Membrane systems and time Petri nets

P. Battyányi[1], Gy. Vaszil[1]

[1]Department of Computer Science, Faculty of Informatics, University of Debrecen, Debrecen, Hungary

Outline

1. Time Petri nets
2. Connections of membrane systems and time Petri nets

Time Petri nets

- The Petri nets are state/transition systems: places are used to convey information and transitions represent events that can modify the information.

- A Petri net is a bipartite graphs: arcs point from places to transitions and from transitions to places. Every arc possesses a multiplicity, which is a positive integer.

Time Petri nets

- A transition is ready to fire, when each of its preplaces, that is the place endpoints of the incoming edges, contains as many tokens as the multiplicity of the arc coming from that preplace.

- Firing a transition means removing as many tokens from the preplaces as prescribed by the multiplicities of the incoming arcs and adding as many tokens to the postplaces as determined by the multiplicities of the outgoing arcs.

Example

In the figures below we illustrate a firing sequence of a Petri net[1]:

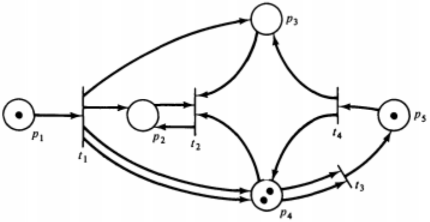

Figure 2.15 A marked Petri net to illustrate the firing rules. Transitions t_1, t_3, and t_4 are enabled.

[1] J. L. Peterson, *Petri Net Theory and the Modelling of Systems*, Prentice Hall, N.J., 1981.

Example

Continuing the example:

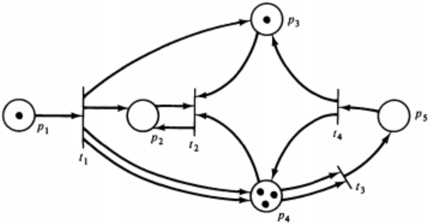

Figure 2.16 The marking resulting from firing transition t_4 in Figure 2.15.

Example

One step more:

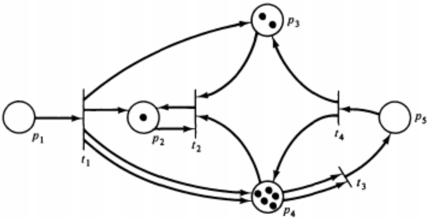

Figure 2.17 The marking resulting from firing transition t_1 in Figure 2.16.

Time Petri nets

Formally: a Petri net is a tuple $N = (P, T, F, V, m_0)$ such that

- P, T, F are finite, where $P \cap T = \emptyset$, $P \cup T \neq \emptyset$ and $F \subseteq (P \times T) \cup (T \times P)$,
- $V : F \to \mathbb{N}_{>0}$,
- $m_0 : P \to \mathbb{N}$.

The elements of P are called places and the elements of T are called transitions. The elements of F are the arcs and F is the flow relation of N. The function V is the multiplicity (weight) of the arcs and m_0 is the initial marking. We may occasionally omit the initial marking and simply refer to a Petri net as the tuple $N = (P, T, F, V)$. We stipulate that, for, every transition t, there is a place p such that $V(p, t) \neq 0$.

Time Petri nets

A Time Petri net (TPN)[2] is a 6-tuple $N = (P, T, F, V, m_0, I)$ such that

- the 5-tuple $S(N) = (P, T, F, V, m_0)$ is a Petri net,
- $I : T \to Q_{\geq 0} \times Q_{\geq 0}$ and, for each $t \in T$, $I(t)_1 \leq I(t)_2$ holds, where $I(t) = [I(t)_1, I(t)_2]$.

We call $I(t)_1$ and $I(t)_2$ earliest and latest firing times belonging to t, respectively. Notation: $eft(t)$, $lft(t)$.

[2] L. Popova-Zeugmann, *Time and Petri Nets*, Springer Verlag, Berlin, 2013.

Time Petri nets

- A transition marking with respect to the strong semantics (or t-marking) is a function $h : T \to \mathbb{R}_{\geq 0} \cup \{\#\}$.
- Let $N = (P, T, F, V, m_o, I)$ be a Time Petri net, m a p-marking and h a t-marking in N. A state in N is a pair $u = (m, h)$ such that
  - $(\forall t \in T)(t^- \not\leq m \supset h(t) = \#)$,
  - $(\forall t \in T)(t^- \leq m \supset h(t) \in \mathbb{R}_{\geq 0} \wedge h(t) \leq lft(t))$.
- A transition marking with respect to the weak semantics is a function $t : T \to \mathbb{R}_{\geq 0}$. A state in N concerning the weak semantics is a pair $u = (m, h)$.

Time Petri nets

Let t be a transition and $u = (m, h)$ be a state such that $u \longrightarrow^{t}$. Then the result of the firing of t is a new state $u' = (m', h')$, such that $m' = m + \triangle t(p)$, where $\triangle t(p) = t^{+}(p) - t^{-}(p)$, and

$$h'(s) \;=\; \begin{cases} h(s), & \text{if } s^{-}(p) \leq m(p),\; s^{-}(p) \leq m'(p) \text{ for all } p \in {}^{\bullet}s, \\ 0, & \text{if } s^{-}(p) > m(p) \text{ for some } p \in {}^{\bullet}s, \text{ but} \\ & \quad s^{-}(p) \leq m'(p) \text{ for all } p \in {}^{\bullet}s, \\ \#, & \text{if } s^{-}(p) > m'(p) \text{ for some } p \in {}^{\bullet}s. \end{cases}$$

Therefore, the firing of a transition has multiple effects. It changes the p-marking of the system as in simple Petri nets, but the time values attached to the transitions may also change. If a transition $s \in T$ which was enabled before the firing of t remains enabled after the firing, then the value $h(s)$ remains the same. If an $s \in T$ is newly enabled with the firing of transition t, then we have $h(s) = 0$. Finally, if s is not enabled after firing of transition t, then $h(s) = \#$.

Time Petri nets

Besides the firing of a transition there is another possibility for a state to alter, and this is the time delay step. Let t be a transition and $u = (m, h)$ be a state and $\tau \in \mathbb{R}^+$. Then elapsing of time with τ is possible in the strong semantics for the state u (in notation: $u \longrightarrow^{\tau}$), if

$$(\forall t \in T)(h(t) \neq \# \supset h(t) + \tau \leq lft(t)).$$

The result of the elapsing of time by τ is defined as follows: $u \longrightarrow^{\tau} u' = (m', h')$, where $m = m'$ and

$$h'(t) = \begin{cases} h(t) + \tau, & \text{if } t + \tau \leq lft(t) \text{ for an arbitrary } t \in T, \\ \# & \text{otherwise.} \end{cases}$$

Observe that the definition of a time elapse with respect to the strong semantics ensures that we are not able to skip a transition when it is enabled.

Time Petri nets

Let t be a transition and $u = (m, h)$ be a state and $\tau \in \mathbb{R}^+$. Then elapsing of time with τ is always possible in the weak semantics. Then the result of the elapsing of time at state $u = (m, h)$ by τ is defined as $u \longrightarrow^{\tau} u' = (m', h')$, where $m = m'$ and $h'(t) = h(t) + \tau$ for every transition t.

Petri nets with time can generate recursively enumerable sets, hence it is a reasonable task to try to represent P systems with time Petri nets. Formally, the simulation of symbol object membrane systems with time Petri nets was elaborated,[3] and the notion of time membrane systems was defined. Now we try to give a sketch for the simulation of other features of membrane systems, like promoters, inhibitors, priority, by time Petri nets.

[3] B. Aman, P. Battyányi, G. Ciobanu, G. Vaszil, *Local time membrane systems and time Petri nets*, Theoretical Computer Science, 2018.

Correspondences between P systems and time Petri nets

Among the others, the following papers establish connections between membrane systems and Petri nets:

- J. H. C. M. Kleijn, M. Koutny and G. Rozenberg, Towards a Petri Net Semantics for Membrane Systems. *Lecture Notes in Computer Science*, volume 3850
- J. H. C. M. Kleijn, M. Koutny and G. Rozenberg, Petri Nets for Biologically Motivated Computing. *Scientific Annals of Computer Science*, vol. 21, 2011, pp. 199225
- J. Kleijn, M. Koutny, A Petri net model for membrane systems with dynamic structure. *Natural Computing*, vol. 8, 2009, pp. 781796

In these articles Petri net steps are considered with maximal parallel multiset of transitions. The distinct membranes are modeled by taking localities, and, altogether, simulating membrane dissolution and promoters/inhibitors are quite complicated in the third paper.

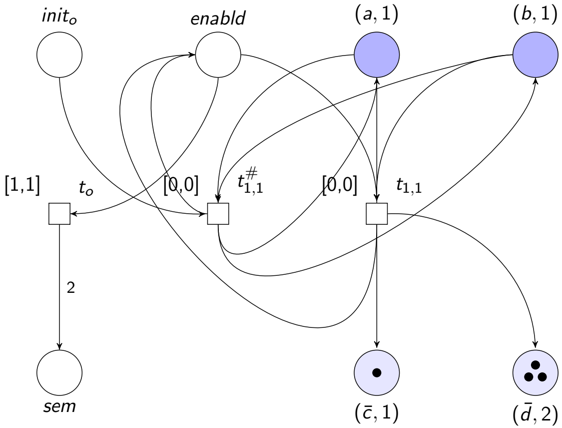

Figure: The Petri net simulating the rule application phase of a membrane computational step. Assume $a, b \in w_1$ and $r_{1,1} = ab \to c(d, in_2)^3$.

Correspondences between P systems and time Petri nets

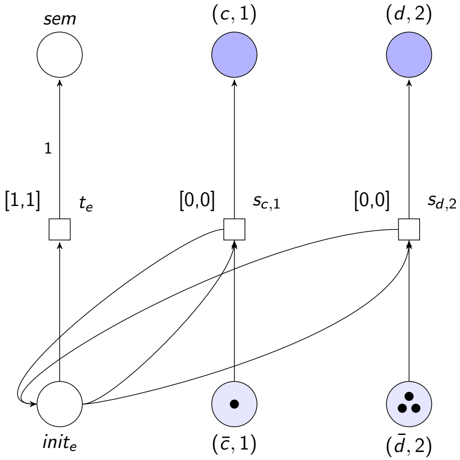

Figure: The Petri net simulating the communication phase of a membrane computational step. When the simulation of a maximal parallel rule application step is finished, a token is given to the semaphore *sem*.

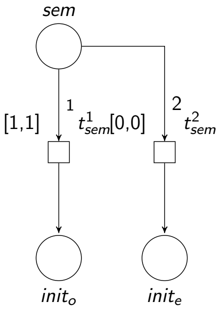

Figure: The semaphore for the Petri net. When the simulation of the rule application phase of a computational step of the membrane system is complete, two tokens appear at *sem*, and then sent to st_e, activating the simulation of the communication phase of the computational step. When the simulation of the communication phase is completed, one token appears at *sem*, which is then sent to st_0, activating the simulation of the rule application phase of a next computational step.

First we give a sketch how our basic model can be altered so that we can simulate promoters and inhibitors for rules. We understand the enability of rule $r \in R_i$ as follows:

- $lhs(r) \leq w_i$,
- $prom^r \leq w_i$,
- $w_i(a) < inhib^r(a)$ for every $a \in O$.

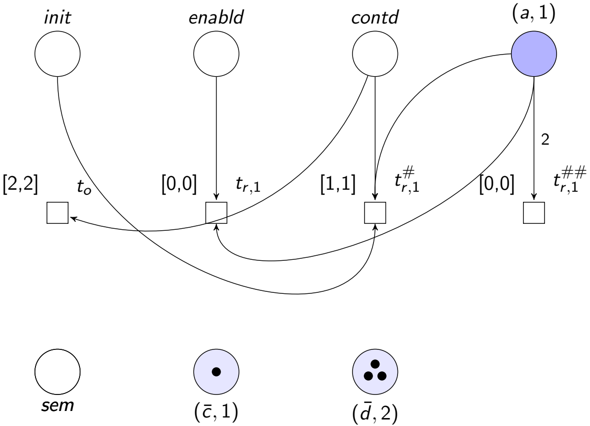

Figure: Input arcs for the transitions for the Petri net, where $a \in w_1$ and $r = a \to c(d, in_2)^3$ and $prom^r(a) = inhib^r(a) = 1$.

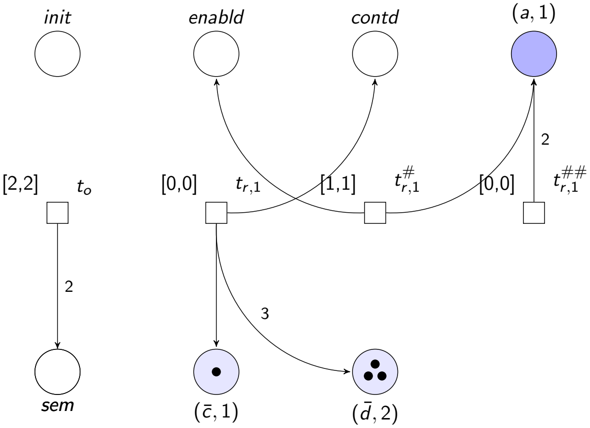

Figure: Output arcs for the transitions for the Petri net, where $a \in w_1$ and $r = a \to c(d, in_2)^3$ and $prom^r(a) = inhib^r(a) = 1$.

Now we construct a simple time Petri net that simulates membrane dissolution. Assume $r \in R_k$ and $parent(k) = l$. The rule application phase is as in our basic construction, only a new place δ_k is added to the output places of t_r, for every $r \in R_k$, in order to indicate the dissolution of m_k. In the figure we omit all auxiliary places, only the places representing objects and δ_k are indicated.

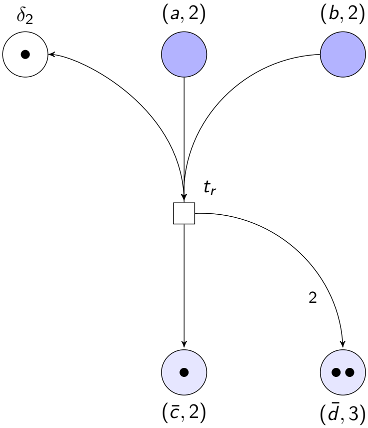

Figure: Assume $a \in w_2$ and $r = ab \to c(d, in_3)^2\delta$. The transition t_r corresponds to the rule application.

We continue with the above construction: in the communication phase the place δ_k controls whether the tokens of $\bar{p}(j,k)$ are transferred back to $p(j,k)$ or moved (recursively) towards the places for the parent membranes, namely, $\bar{p}(j,l)$. To this end, if $s_{j,k}$ is the transition connecting $\bar{p}(j,k)$ and $p(j,k)$, then we introduce a new transition $s^{\#}_{j,k}$ with time interval $[0,0]$ and set the time interval of $s_{j,k}$ to $[1,1]$. The transition $s^{\#}_{j,k}$ is connected with $\bar{p}(j,k)$ together with δ_k and $\bar{p}(j,l)$, as the next figure shows.

Correspondences between P systems and time Petri nets

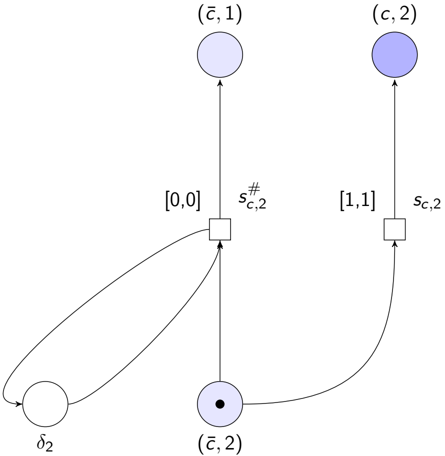

Figure: The Petri net simulating the communication phase of a membrane computational step. Membrane m_2 is dissolved, hence $s^{\#}_{c,2}$ can be activated moving the elements of $\bar{p}(c, 2)$ to $\bar{p}(c, 1)$.

Priority of rules can also be realized by imposing a new condition on the applicability of the rules demanding that it is necessary for a rule to be applicable that all rules of higher priority are non applicable.

Remarks, conjectures

Time Petri nets seem to be a very flexible and convenient tool for modelling P systems with different additional features. Hence, the notions and results already existing for Petri nets can be transferred to membrane systems.

Thank you for your attention!

- B. Aman, P. Battyányi, G. Ciobanu, Gy. Vaszil: Local time membrane systems and time Petri nets, Theoretical Computer Science (to appear)
- J. H. C. M. Kleijn and M. Koutny and G. Rozenberg, Towards a Petri Net Semantics for Membrane Systems. *Lecture Notes in Computer Science*, volume 3850, International Workshop on Membrane Computing, WMC 2005, 292–309, Springer Verlag, Berlin, 2005.
- G. Păun, Computing with membranes. *Journal of Computer and System Sciences*, 61(1) (2000) 108–143.
- J. L. Peterson, *Petri Net Theory and the Modelling of Systems*, Prentice Hall, N.J., 1981.
- L. Popova-Zeugmann, *Time and Petri Nets*, Springer Verlag, Berlin, 2013.