From NP-completeness to DP-completeness: A membrane computing perspective

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- If $\mathbf{P} \neq \mathbf{NP}$ then every presumably efficient computing model is an efficient one.

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This lower bound for $\mathbf{PMC}_{\mathcal{R}}$ can be **improved**.

The complexity class DP (difference class)

Introduced by C.H. Papadimitriou and M. Yannakakis[1]

- ⋆ A language L is in the class **DP** iff there are two languages L_1 and L_2 such that $L_1, L_2 \in \mathbf{NP}$ and $L = L_1 \setminus L_2$.

Then, $L \in \mathbf{DP}$ iff there are $L_1 \in \mathbf{NP}$ and $L_2 \in \textbf{co-NP}$ such that $L = L_1 \cap L_2$.

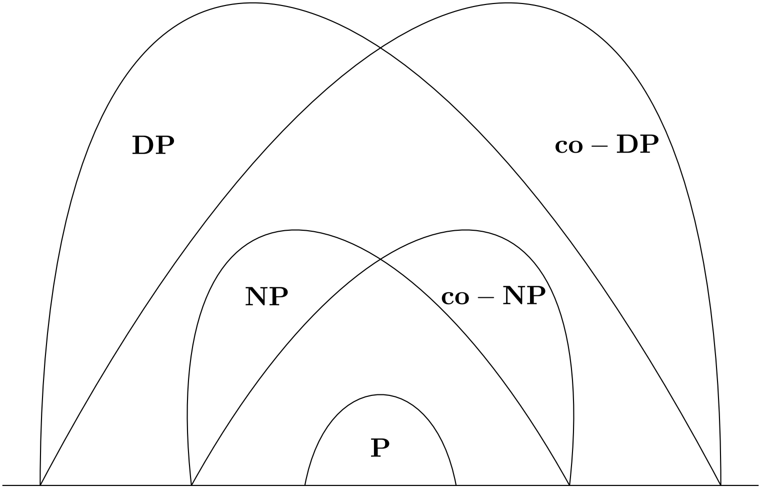
Class **DP**:

- ∗ lies between the first two levels of the **polynomial hierarchy**.
- ∗ is the second level in the **Boolean hierarchy**.

$\mathbf{NP} \subseteq \mathbf{DP} \subseteq \mathbf{P}^{\mathbf{NP}}$.

$\mathbf{NP} \cup \textbf{co-NP} \subseteq \mathbf{DP} \cap \textbf{co-DP}$.

[1] C.H. Papadimitriou, M. Yannakis. The complexity of facets (and some facets of complexity). **Proceedings of the 24th ACM Symposium on the Theory of Computing**, 1982, pp. 229-234.


DP
co – DP
NP
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P

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Proposition: If X_1 is an **NP** complete problem and X_2 is a **co-NP** complete problem then $X_1 \otimes X_2$ is a **DP** complete problem.

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Corollary: If X is an **NP** complete problem, then $X \otimes \overline{X}$ is a **DP** complete problem.

Main result

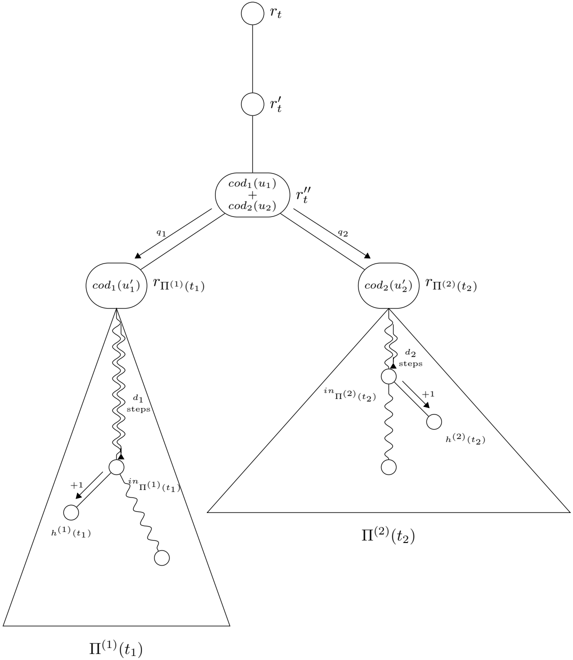
Let $\mathcal{R}$ be a computing model of recognizer non-cooperative P systems allowing **dissolution**, **object evolution** *and* **communication** *rules.*

- *If $X_1 \in \mathbf{PMC}_{\mathcal{R}}$ and $X_2 \in \mathbf{PMC}_{\mathcal{R}}$ then $X_1 \otimes X_2 \in \mathbf{PMC}_{\mathcal{R}}$.*

Sketch:
For $i = 1, 2$,

- Let $\mathbf{\Pi}^{(i)} = \{\Pi^{(i)}(t) \mid t \in \mathbf{N}\}$ a family of systems from $\mathcal{R}$ solving X_i in polynomial-time.
- Let (cod_i, s_i) be a polynomial encoding from X_i into $\mathbf{\Pi}^{(i)}$.

A family $\mathbf{\Pi} = \{\Pi(t) \mid t \in \mathbf{N}\}$ of membrane systems from $\mathcal{R}$ will be defined from $\mathbf{\Pi}^{(1)}$ and $\mathbf{\Pi}^{(2)}$, providing a uniform and polynomial-time solution to $X_1 \otimes X_2$.


r_t
r'_t
$cod_1(u_1) + cod_2(u_2)$
r''_t
q_1
q_2
$cod_1(u'_1)$
$r_{\Pi^{(1)}(t_1)}$
$cod_2(u'_2)$
$r_{\Pi^{(2)}(t_2)}$
d_1 steps
$+1$
$in_{\Pi^{(1)}(t_1)}$
$h^{(1)}(t_1)$
d_2 steps
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Proof: If X is an **NP**-complete problem such that $X \in \mathbf{PMC}_{\mathcal{R}}$, then $X \otimes \overline{X}$ is a **DP**-complete problem such that $X \otimes \overline{X} \in \mathbf{PMC}_{\mathcal{R}}$.

THANK YOU

FOR YOUR ATTENTION!