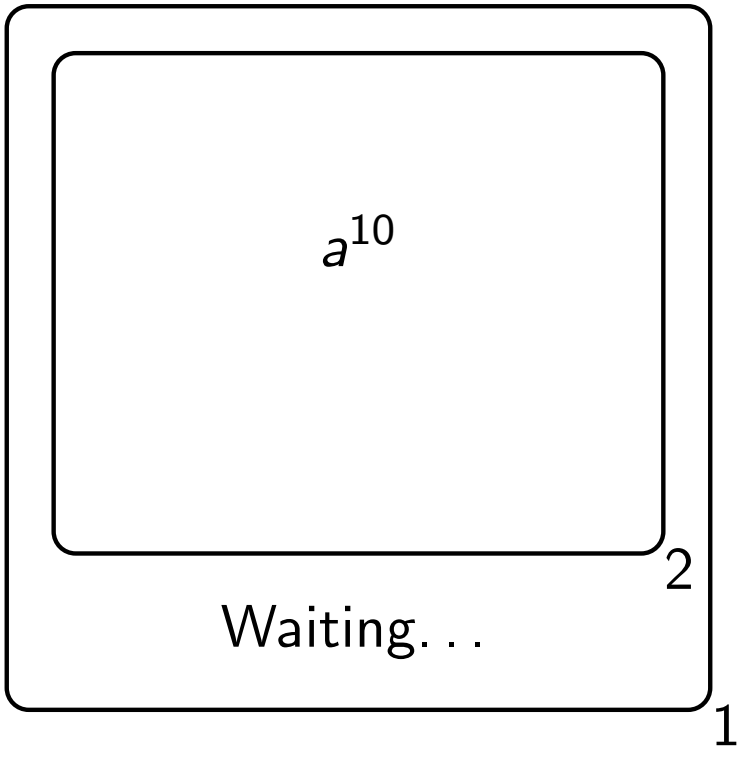

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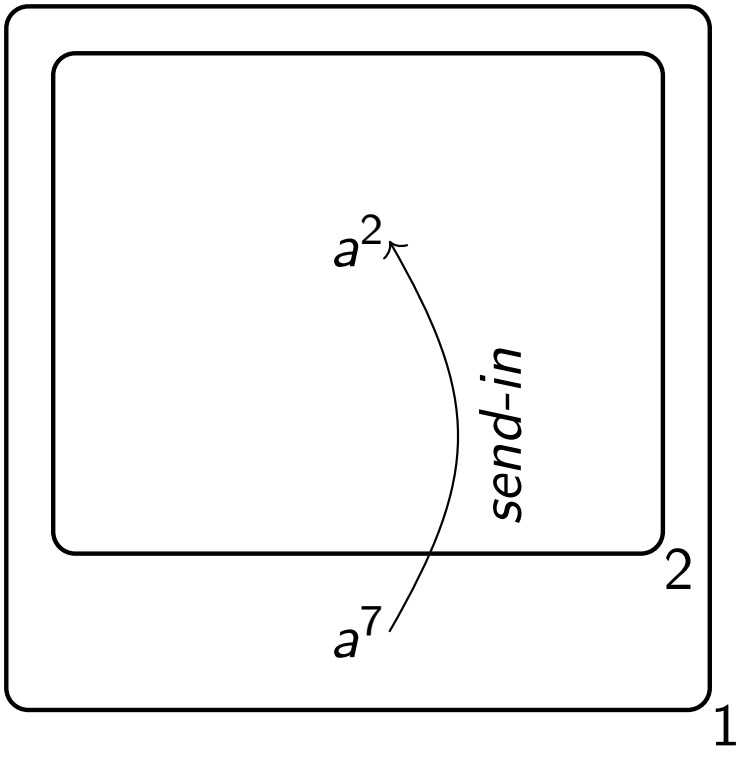

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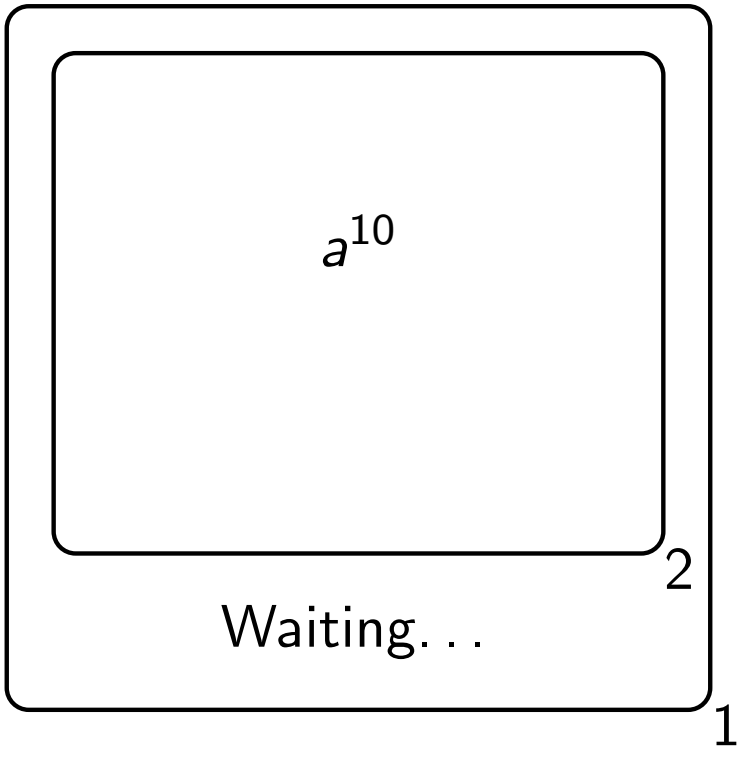

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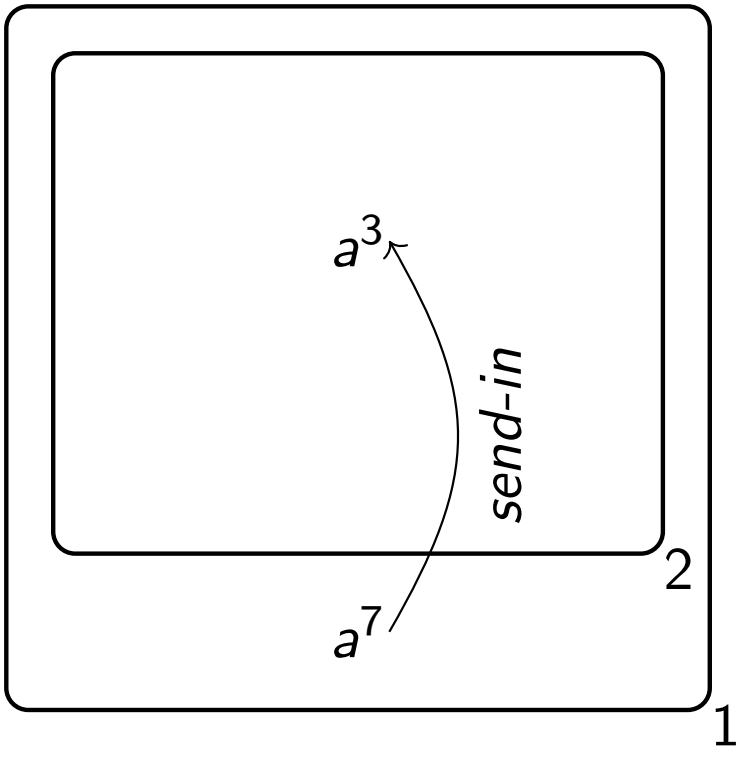

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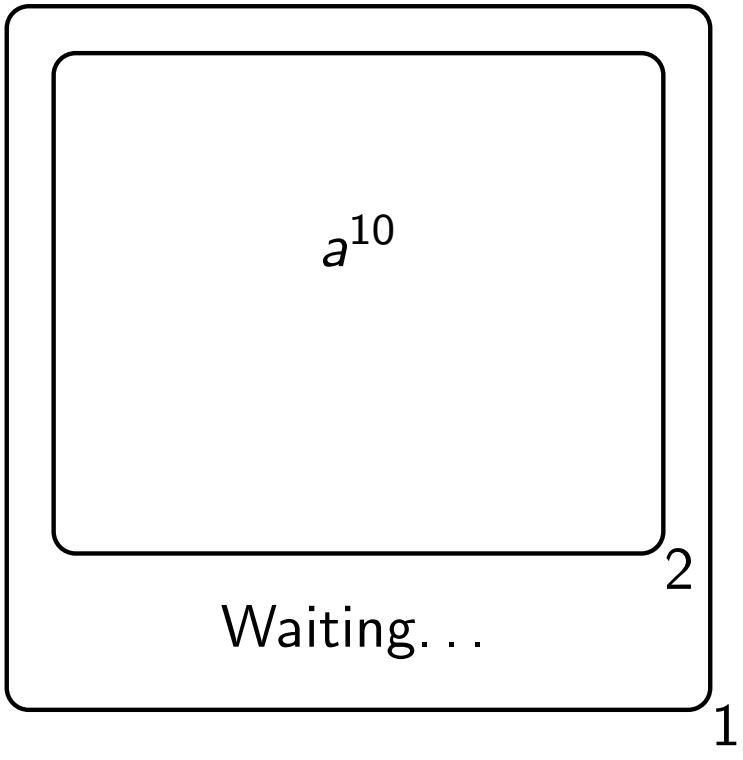

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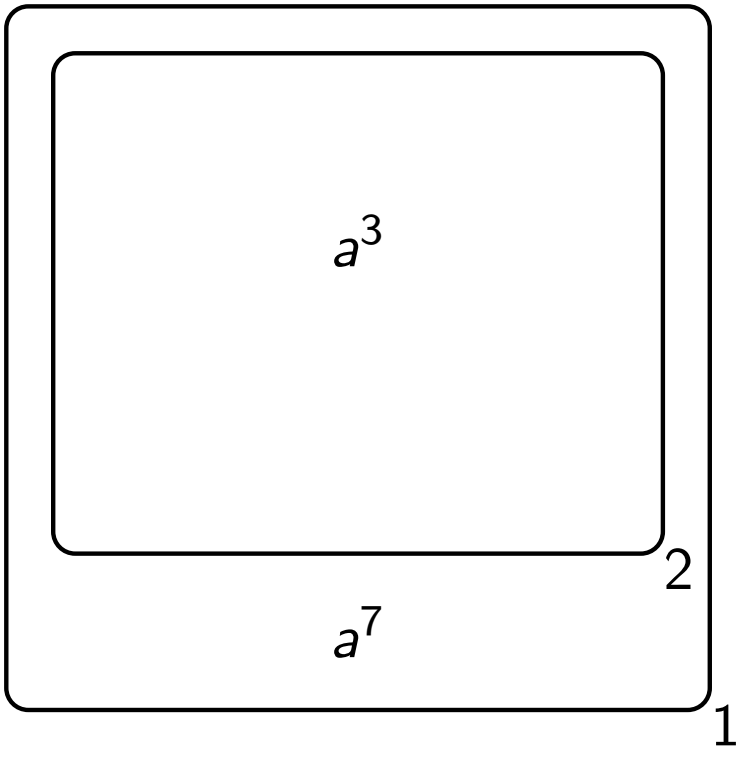

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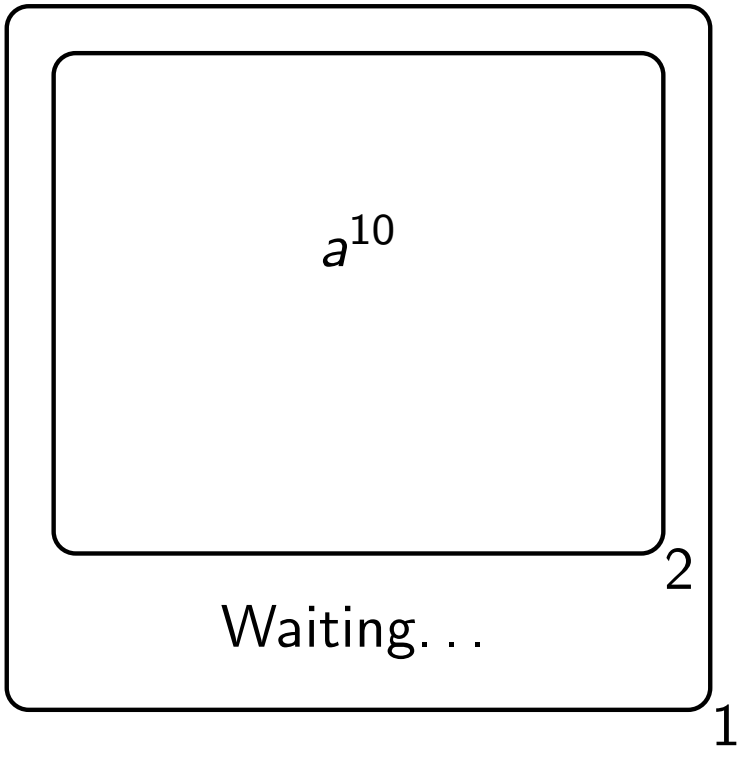

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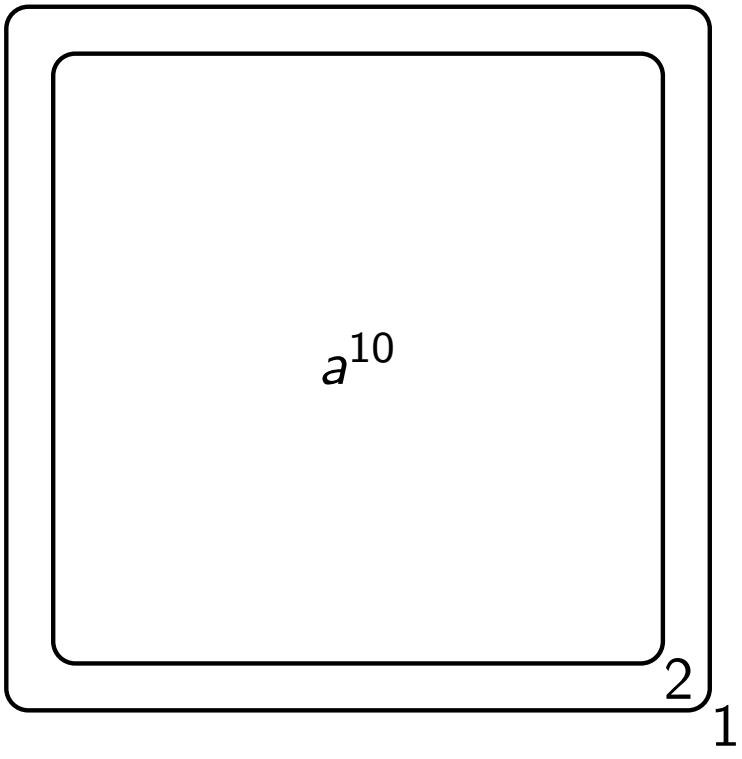

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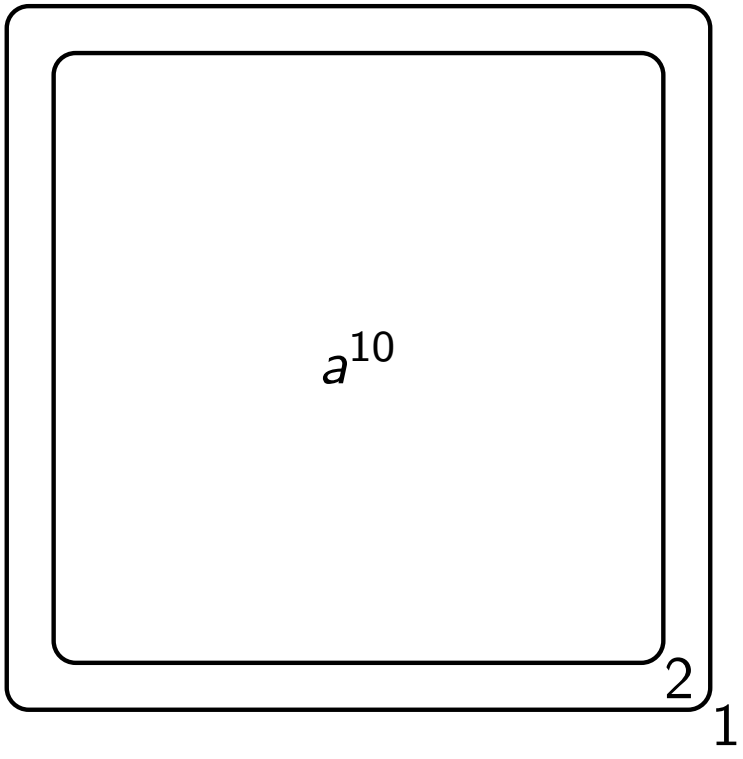

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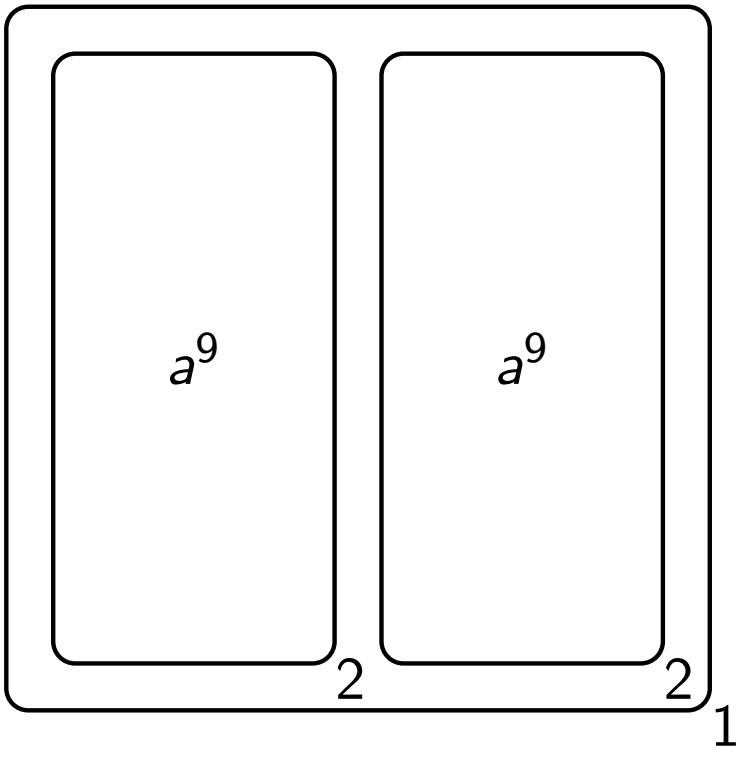

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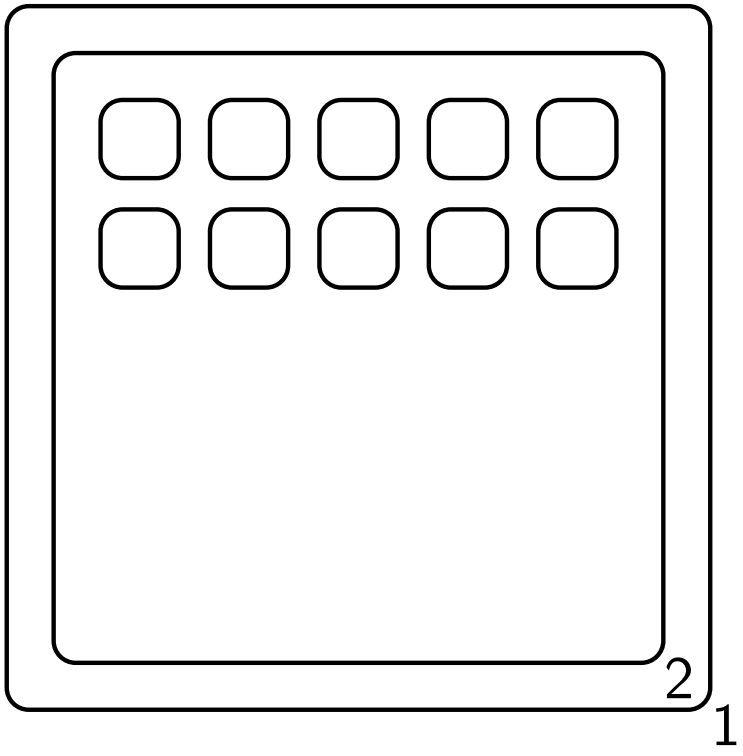

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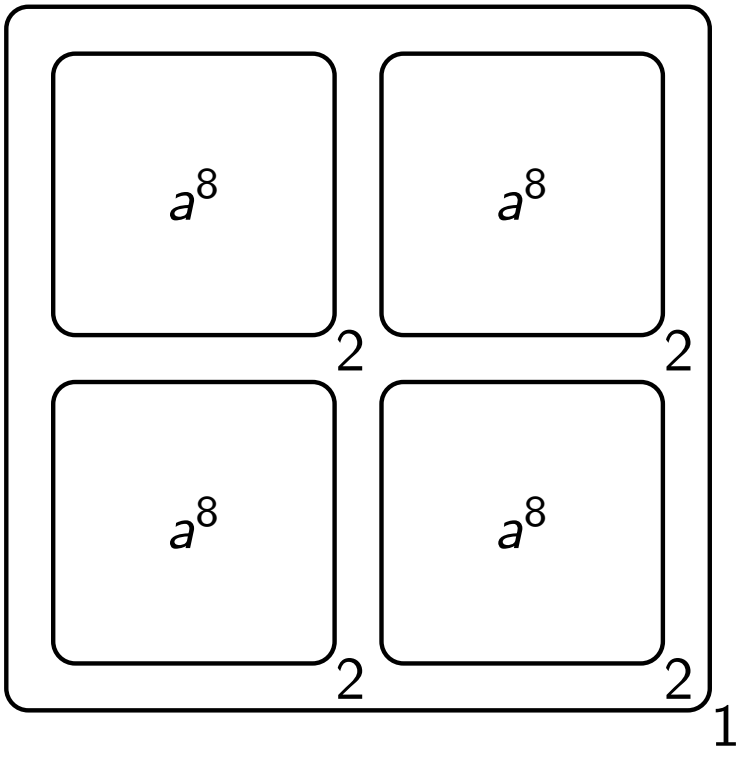

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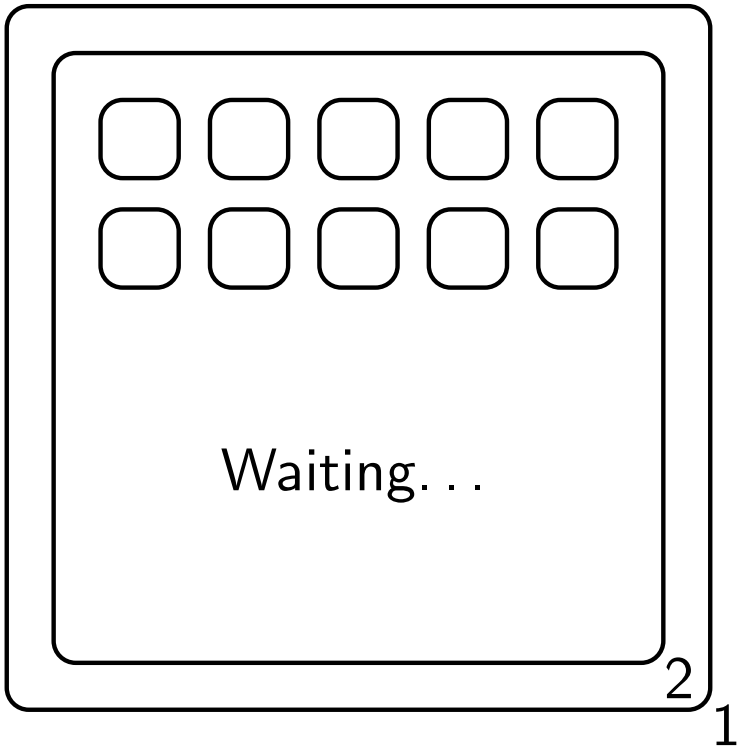

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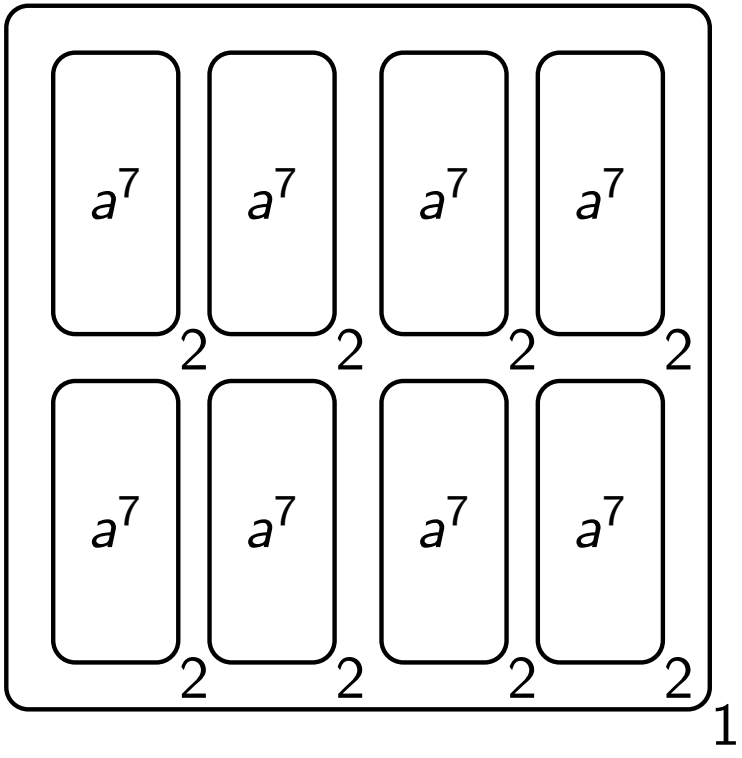
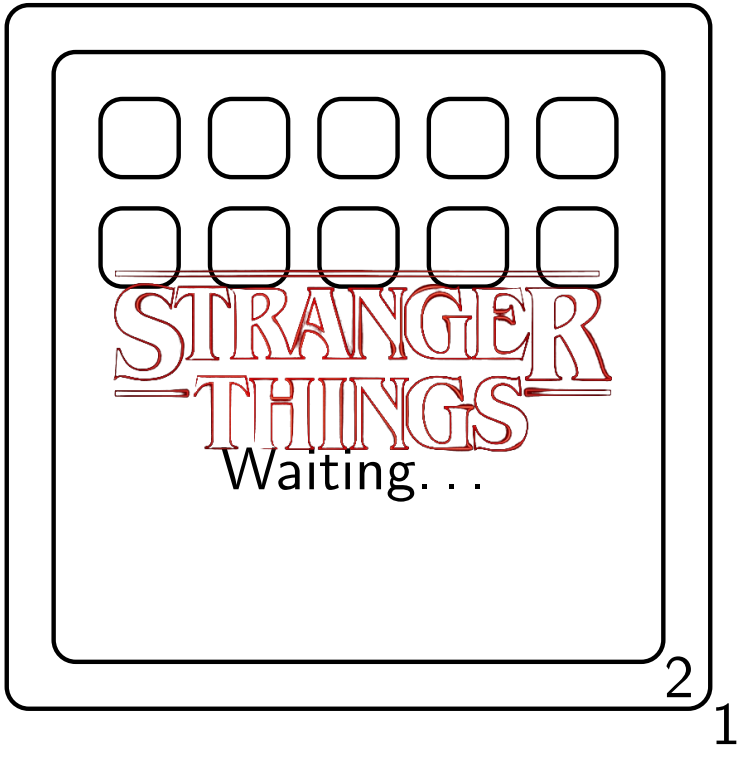


$\mathcal{DAM}^0$



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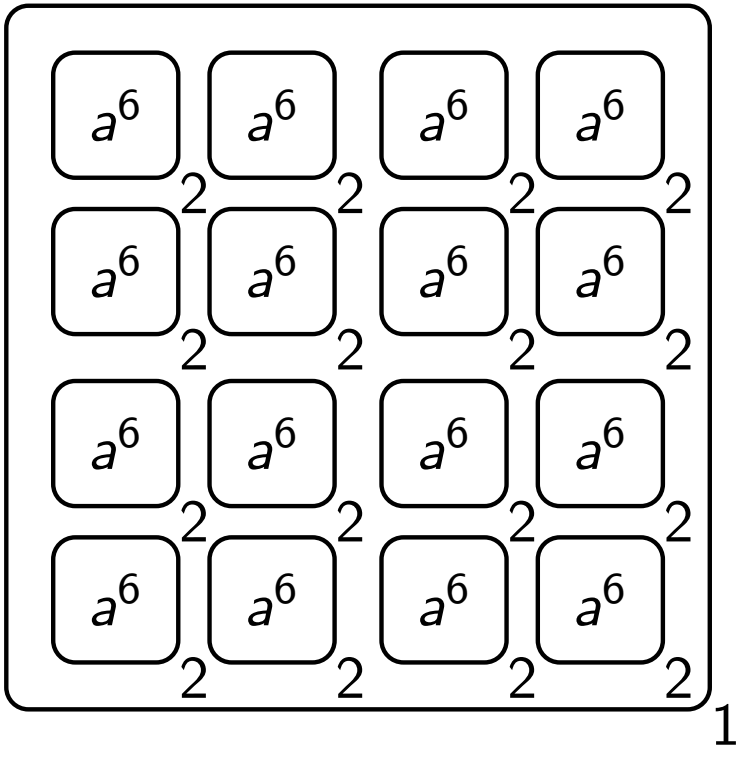

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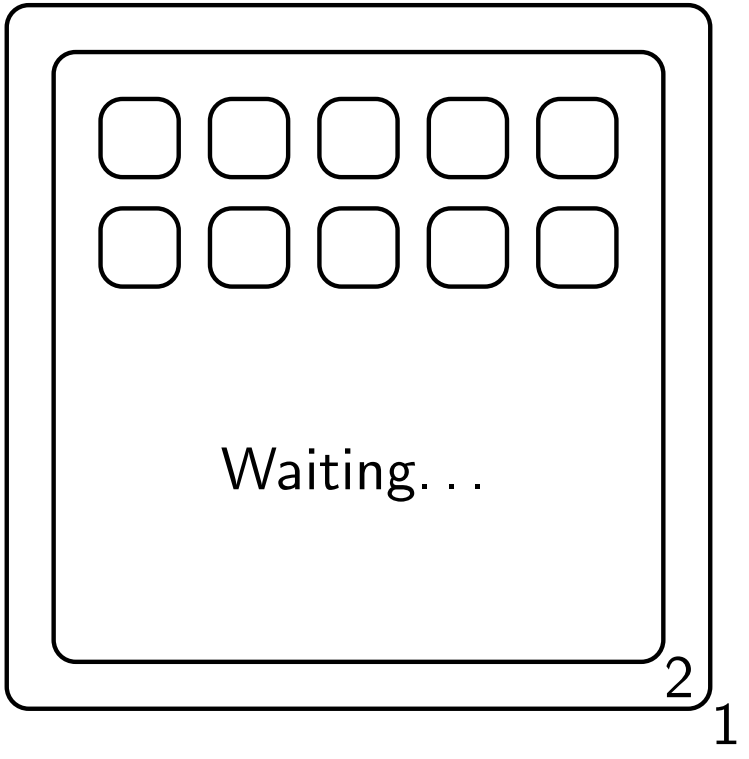

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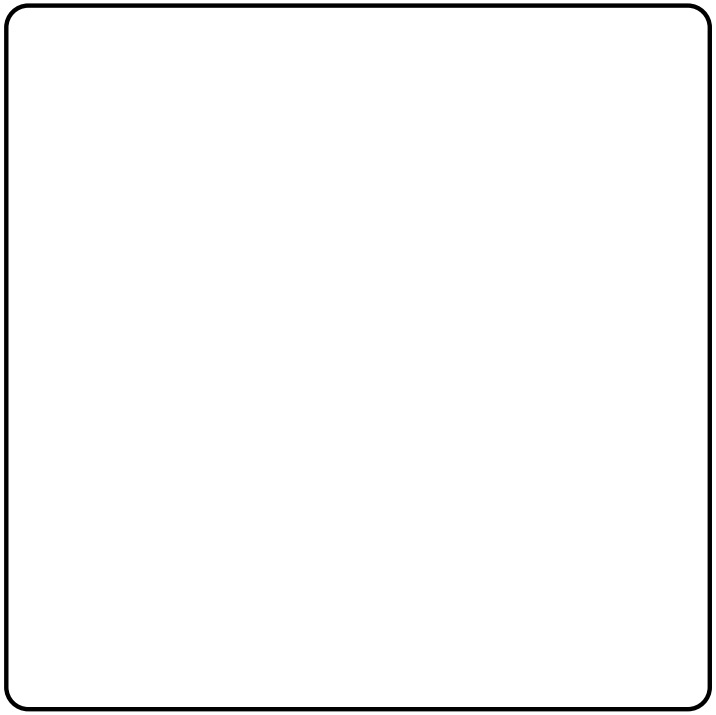

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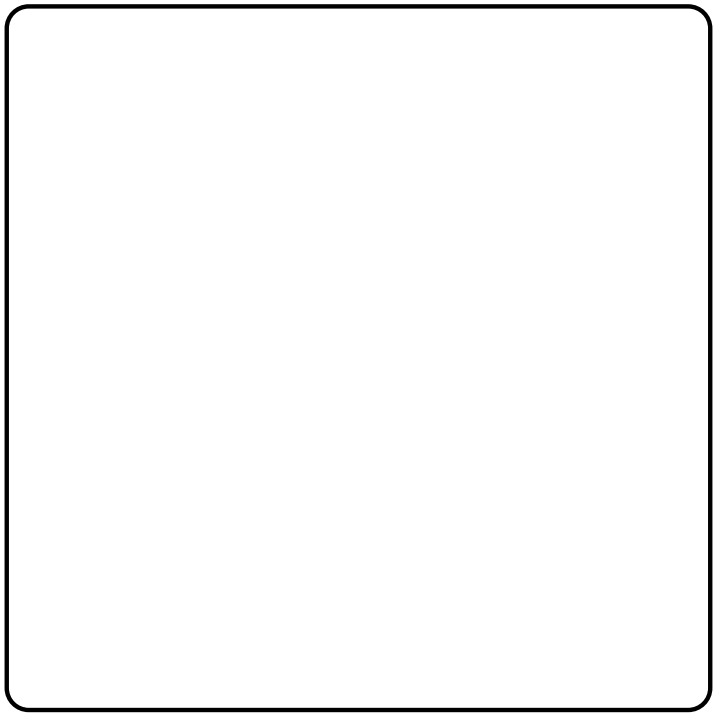
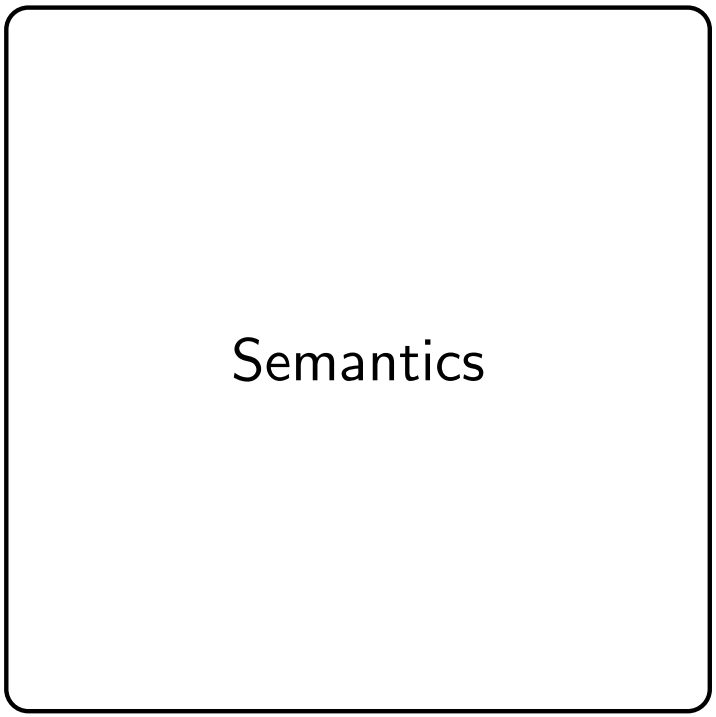
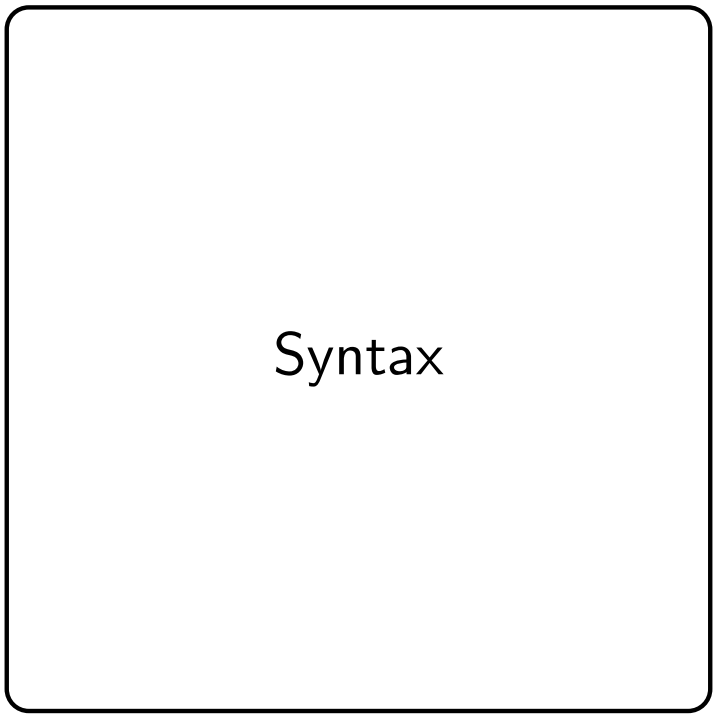


$\mathcal{CAM}^0_{max}$

Previously, on The Computational Complexity Theory





$\mathcal{DAM}^0$

$\mathcal{CAM}^0_{max}$

Teaching an old dog old tricks

- $\mathcal{CAM}^0_{min}(\delta, \gamma)$

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- $\mathcal{CAM}^0_{min}(\delta, \gamma)$
- $(a_0)\ [\, a \to u \,]_h$
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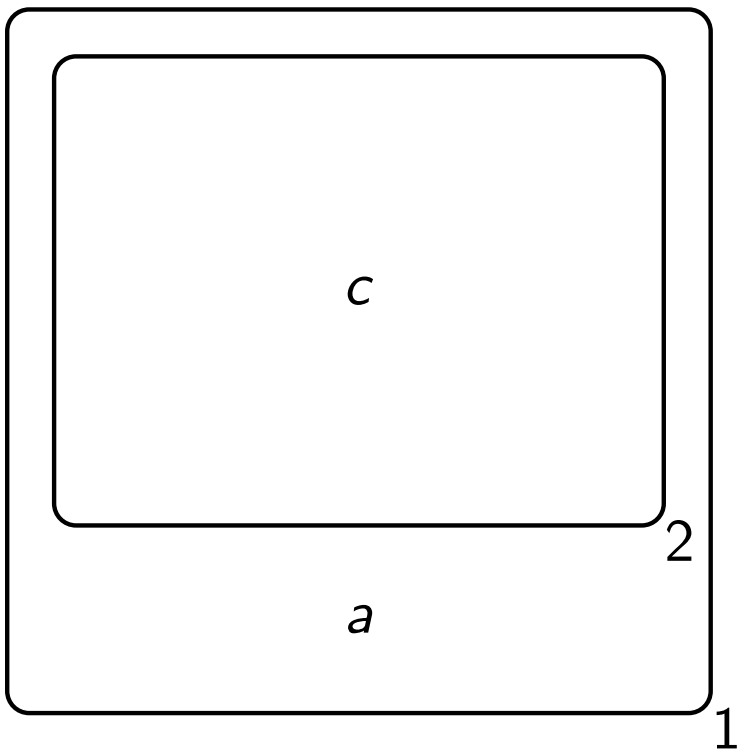
Teaching an old dog old tricks



$[\, c \rightarrow [\, de \,]_3 \,]_2$

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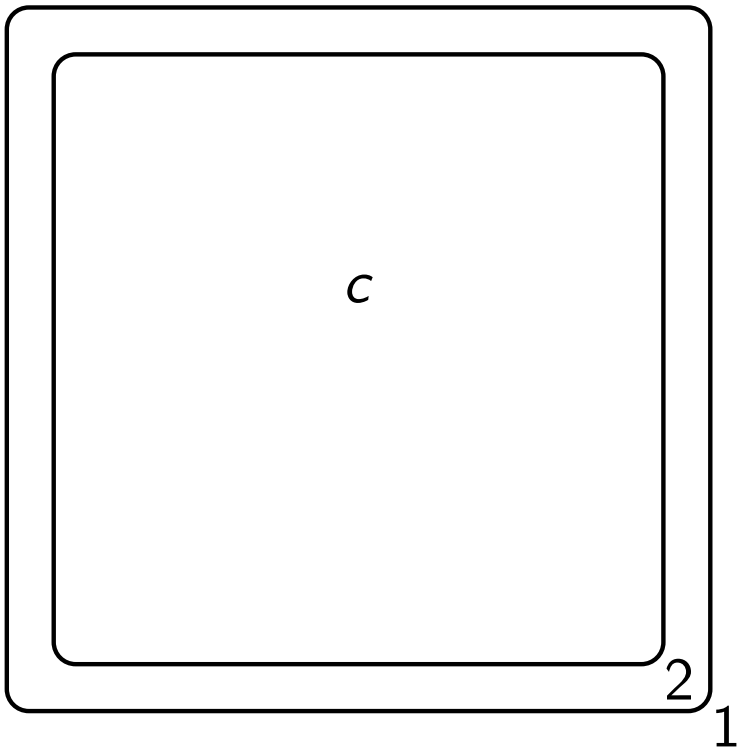

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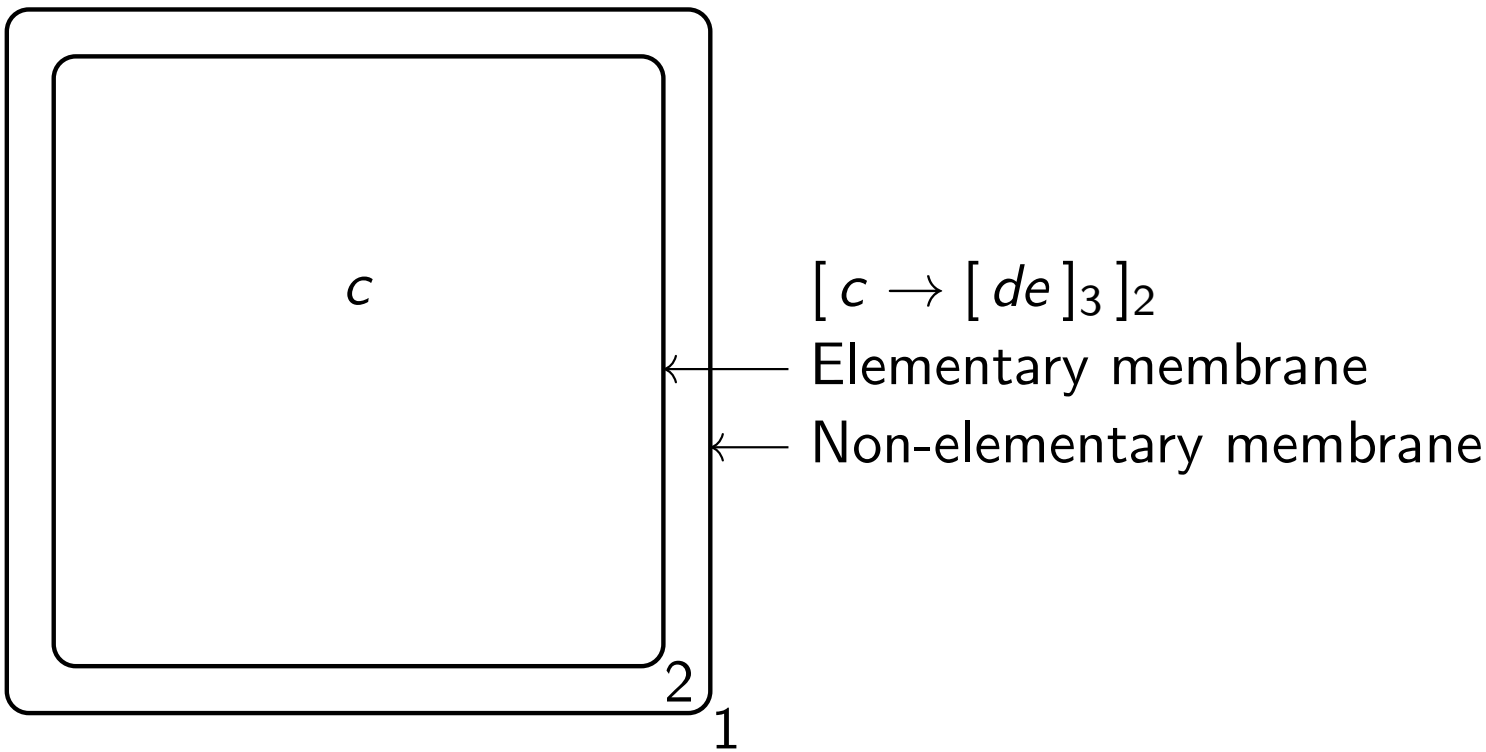

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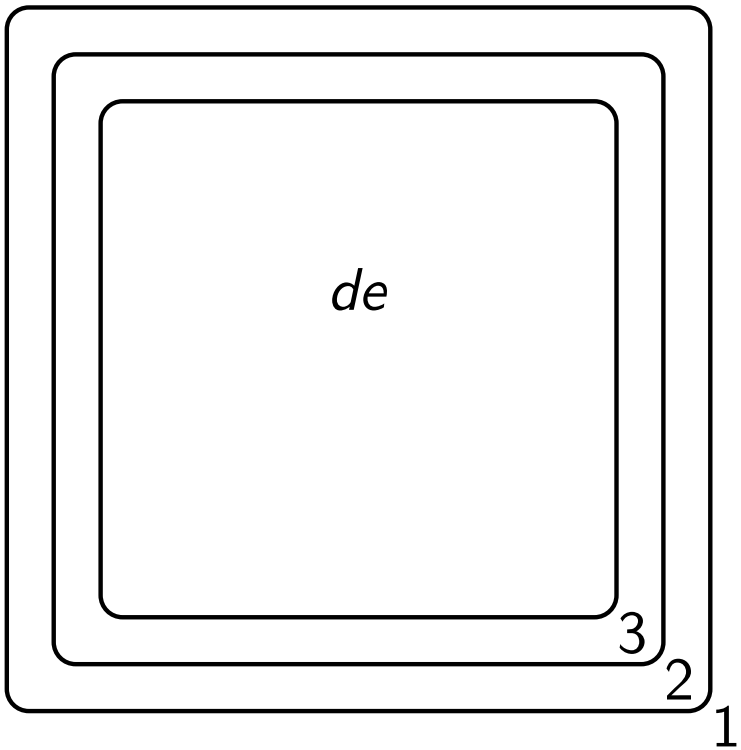

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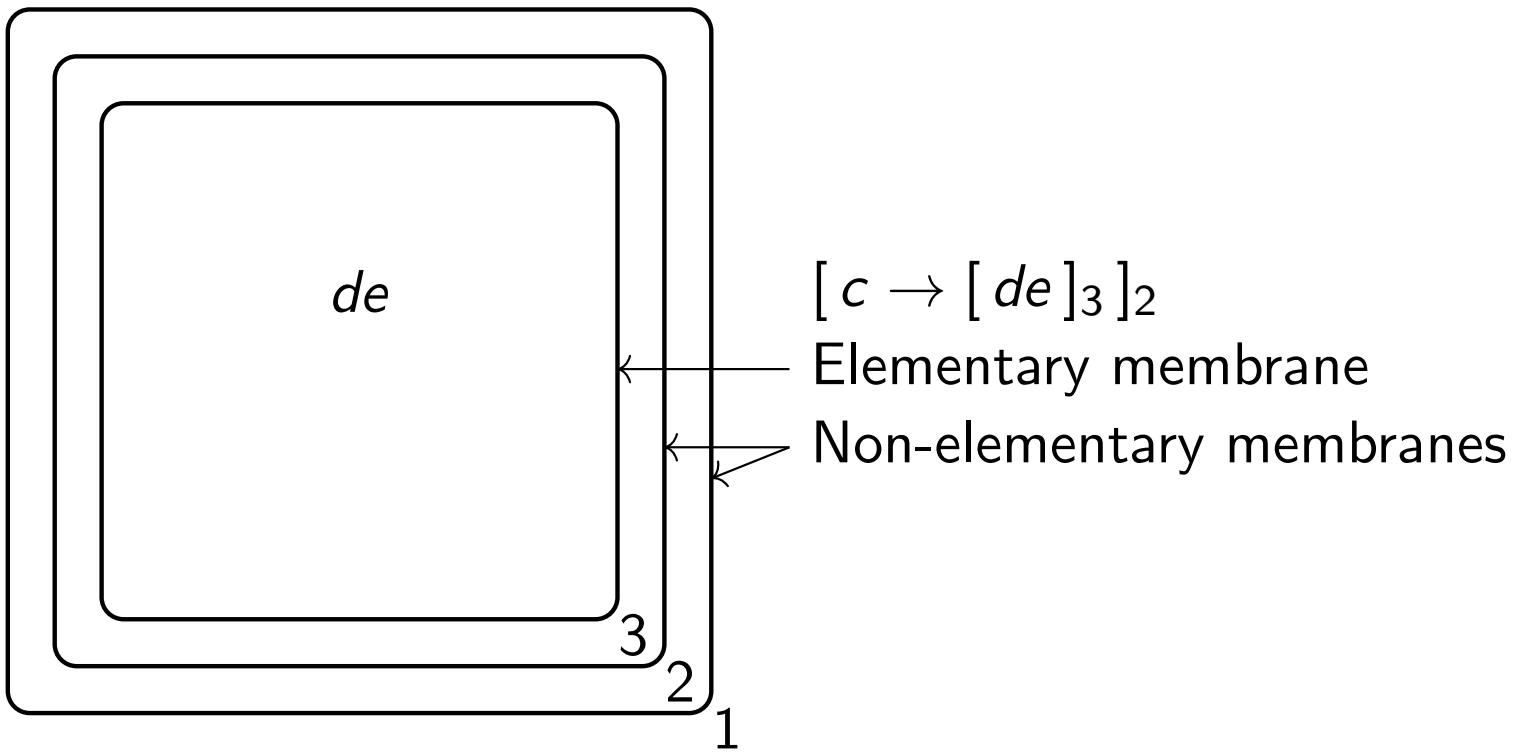

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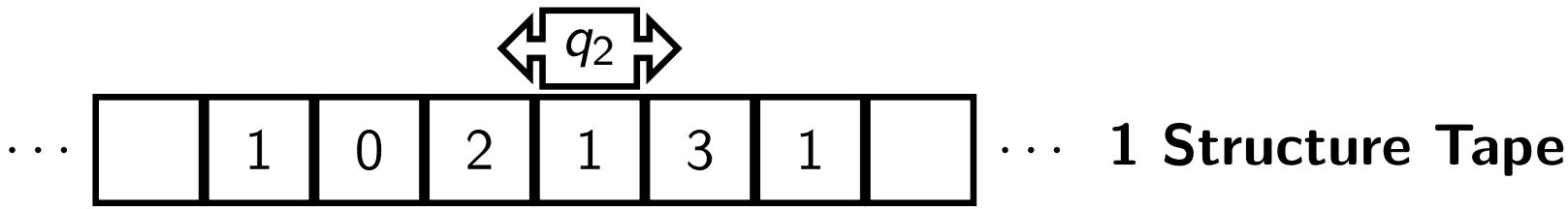
- Similar to $\mathbf{P} \subseteq \mathbf{PMC}_{\mathcal{T}}$ [2]

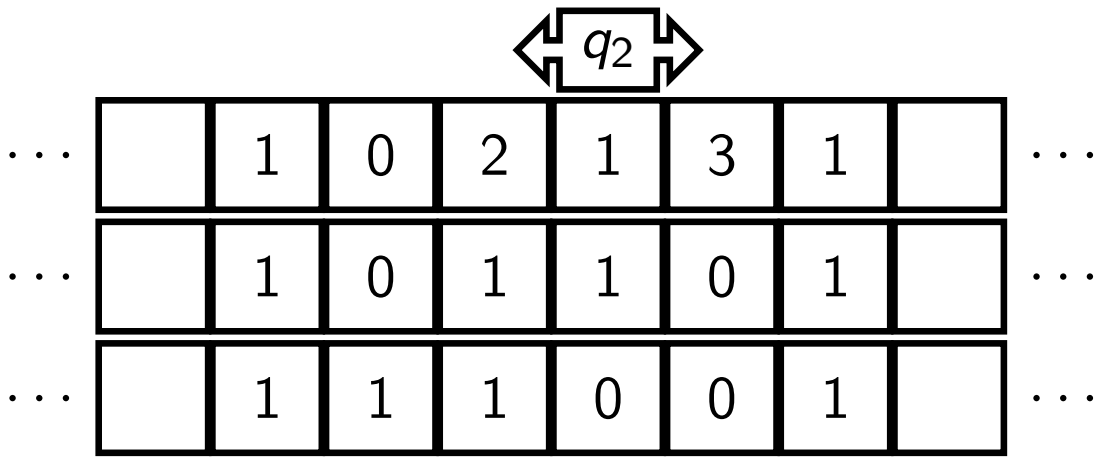
[2] M.Á. Gutíerrez-Naranjo, M.J. Pérez-Jiménez, A. Riscos-Núñez, F.J. Romero-Campero, Á. Romero-Jiménez. Characterizing tractability by cell-like membrane systems. In K.G. Subramanian, K. Rangarajan, M. Mukun (eds.) *Formal models, languages and applications*, World Scientific, Singapore, 2006, pp. 137–154.

Complexity results



1 Structure Tape

Complexity results



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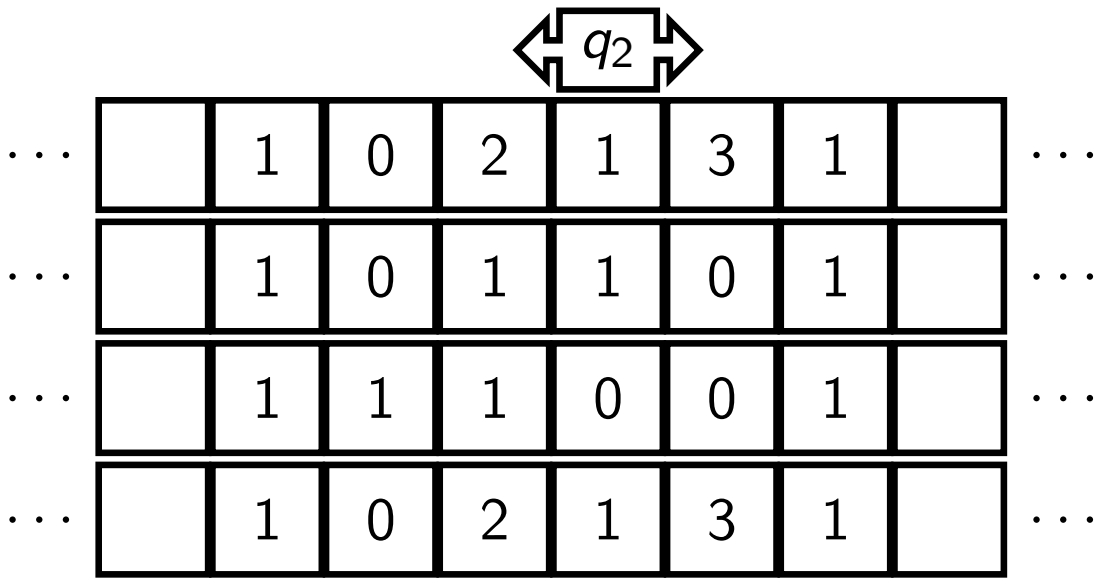
$O(|\Gamma| \cdot |H|)$ **Multipl. Tapes**

$O(|\Gamma| \cdot |H|)$ **Auxiliary Tapes**

Complexity results



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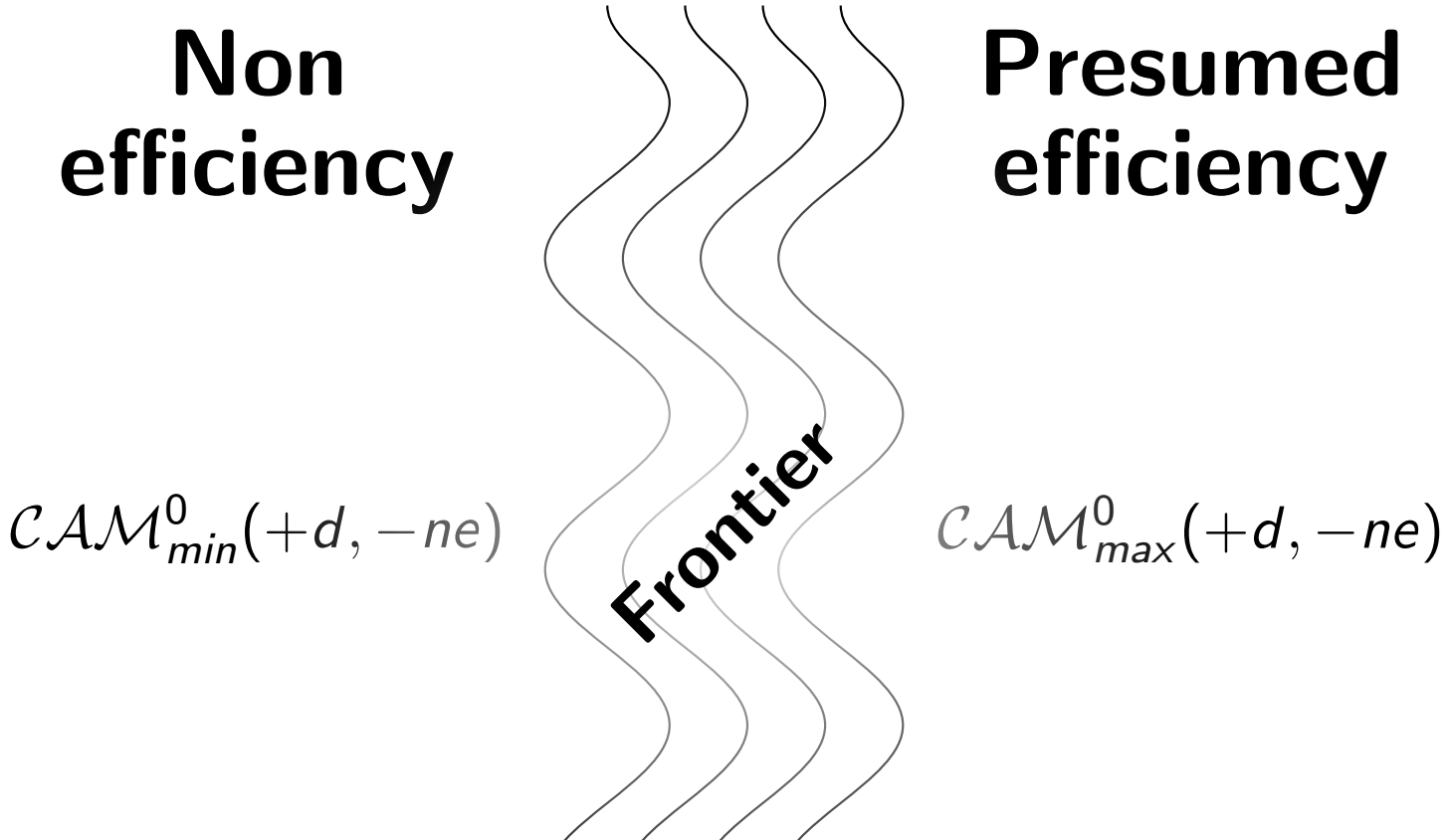

Future work

- **PMC**$_{\mathcal{CAM}^0_{min}(+d,+ne)}$

Future work

- $\mathbf{PMC}_{\mathcal{CAM}^0_{min}(+d,+ne)}$
- $\mathbf{PMC}_{\mathcal{CAM}^0_{min}(-d,+ne)}$

Future work

- $\mathbf{PMC}_{\mathcal{CAM}^0_{min}(+d,+ne)}$
- $\mathbf{PMC}_{\mathcal{CAM}^0_{min}(-d,+ne)}$
- $\mathbf{PMC}_{\mathcal{CAM}^0_{max}(-d,-ne)}$

DANKE

Gràcies Merci

THANKYOU

谢谢

Obrigado Gracias Dankon

MULTUMESC