Parallel Communicating ET0L Systems vs. Polymorphic P Systems

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BWMC 2025, Sevilla, January 22, 2025

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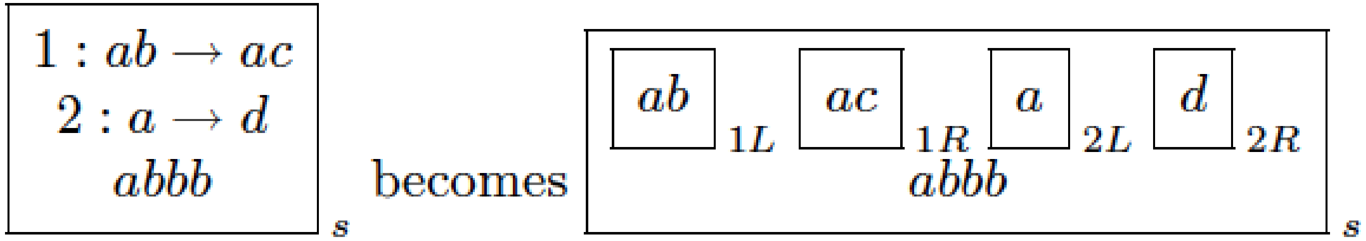
Polymorphic P systems - The idea

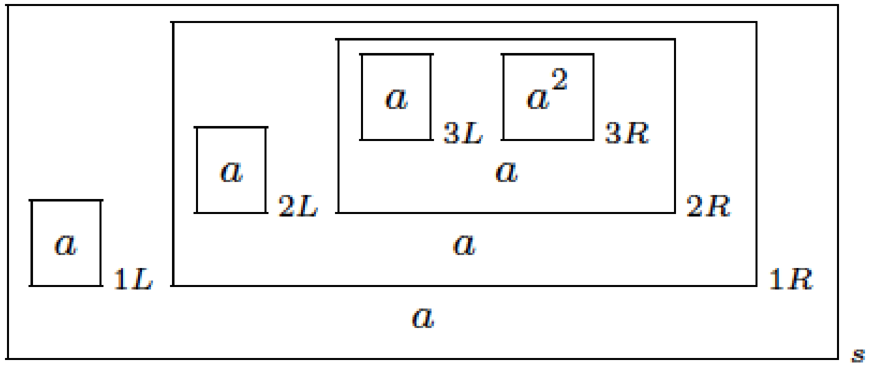
- Artiom Alhazov, Sergiu Ivanov, Yurii Rogozhin: Polymorphic P Systems. In: *CMC 2010*, Vol. 6501 of *LNCS*, pp. 81-94, 2010
- Sergiu Ivanov: Polymorphic P Systems with Non-cooperative Rules and No Ingredients.
  In: *CMC 2014*, Vol. 8961 of *LNCS*, pp. 258-273, 2014

Polymorphic P systems - The idea

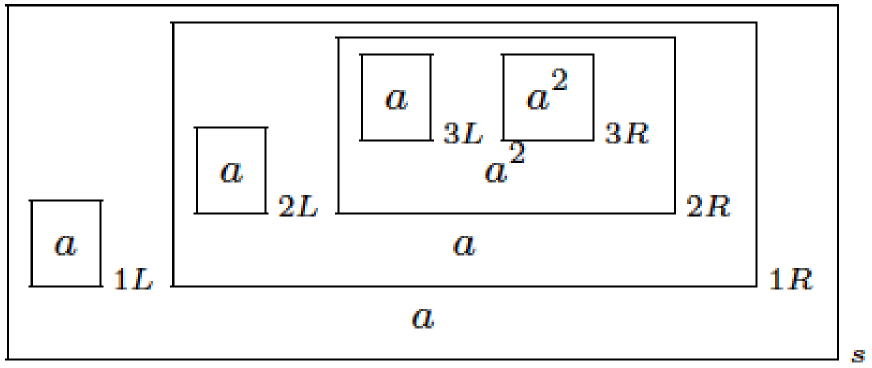
- To manipulate the **rules** during a computation: **represent** them **as data**



For example



$3 : a \rightarrow a^2 \text{ in } 2R$
$2 : a \rightarrow a \text{ in } 1R$
$1 : a \rightarrow a \text{ in } s$
$\Rightarrow$



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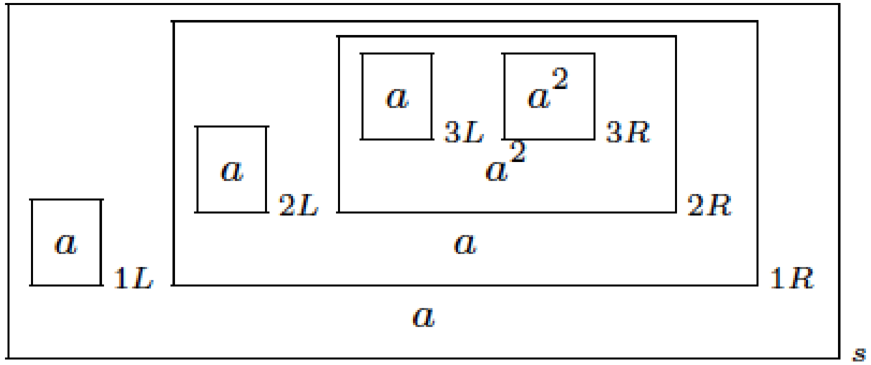

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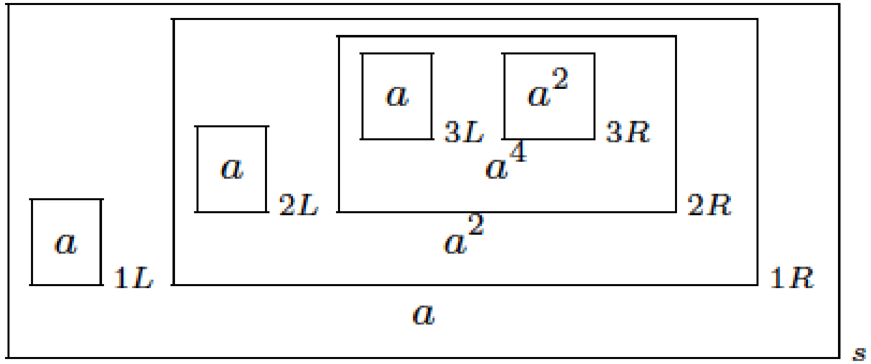

$3 : a \rightarrow a^2 \text{ in } 2R$
$2 : a \rightarrow a^4 \text{ in } 1R$
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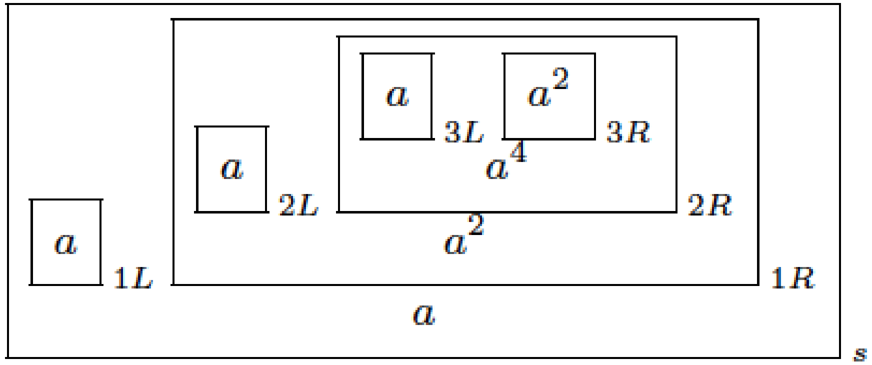
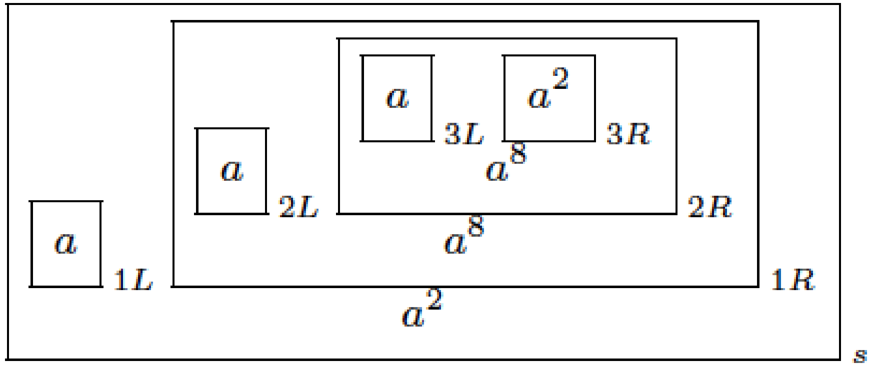
$\Rightarrow$

For example

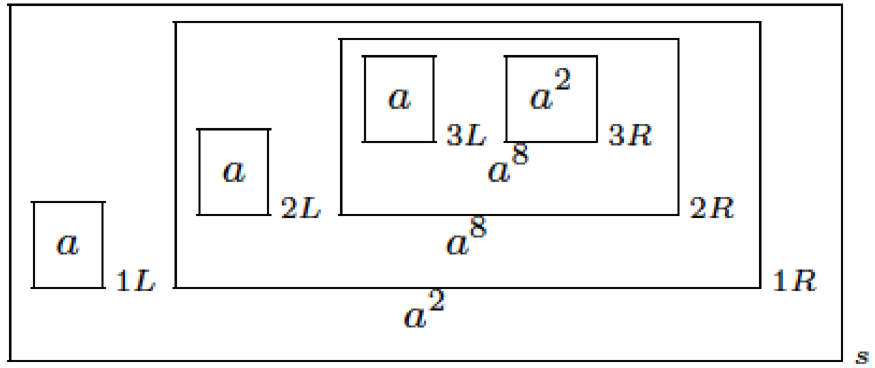


$3 : a \rightarrow a^2$ in $2R$
$2 : a \rightarrow a^4$ in $1R$
$1 : a \rightarrow a^2$ in s
$\Rightarrow$



$3 : a \rightarrow a^2$ in $2R$
$2 : a \rightarrow a^8$ in $1R$
$1 : a \rightarrow a^8$ in s
$\Rightarrow$

For example



$3 : a \rightarrow a^2$ in $2R$
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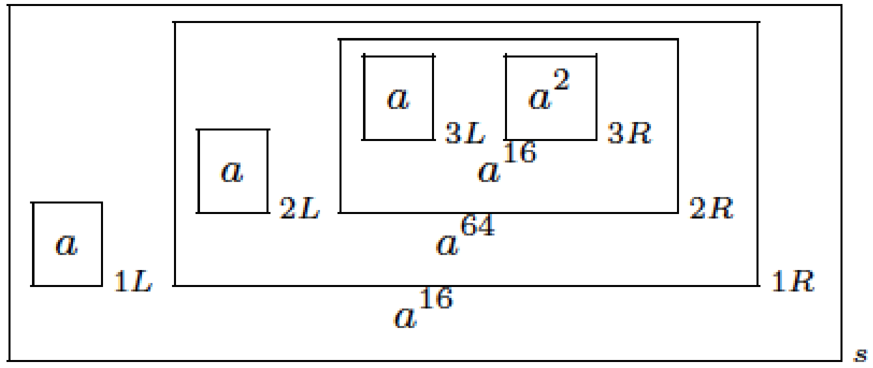
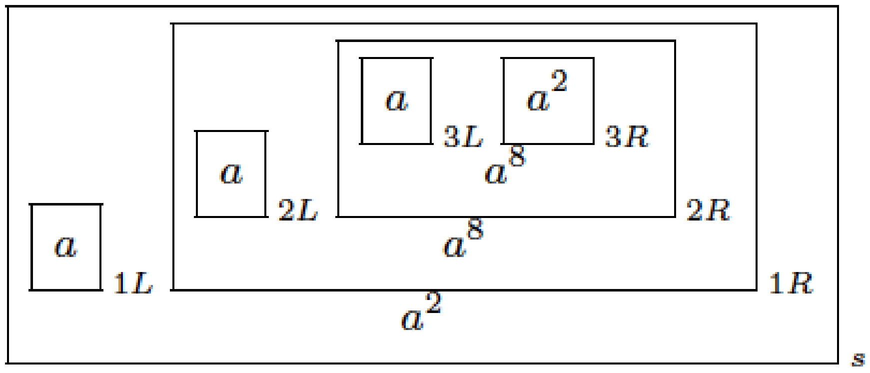
$\Rightarrow$



$3 : a \rightarrow a^2$ in $2R$
$2 : a \rightarrow a^{16}$ in $1R$
$1 : a \rightarrow a^{64}$ in s
$\Rightarrow \cdots$

For example



$3 : a \to a^2$ in $2R$
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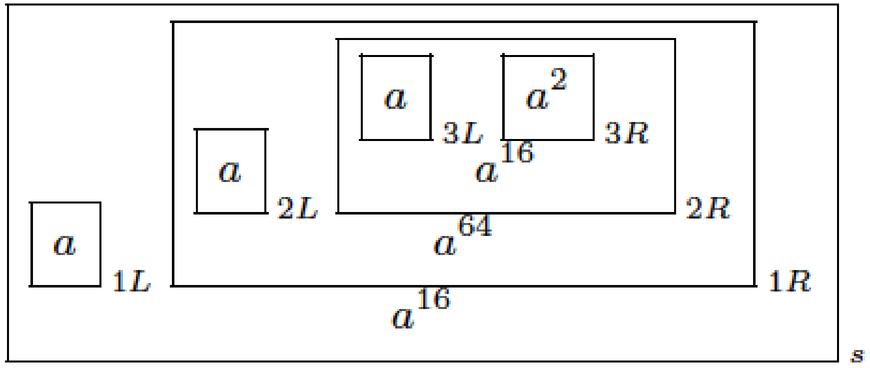
$\Rightarrow$



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$\Rightarrow \cdots$

$$(2, 2^n, 2^{n(n-1)/2}, 2^{n(n-1)(n-2)/6})$$

Non-cooperative polymorphic P systems

- **Strong non-cooperative** systems: left membranes contain at most one symbol
- **Weak non-cooperative systems**: all rules which are actually applied have one symbol on their left-hand side

Theorem 2. $NOP_*(polym_{+d}(ncoo_w)) = NOP_*(polym_{+d}(ncoo_s))$.

Systems with finite sets of instances of dynamic rules

- **Non-cooperative** rules → Left-membranes have **finitely many** possible **membrane contents** in any computation

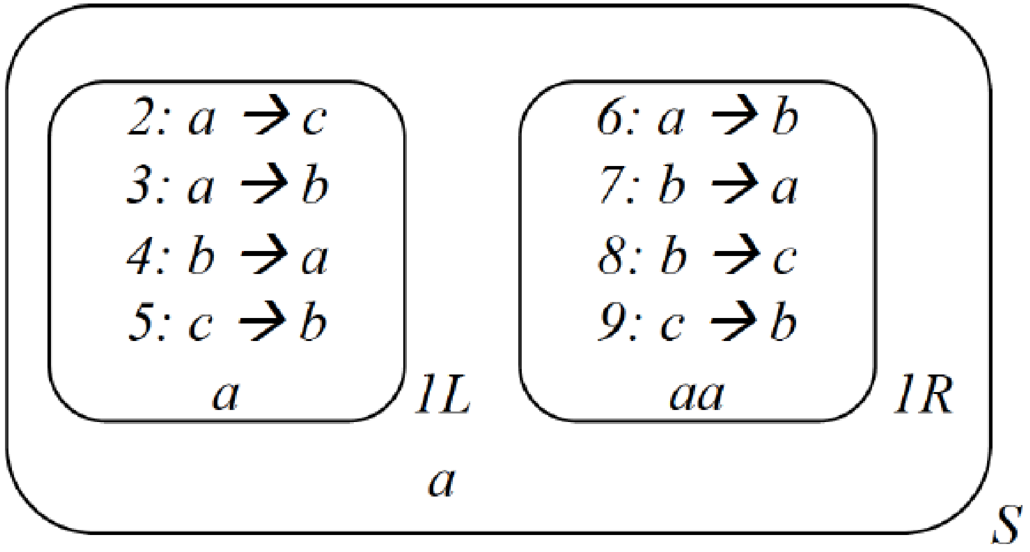
→left-membranes are always "**finitely representable**"

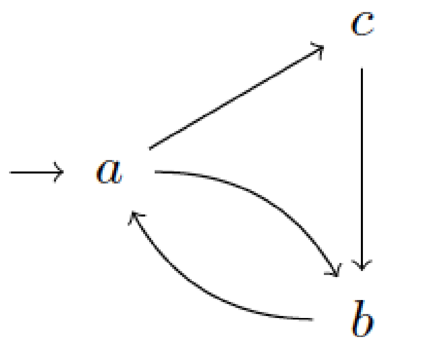
- What about "finitely representable" **right-membranes**?

Finite representability

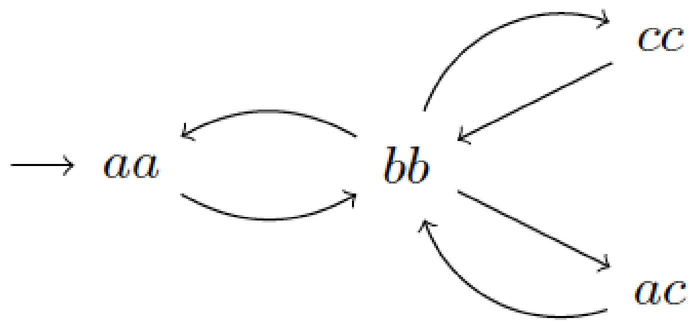
Region ***h*** **is FIN-representable** if the **set of successor multisets** of the initial contents w_h of region *h* is **finite**.

FIN-representability, an example





$$\sigma^*_{0,1L}(a) = \{a, b, c\}$$



$$\sigma^*_{0,1R}(aa) = \{aa, bb, cc, ac\}$$

Region *S* is not FIN-representable

Theorem: $\mathcal{L}(NOP(polym\ ,ncoo, fin)) = PsET0L.$

All *Right* membranes are FIN-representable

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What is ET0L?

ET0L example

$$L = \{ww \mid w \in \{a,b\}^*\}$$

$$G = (\{a,b,A\}, \{a,b\}, S, AA)$$

- $A \to aA$, $a \to a$, $b \to b$ — 1. table
- $A \to bA$, $a \to a$, $b \to b$ — 2. table
- $A \to \lambda$, $a \to a$, $b \to b$ — 3. table

Theorem 0.2.11. i)

$$\begin{array}{ccccc} & \subseteq & \mathcal{L}(\mathrm{T0L}) & \subseteq & \\ \mathcal{L}(\mathrm{DT0L}) & & & & \mathcal{L}(\mathrm{ET0L}) \subseteq \mathcal{L}(\mathrm{CS}), \\ & \subseteq & \mathcal{L}(\mathrm{EDT0L}) & \subseteq & \end{array}$$

and $\mathcal{L}(\mathrm{T0L})$ *and* $\mathcal{L}(\mathrm{EDT0L})$ *are incomparable.*

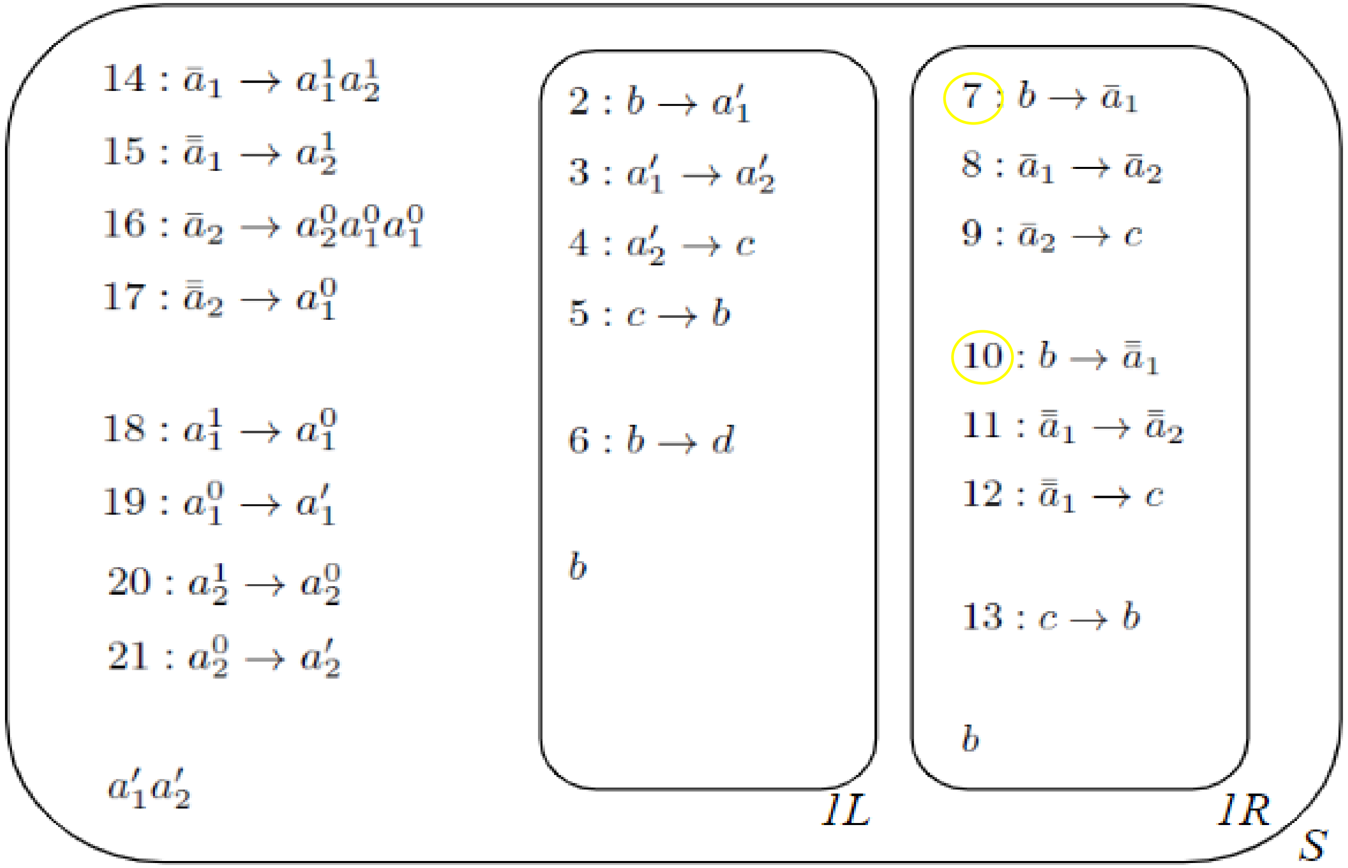
ii) $\mathcal{L}(\mathrm{CF}) \subset \mathcal{L}(\mathrm{ET0L})$, *and* $\mathcal{L}(\mathrm{CF})$ *is incomparable with* $\mathcal{L}(\mathrm{EDT0L})$, $\mathcal{L}(\mathrm{T0L})$, *and* $\mathcal{L}(\mathrm{DT0L})$.

iii) *For any* ET0L (EDT0L) *system* G *there is a propagating* ET0L (EDT0L) *system* G' *such that* $L(G) = L(G')$.

$$PsET0L \subseteq \mathcal{L}(NOP(polym.,ncoo,fin))$$

Proof idea – an example

The membrane system:



The ET0L system:

$$G = (V, T, U, w)$$
$$V = T = \{a_1, a_2\},$$
$$w = a_1 a_2,$$
$$U = (P_1, P_2),$$
$$P_1 = \{a_1 \to a_1 a_2,\ a_2 \to a_2 a_1 a_1$$
$$P_2 = \{a_1 \to a_2,\ a_2 \to a_1\}.$$

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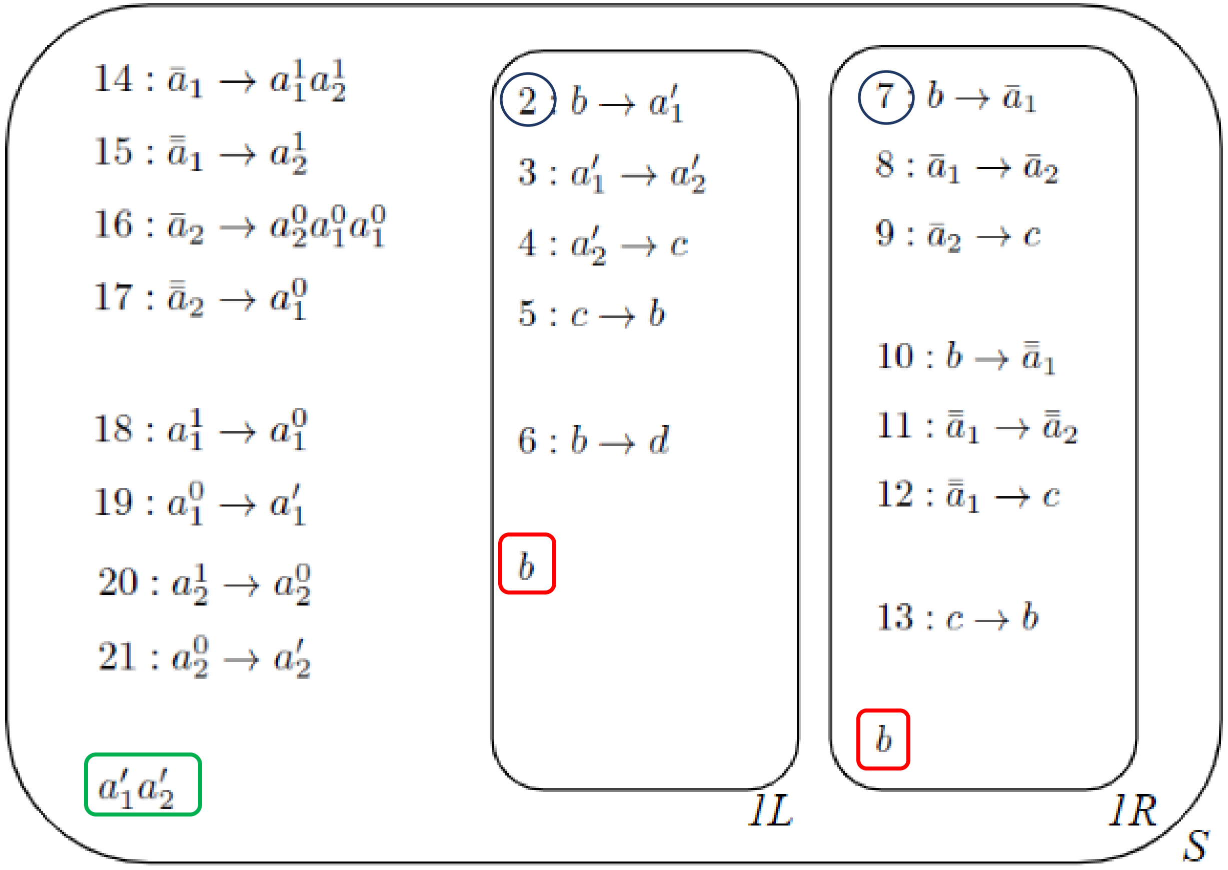
Proof idea

Step	Rule 1	Contents of the Skin
1.	$b \to b$	$a_1' a_2'$
2.	$a_1' \to \bar{a}_1$	$a_1' a_2'$
3.	$a_2' \to \bar{a}_2$	$\bar{a}_1 a_2'$
4.	$c \to c$	$a_1^1 a_2^1 \bar{a}_2$
5.	$b \to b$	$a_1^0 a_2^0 a_2^0 a_1^0 a_1^0$
6.	$a_1' \to \bar{a}_1$ or $a_1' \to \bar{\bar{a}}_1$	$a_1' a_2' a_2' a_1' a_1'$



ET0L tables: $P_1 = \{a_1 \to a_1 a_2,\ a_2 \to a_2 a_1 a_1\}$,
$P_2 = \{a_1 \to a_2,\ a_2 \to a_1\}$.

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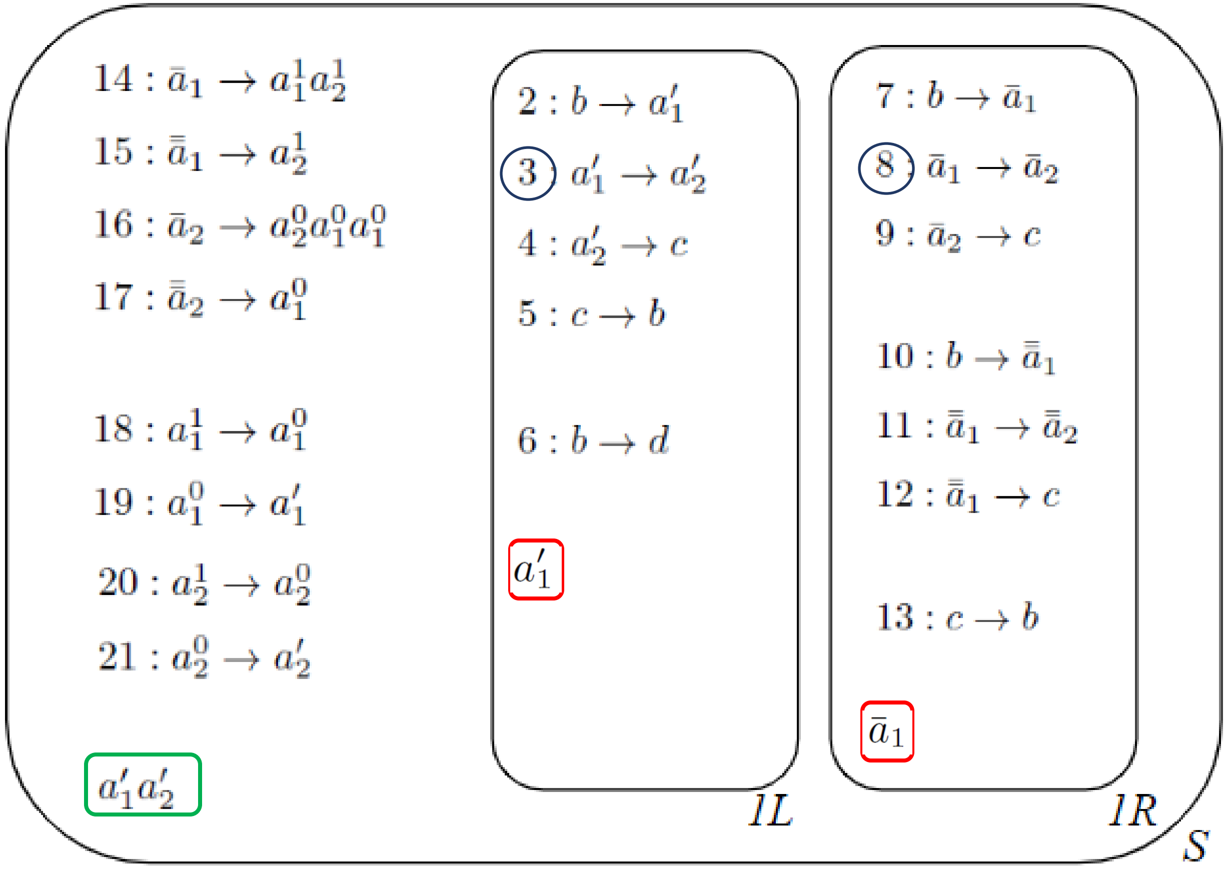

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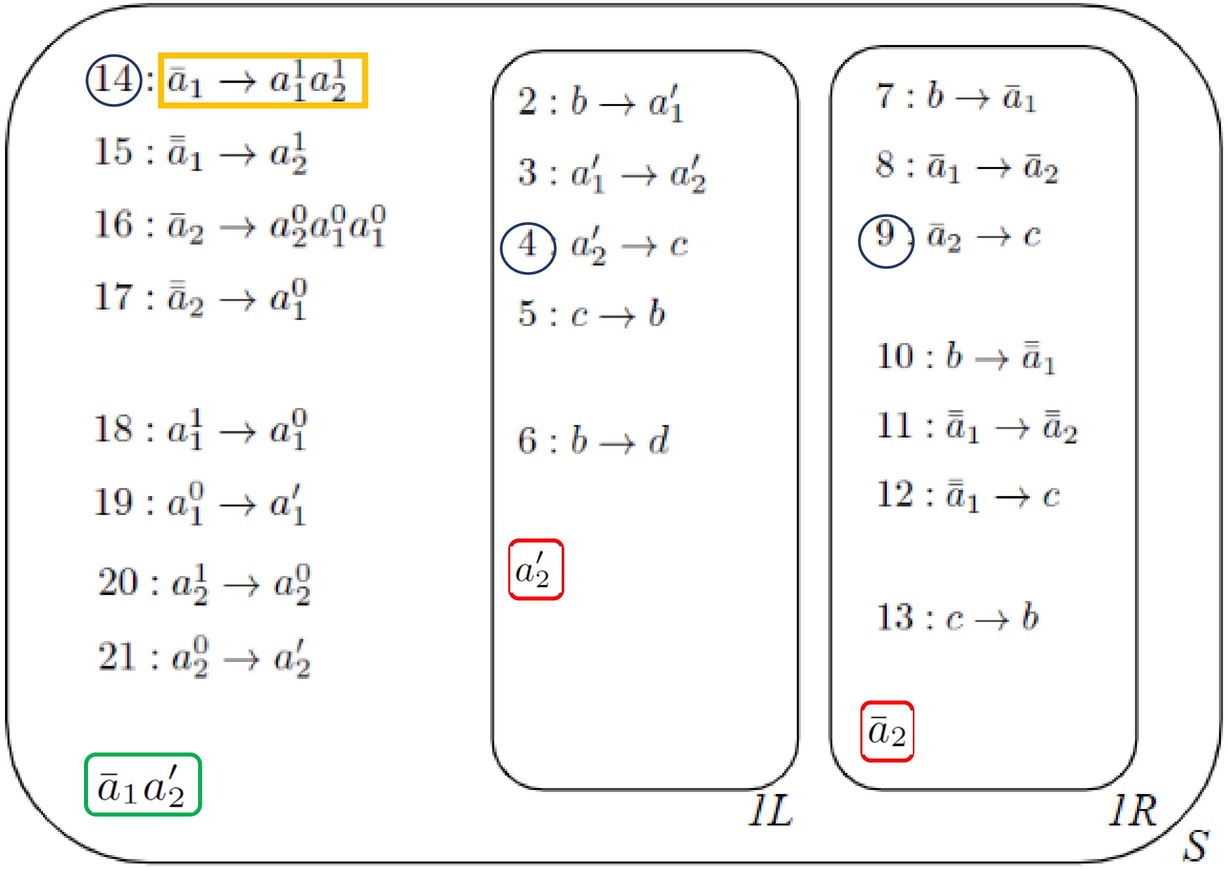

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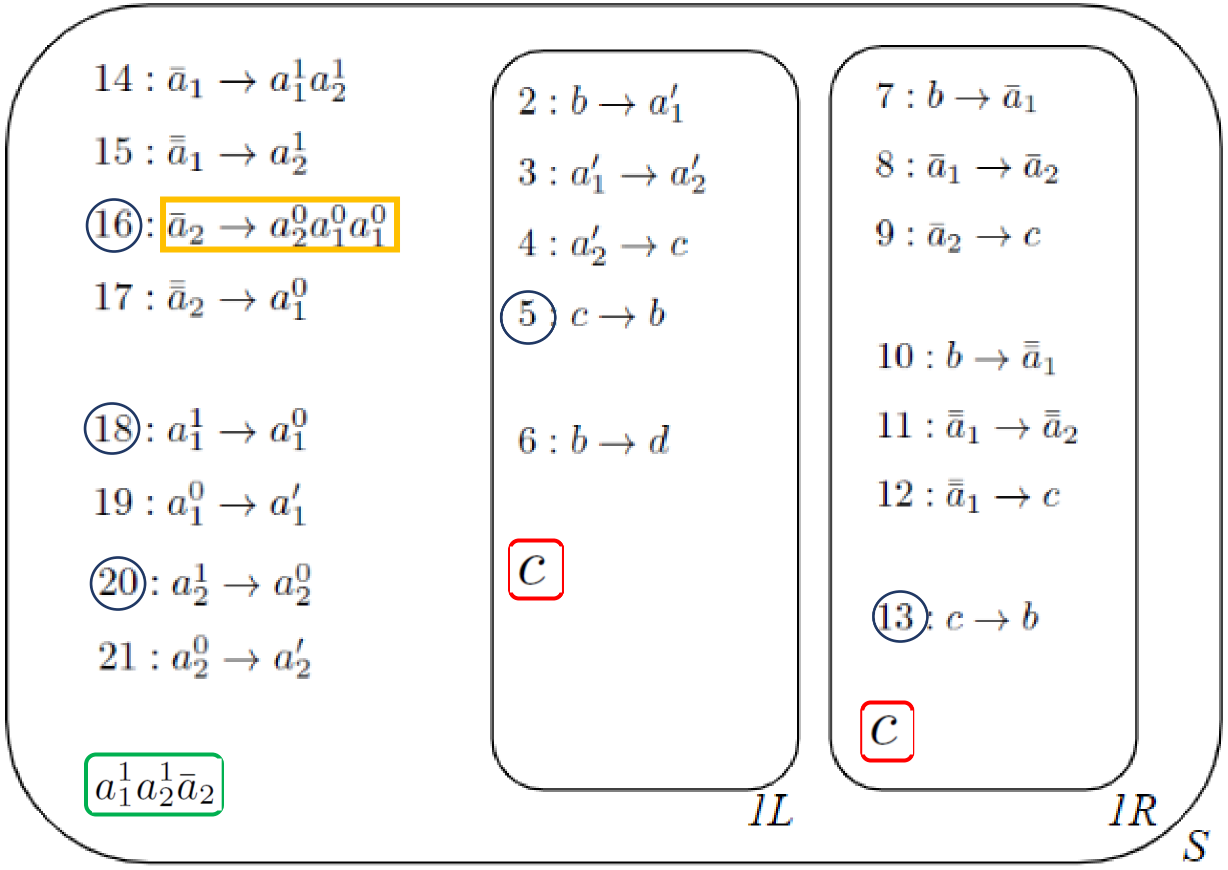

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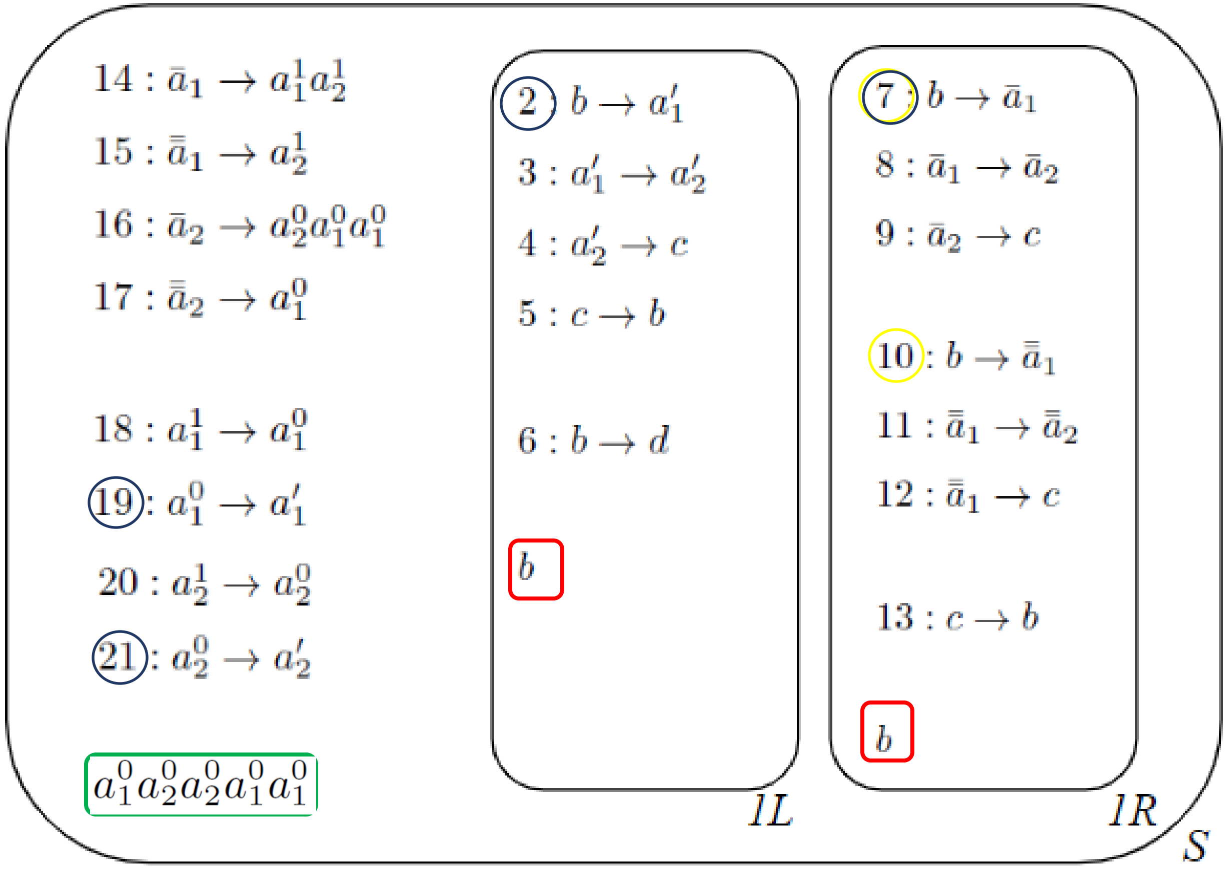

ETOL tables: $P_1 = \{a_1 \to a_1 a_2,\ a_2 \to a_2 a_1 a_1\}$,
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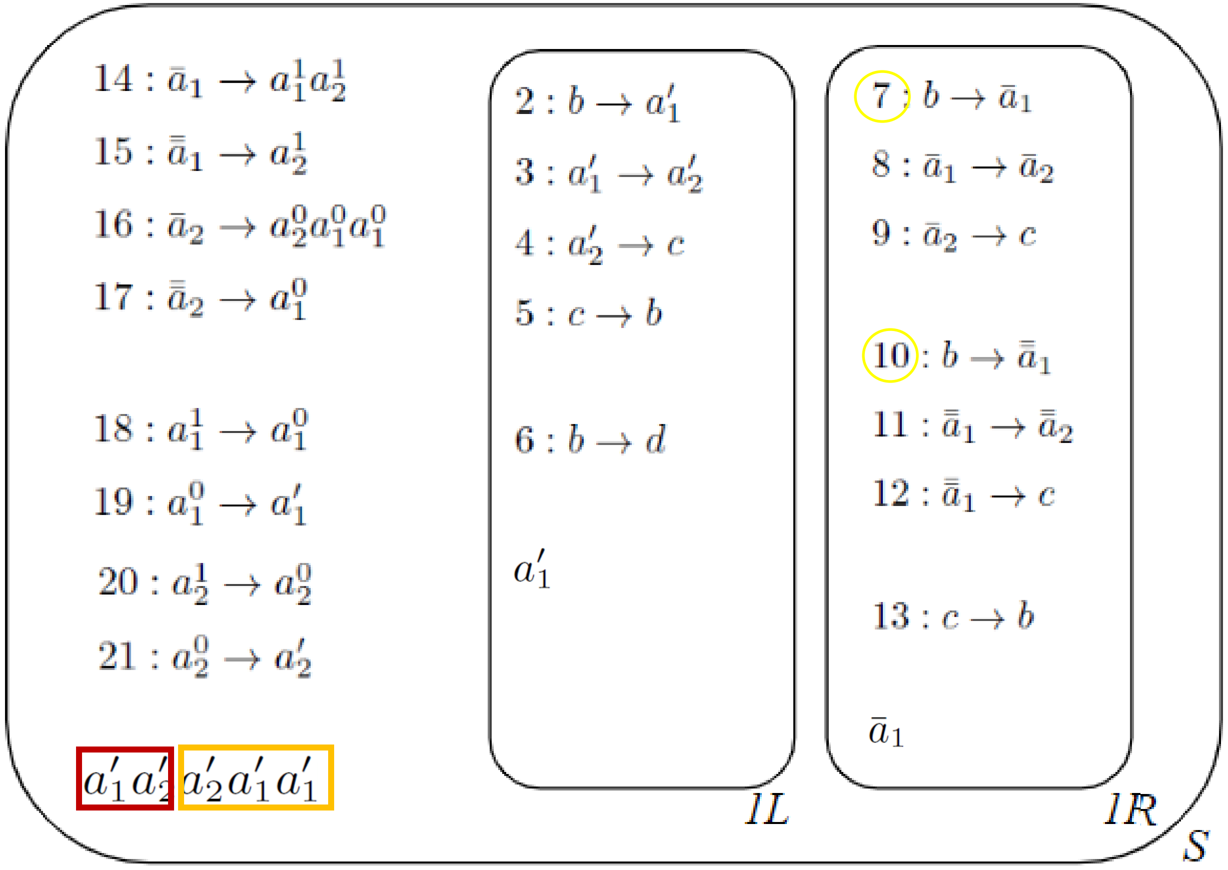
Proof idea

Step	Rule 1	Contents of the Skin
1.	$b \to b$	$a'_1 a'_2$
2.	$a'_1 \to \bar{a}_1$	$a'_1 a'_2$
3.	$a'_2 \to \bar{a}_2$	$\bar{a}_1 a'_2$
4.	$c \to c$	$a_1^1 a_2^1 \bar{a}_2$
5.	$b \to b$	$a_1^0 a_2^0 a_2^0 a_1^0 a_1^0$
6.	$a'_1 \to \bar{a}_1$ or $a'_1 \to \bar{\bar{a}}_1$	$a'_1 a'_2 a'_2 a'_1 a'_1$



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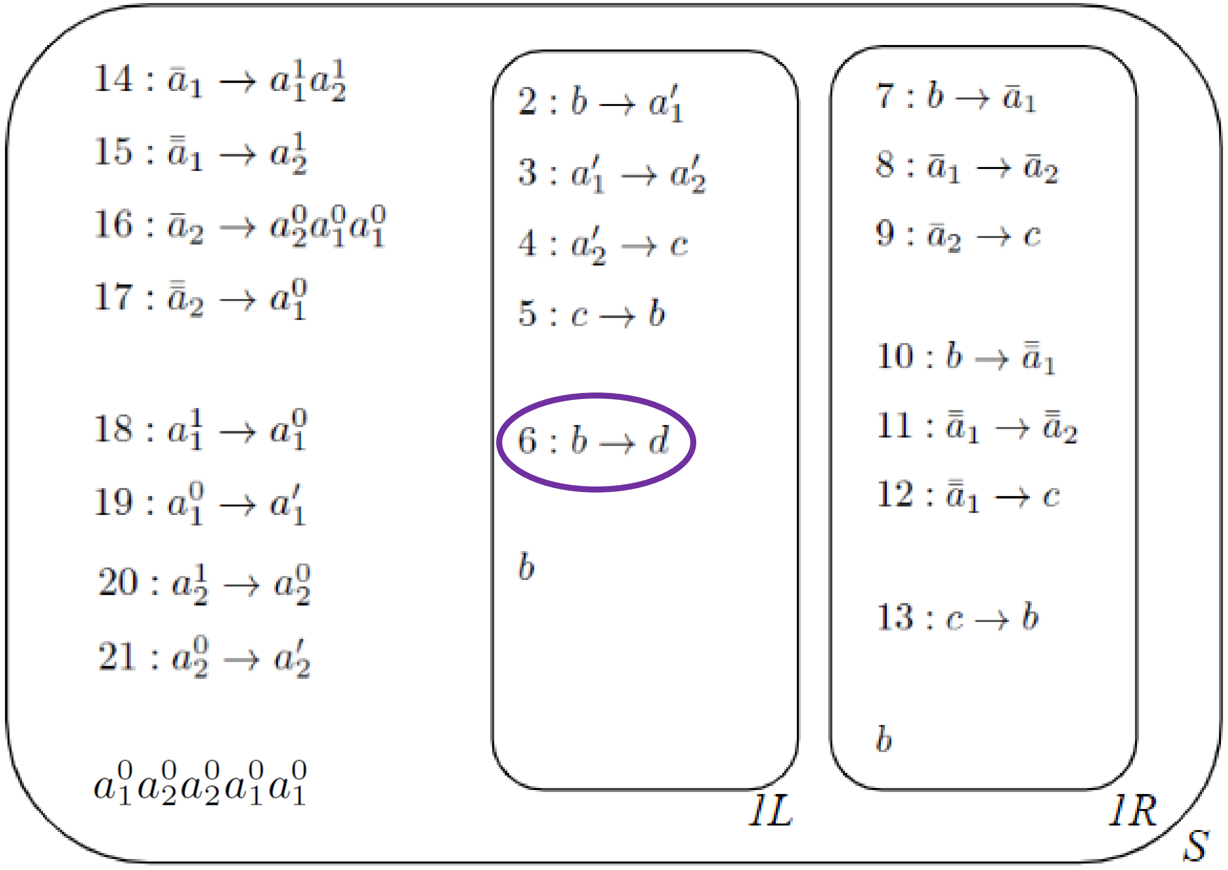

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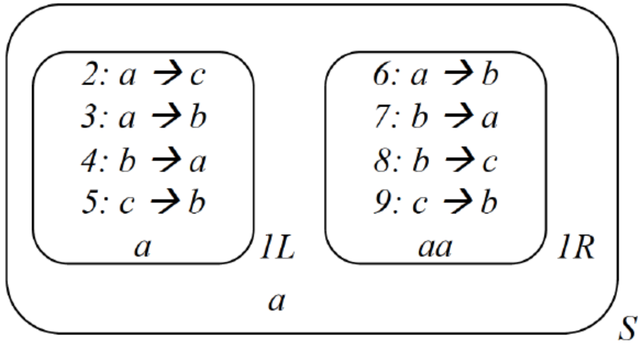
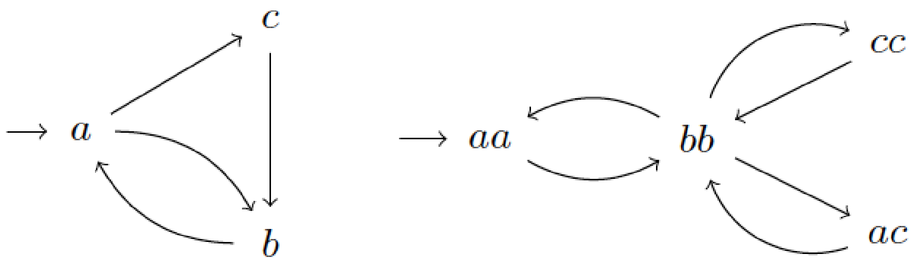
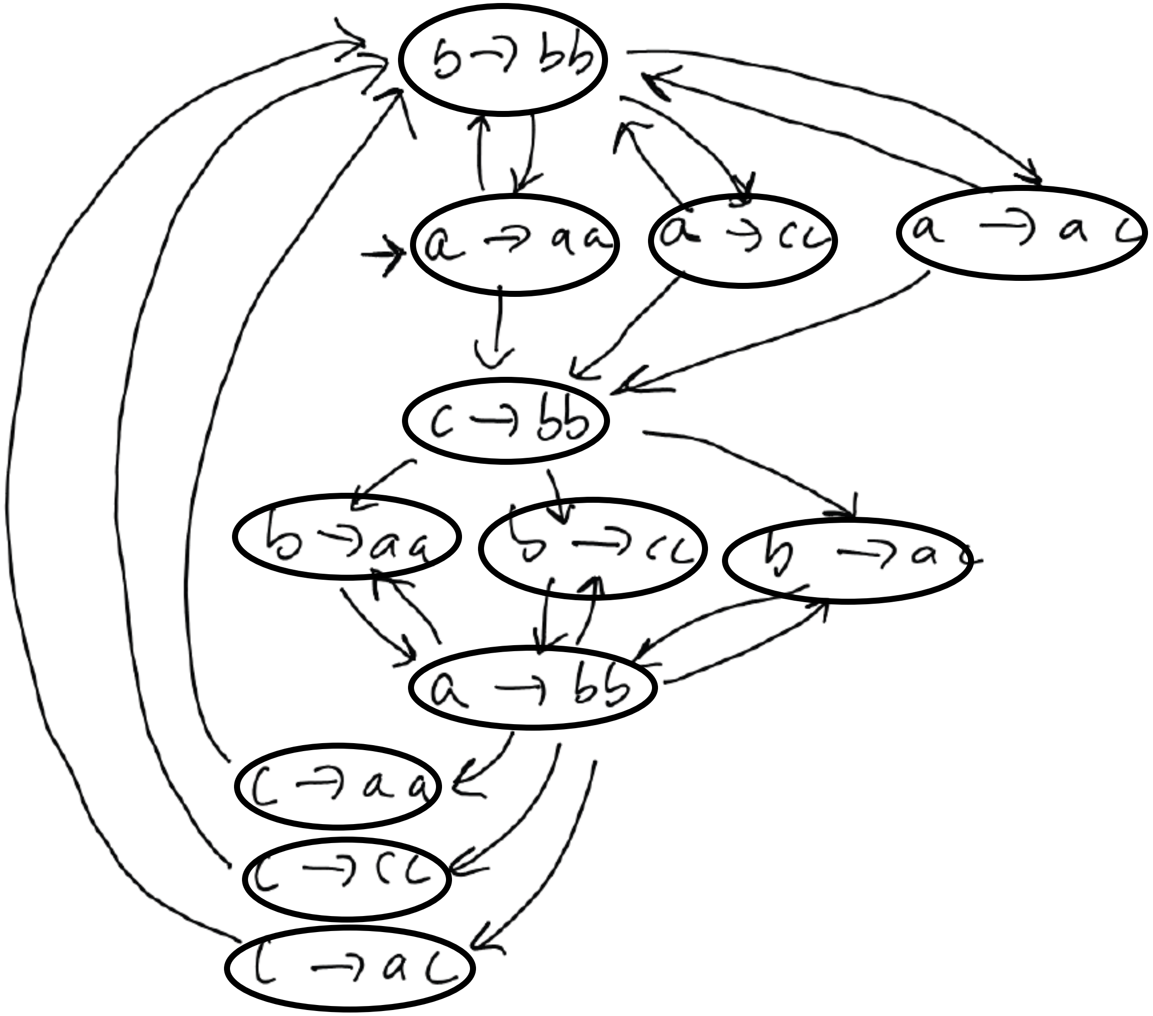
$$\mathcal{L}(NOP(polym\ ,ncoo,fin)) \subseteq PsET0L.$$

The proof idea

We can construct the **finite set** of instances of **rule 1**:





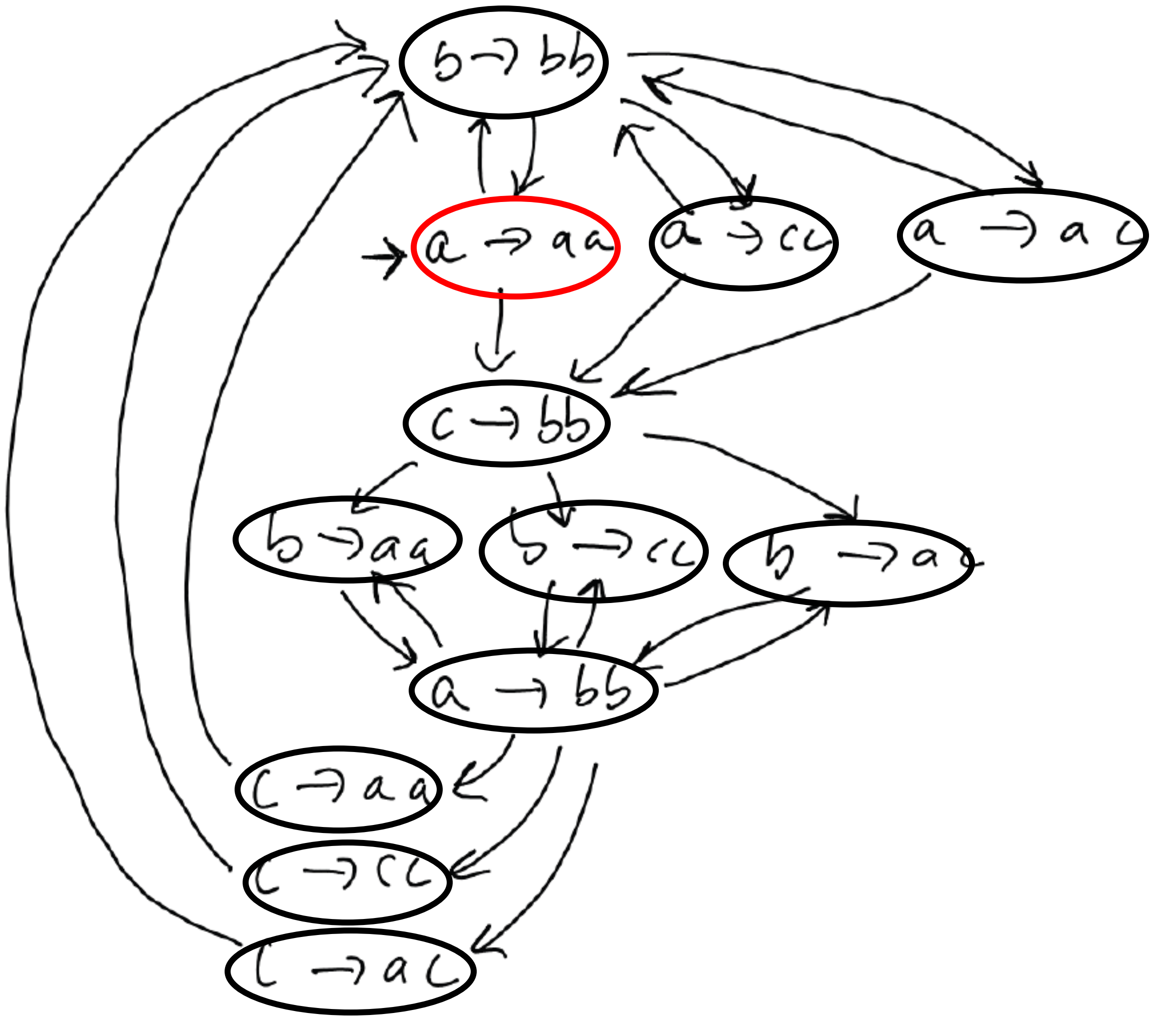


The proof idea

The **construction** of the *ET0L* **tables**:

- initial string: $d_{a\to aa}\ a$
- the table:

  $a \to aa$
  $d_{a\to aa} \to d_{b\to bb}$
  $d_{a\to aa} \to d_{c\to bb}$
  $d_{x\to yz} \to F$
  $F \to F$



The proof idea

The **construction** of the *ET0L* **tables**:

- after the *1st* step: $d_{a \to aa}\ aa$
- the table:

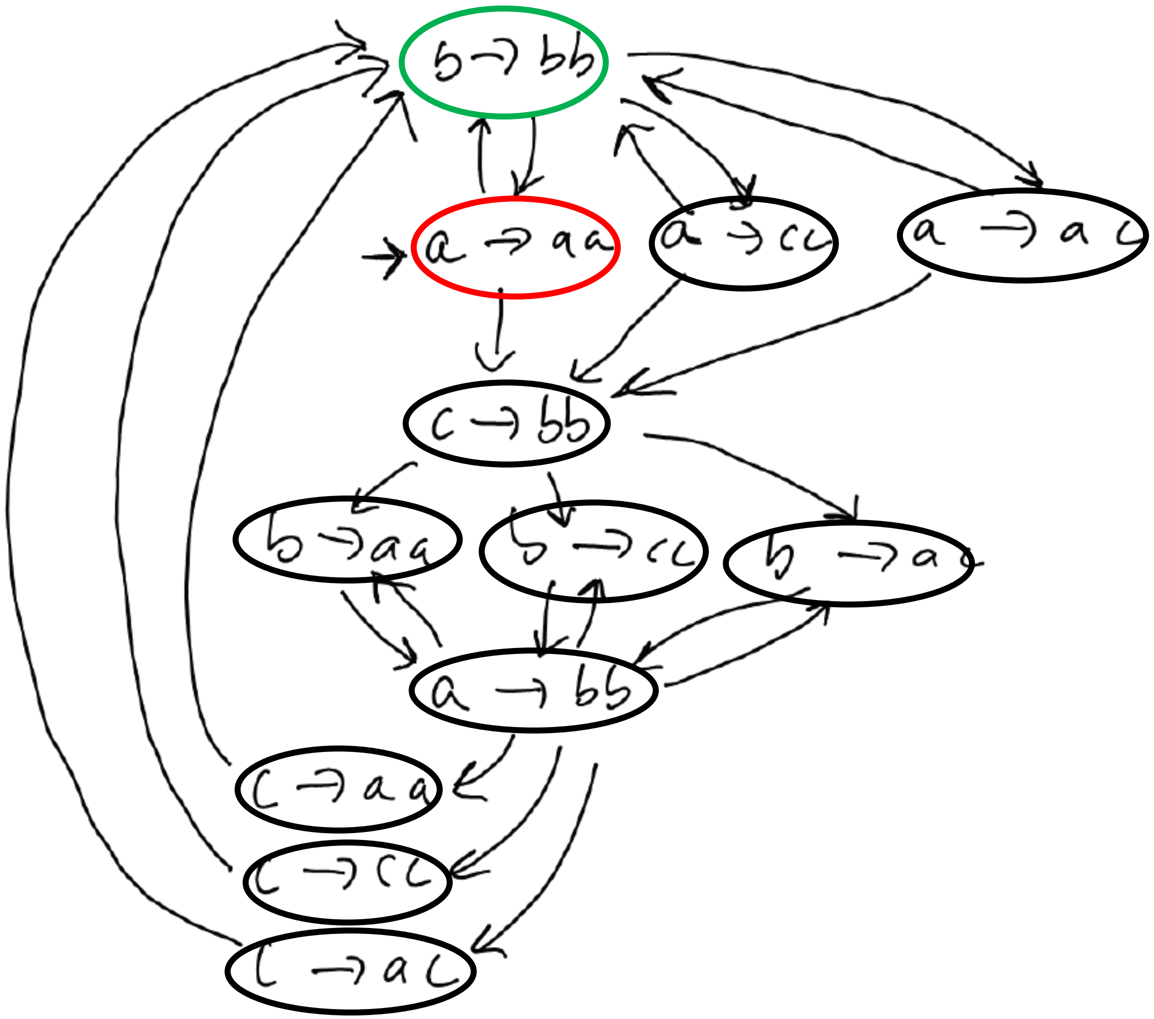
  $a \to aa$

  $d_{a \to aa} \to d_{b \to bb}$

  $d_{a \to aa} \to d_{c \to bb}$

  $d_{x \to yz} \to F$

  $F \to F$



The proof idea

The **construction** of the *ET0L* **tables**:

- after the 1st step: $d_{a \to aa}\ aa$
- the table:
  $b \to bb$
  $d_{b \to bb} \to d_{a \to aa}$
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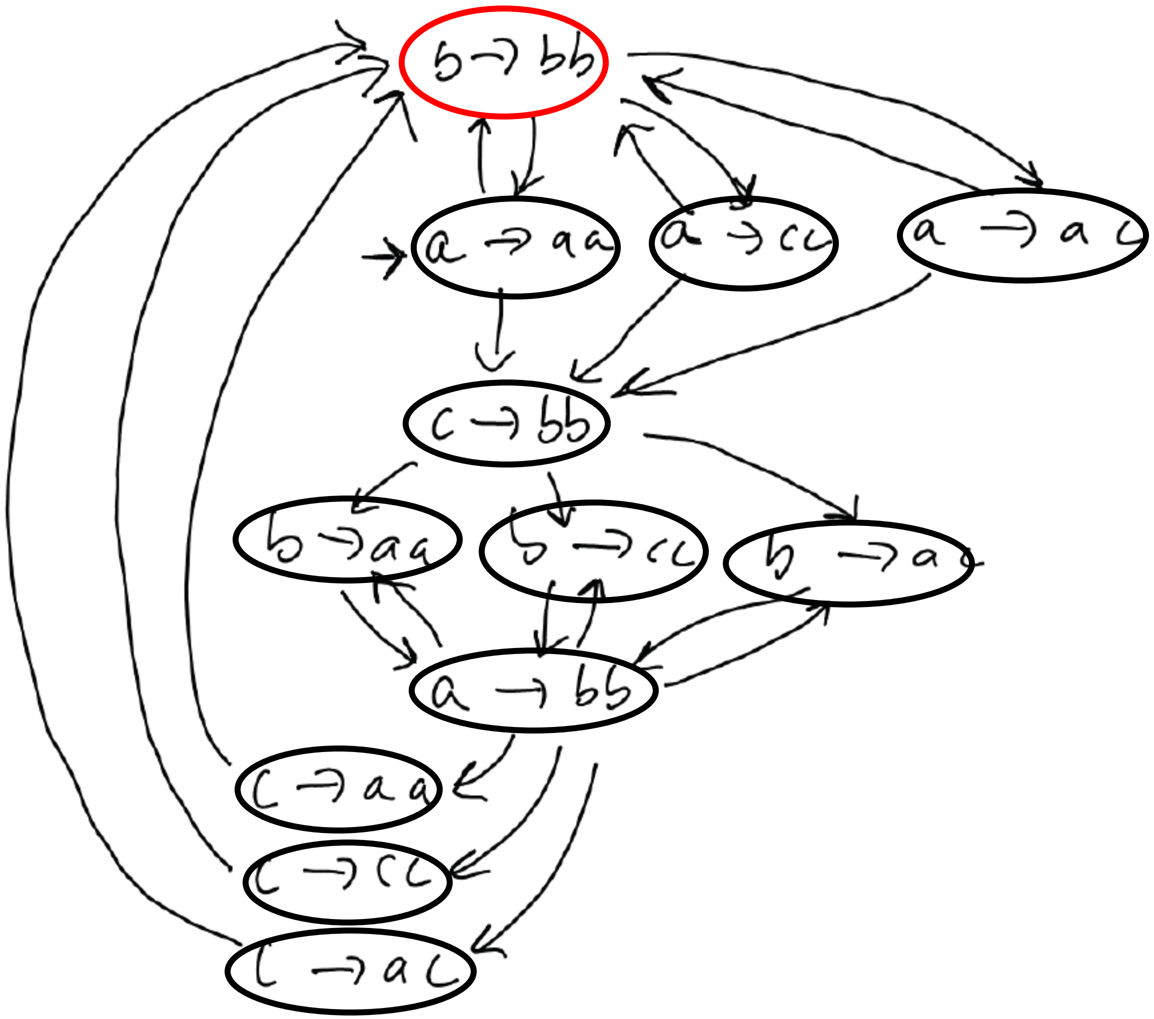
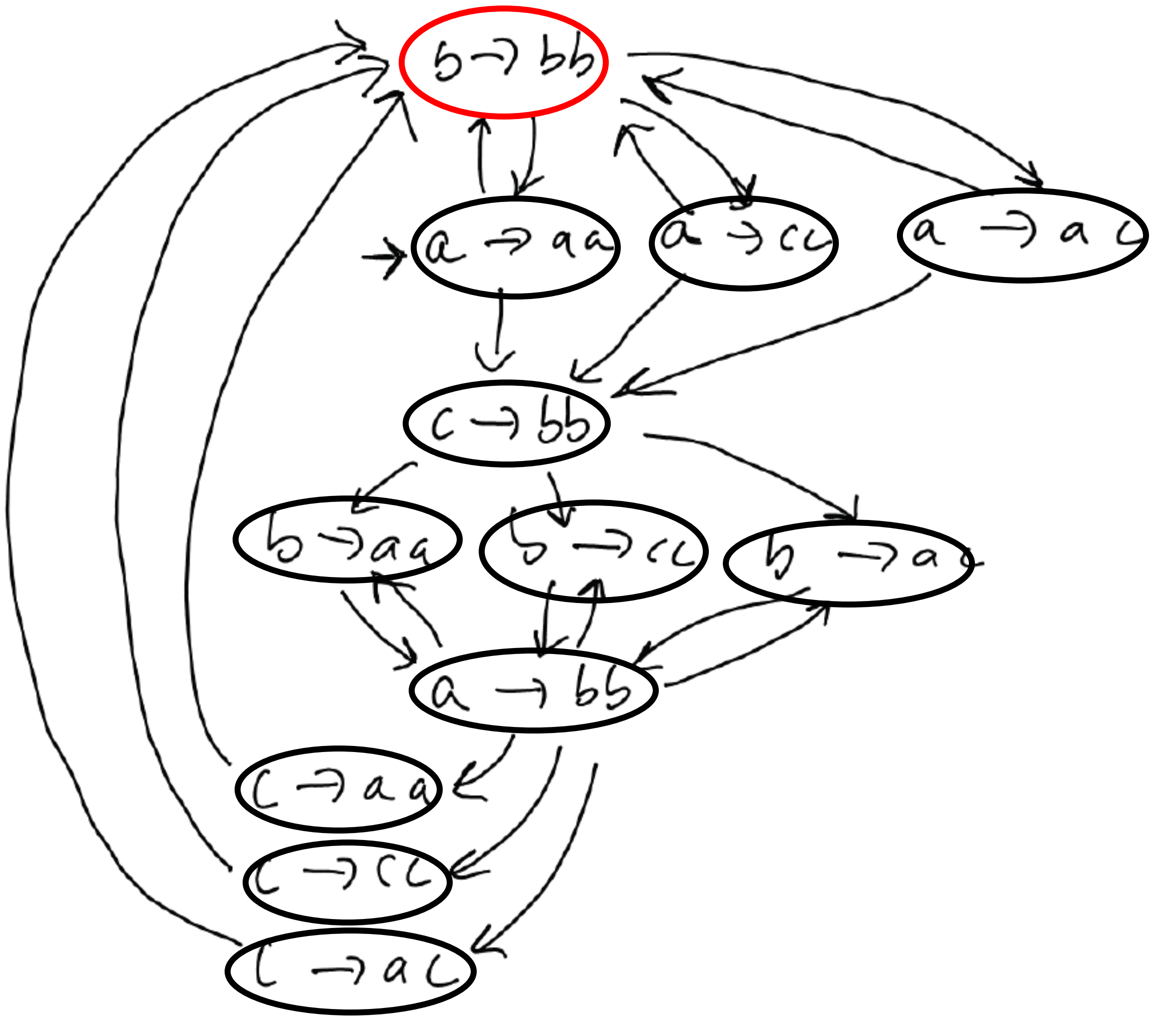
  $d_{b \to bb} \to d_{a \to ac}$
  $d_{x \to yz} \to F$
  $F \to F$



The proof idea

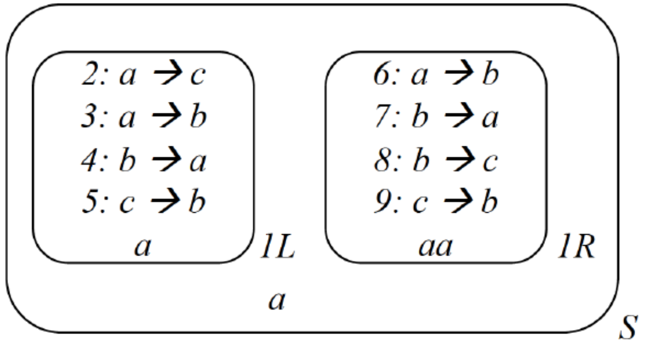
The **construction** of the *ET0L* **tables**:

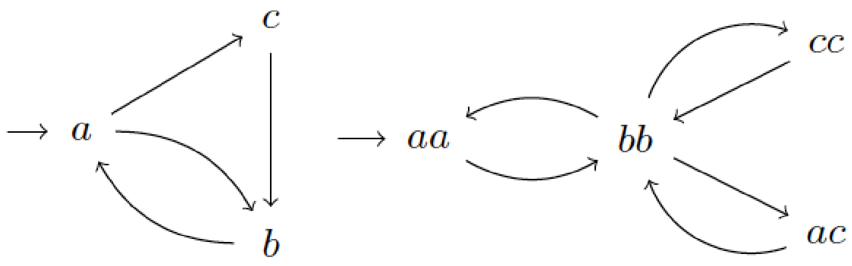
- after the 1st step: $d_{a \to aa}\ aa$
- the table:
  $b \to bb$
  $d_{b \to bb} \to d_{a \to aa}$
  $d_{b \to bb} \to d_{a \to cc}$
  $d_{b \to bb} \to d_{a \to ac}$
  $d_{x \to yz} \to F$
  $F \to F$ and so on…

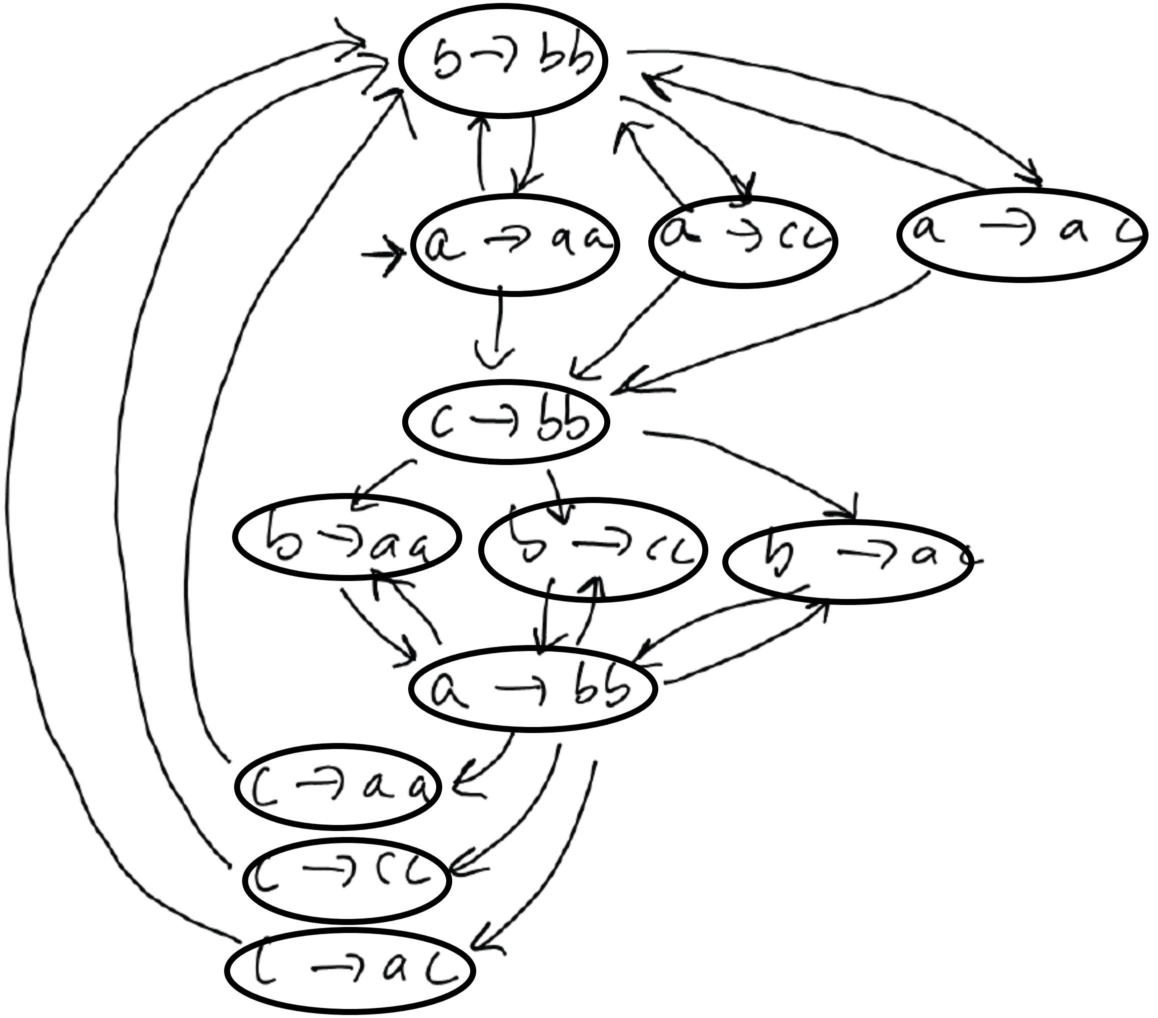


If we have two dynamic rules…

…we can **construct** the finite set of instances of **rule pairs**







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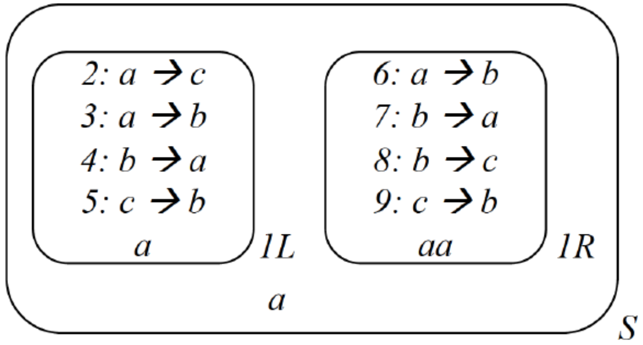
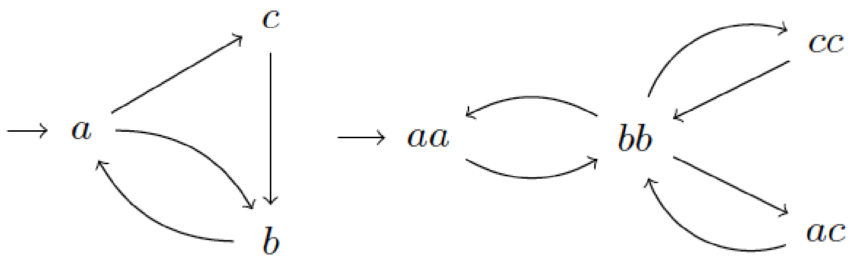
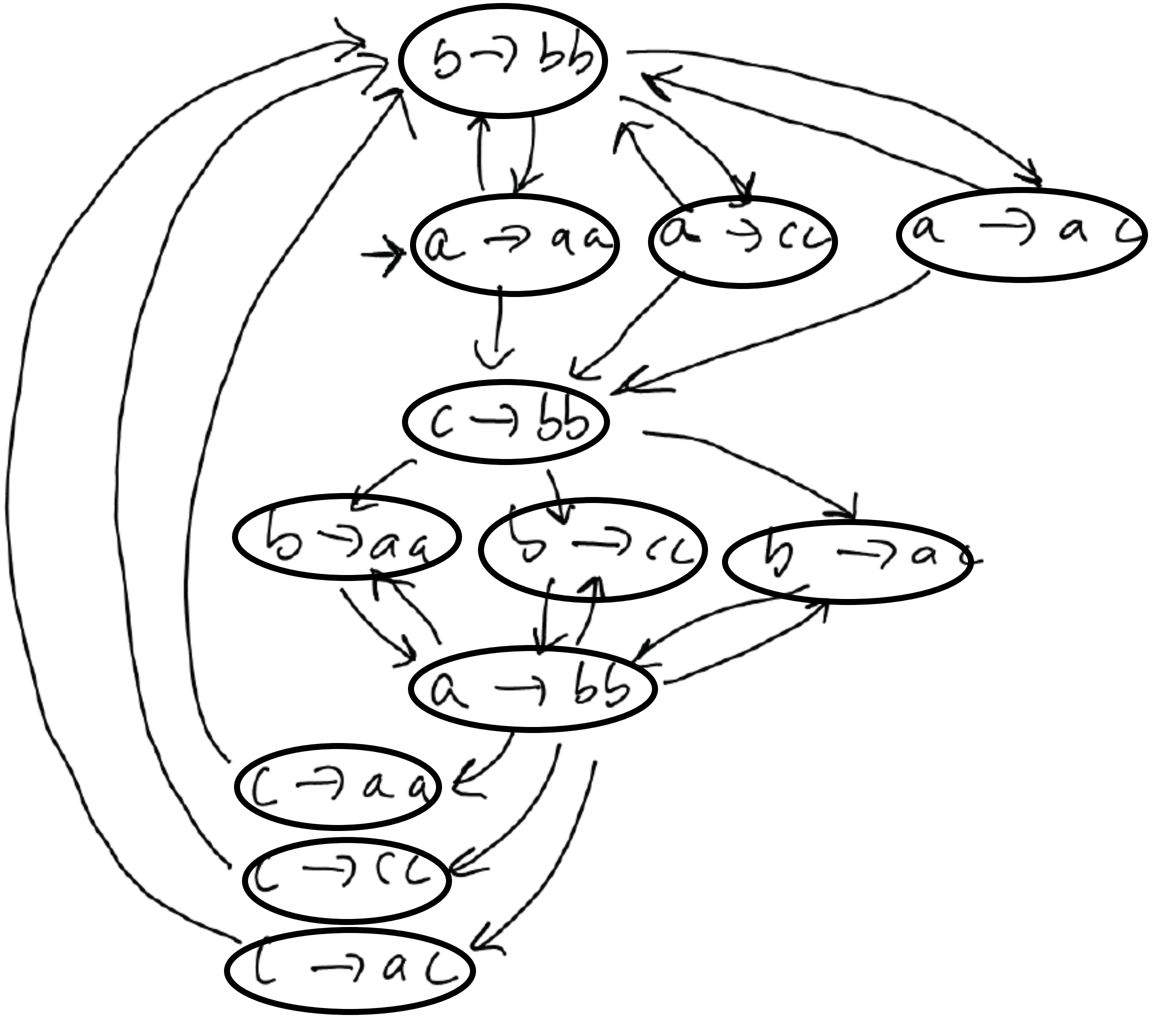
If we have several dynamic rules…

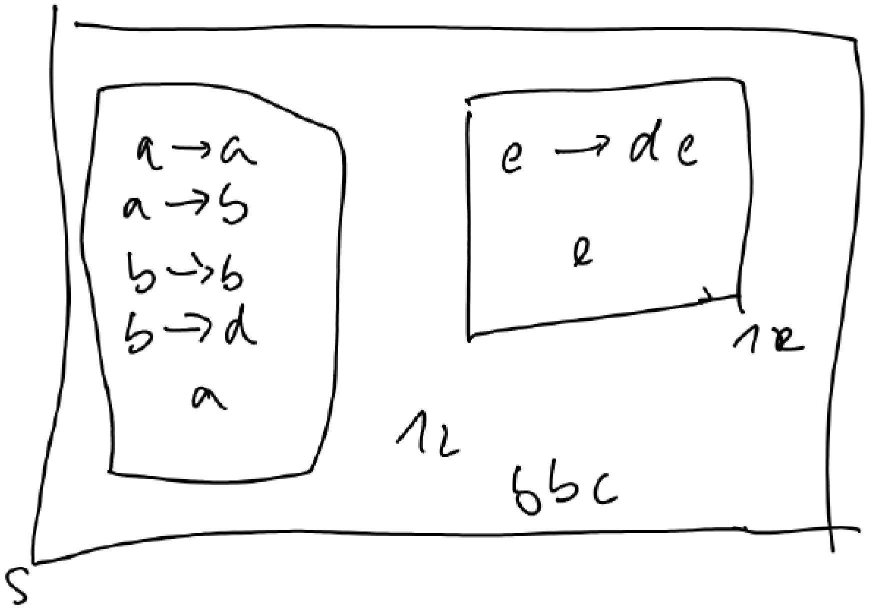
…we can construct the finite set of instances of **groups of rules that can be applied simultaneously**







$$\mathcal{L}(NOP(polym\ , ncoo, fin)) = \ ?$$



What happens if the system is not finitely representable?

The righthand sides of rules are “words” of an infinite language →
→ symbols are replaced with “words” of unbounded length →
→ like **parallel communicating** grammar systems?

A Parallel Communicating E0L system

$$\Gamma = (N, K, T, G_1, G_2, G_3, G_4)$$

with $N = \{a, b, c, d\}$, $T = \{a, c\}$, $K = \{Q_1, Q_2, Q_3, Q_4\}$, and $G_i = (N \cup K, T, \omega_i)$, $1 \leq i \leq 4$, where

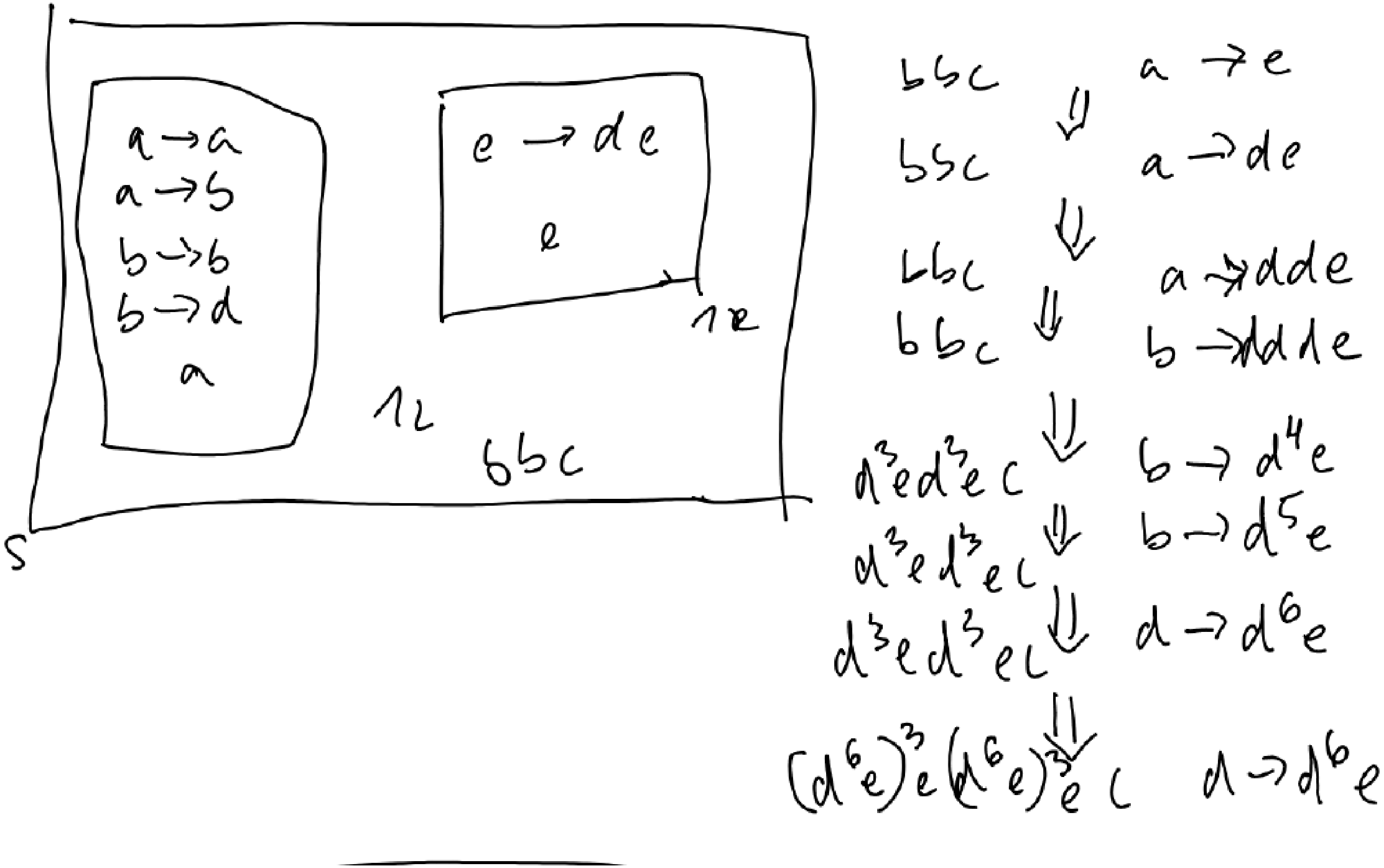
$$\omega_1 = a,\ P_1 = \{a \rightarrow aa\},$$
$$\omega_2 = b,\ P_2 = \{b \rightarrow Q_1 b\},$$
$$\omega_3 = d,\ P_3 = \{b \rightarrow \lambda, d \rightarrow Q_2, d \rightarrow Q_4\},\ \text{and}$$
$$\omega_4 = d,\ P_4 = \{d \rightarrow cd\}.$$

$$\omega_1 = a,\ P_1 = \{a \to aa\},$$
$$\omega_2 = b,\ P_2 = \{b \to Q_1 b\},$$
$$\omega_3 = d,\ P_3 = \{b \to \lambda, d \to Q_2, d \to Q_4\},$$
$$\omega_4 = d,\ P_4 = \{d \to cd\}.$$

G_1	G_2	G_3	G_4
a	b	d	d
aa	$Q_1 b$	Q_4	cd
aa	aab	cd	cd
$aaaa$	$aaQ_1 b$	cQ_4	ccd
$aaaa$	$aa\,aaaa\,b$	$c\,ccd$	ccd
a^8	$aa\,aaaa\,Q_1 b$	$c\,ccQ_2$	$cccd$
a^8	$aa\,aaaa\,a^8 b$	$c\,ccQ_2$	$cccd$
a^8	$aa\,aaaa\,a^8 b$	$c\,cc\,aa\,aaaa\,a^8 b$	$cccd$
a^{16}	$aa\,aaaa\,a^8 Q_1 b$	$c\,cc\,aa\,aaaa\,a^8$	$ccccd$

$$L(\Gamma) = \{c^{\frac{n(n+1)}{2}} a^{2^{n+2}-2} \mid n \geq 0\}$$

Polymorphic P systems vs. PC ET0L systems



Polymorphic P systems vs. PC ET0L systems – a system with 2 components

($a_{1L} \to a_{1L}$
$a_{1L} \to b_{1L}$
$b_{1L} \to F$
$d_{1L} \to F$
$b \to b$
$d \to d$)
⋮

($b_{1L} \to b_{1L}$
$b_{1L} \to d_{1L}$
$b \to Q_{1R}$
$a_{1L} \to F$
$c \to c$)
⋮

($d_{1L} \to d_{1L}$
$a_{1L} \to F$
$b_{1L} \to F$
$d \to Q_{1R}$
⋮)

$S \to 1L$	$1R$
$bbc\,a_{1L}$	S
$bbc\,a_{1L}$	e
$bbc\,a_{1L}$	de
$bbc\,b_{1L}$	dde
$Q_{1R}Q_{1R}c\,b_{1L}$	$ddde$
$d^3e\,d^3e\,c\,b_{1L}$	
$d^3e\,d^3e\,c\,b_{1L}$	d^4e
$d^3e\,d^3e\,c\,d_{1L}$	d^5e
$(Q_{1R})^3e(Q_{1R})^3e\,c\,d_{1L}$	d^6e
$(d^6e)^3e(d^6e)^3e\,c\,d_{1L}$	

($e \to de$)

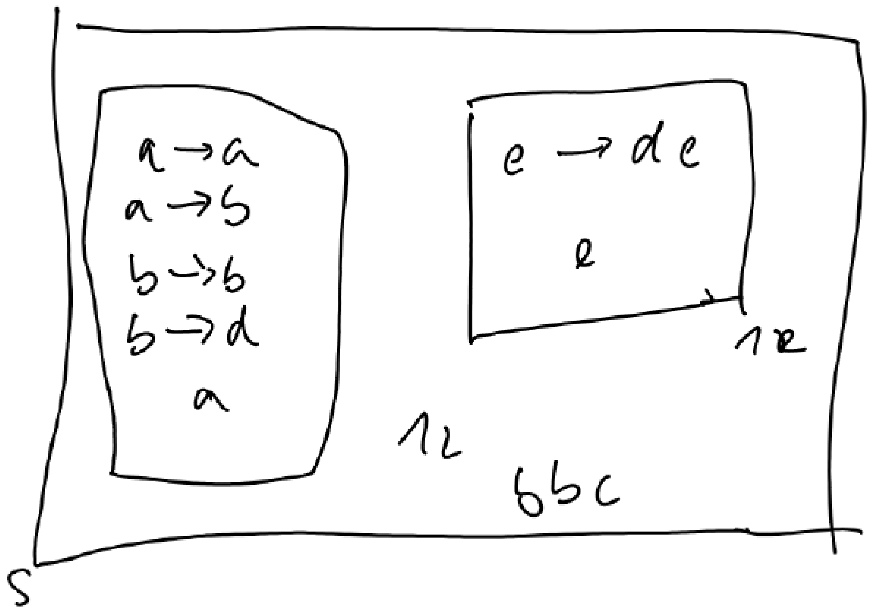
$(a_{1L} \to a_{1L}$
$a_{1L} \to b_{1L}$
$b_{1L} \to F$
$d_{1L} \to F$
$b \to b$
$d \to d)$
⋮

$(b_{1L} \to b_{1L}$
$b_{1L} \to d_{1L}$
$b \to Q_{1R}$
$a_{1L} \to F$
$c \to c)$
⋮

$(d_{1L} \to d_{1L}$
$a_{1L} \to F$
$b_{1L} \to F$
$d \to Q_{1R}$
⋮)

s − 1L	1R
$bbc\,a_{1L}$	s
$bbc\,a_{1L}$	e
$bbc\,a_{1L}$	de
$bbc\,b_{1L}$	dde
$Q_{1R}Q_{1R}c\,b_{1L}$	$ddde$
$d^3e\,d^3e\,c\,b_{1L}$	
$d^3e\,d^3e\,c\,b_{1L}$	d^4e
$d^3e\,d^3e\,c\,d_{1L}$	d^5e
$(Q_{1R})^3e(Q_{1R})^3e\,c\,d_{1L}$	d^6e
$(d^6e)^3e(d^6e)^3e\,c\,d_{1L}$	

$(e \to de)$



This idea can be formalized:

Theorem: $\mathcal{L}(NOP(polym, ncoo)) \subseteq PsNPC(ET0L)$

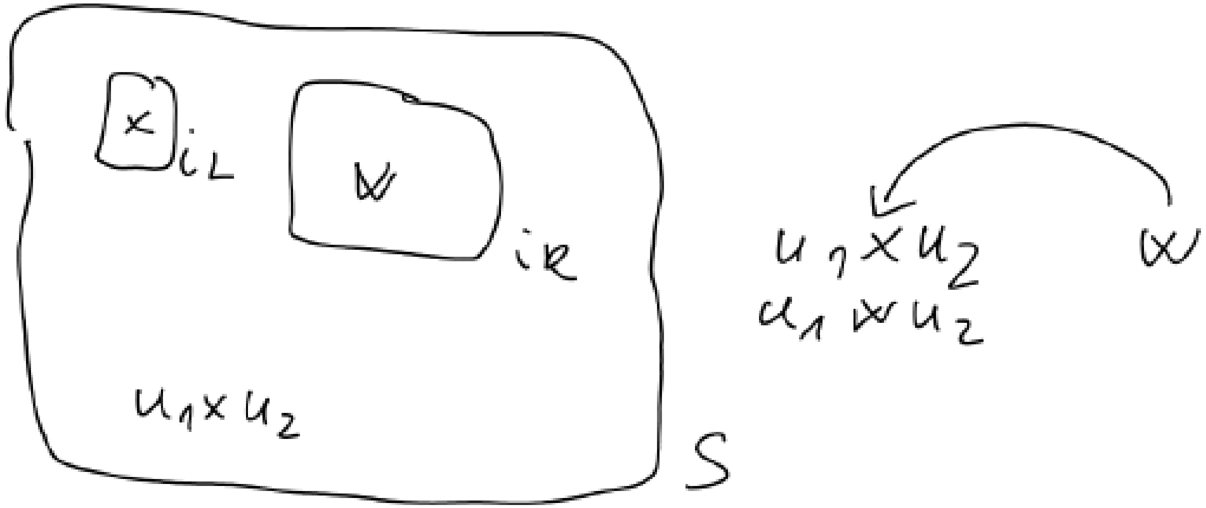
[A. Kuczik, Gy. Vaszil, CMC 2024]

The other way around?

$$\mathcal{L}(NOP(polym, ncoo)) \supseteq PsNPC(ET0L)$$

- The “communication graphs” of P systems are not complex enough

→What can polymorphic P systems do?



The other way around?

$$\mathcal{L}(NOP(polym, ncoo)) \supseteq PsNPC(ET0L)$$

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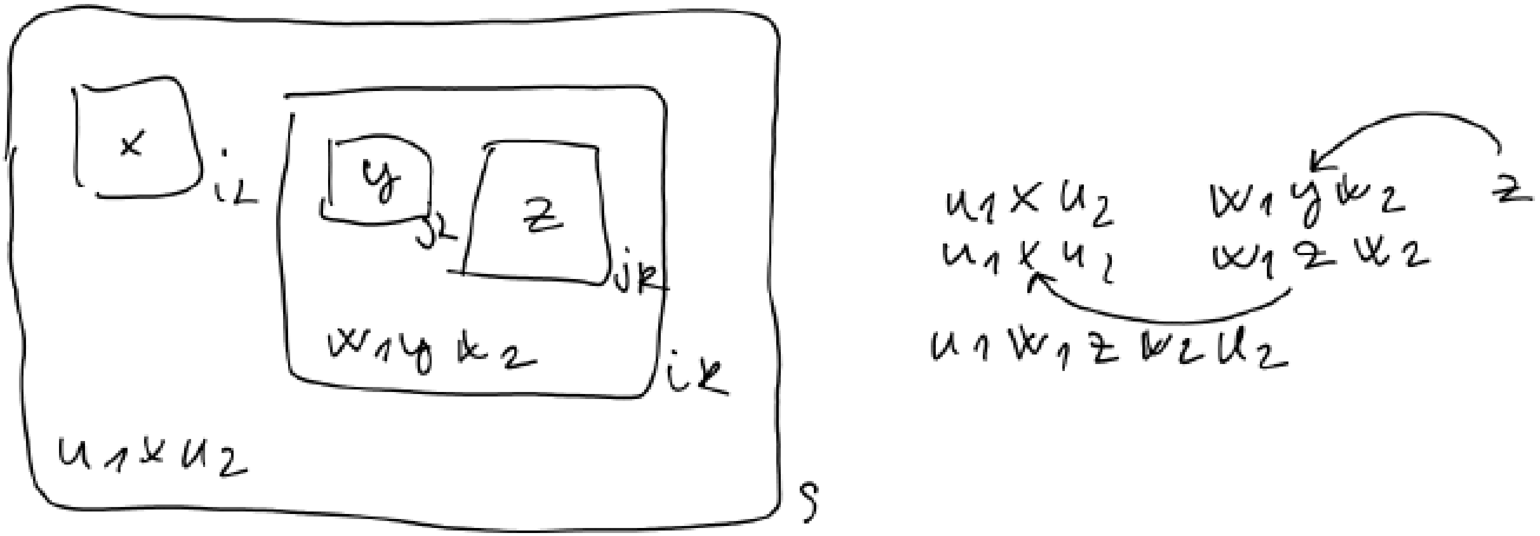


The other way around?

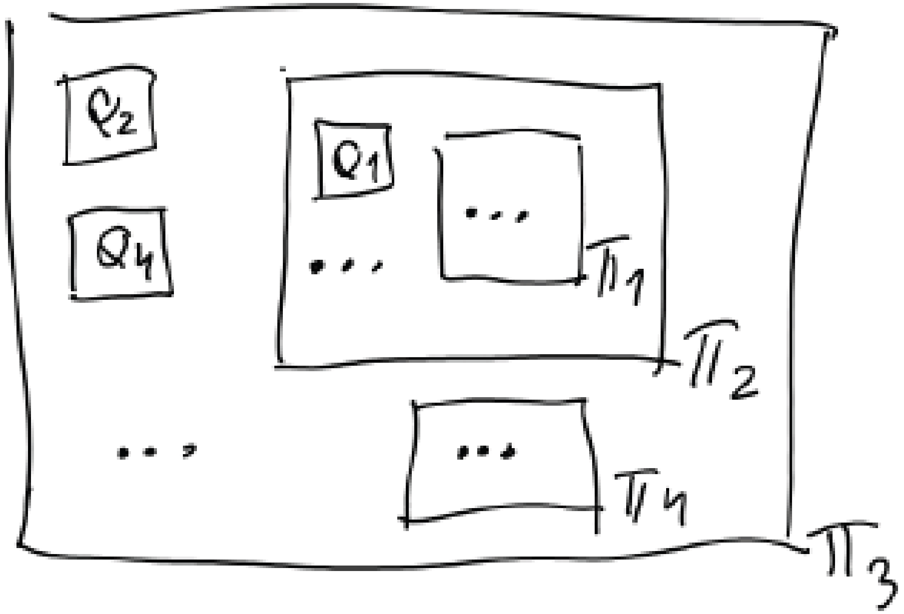
- The “communication graphs” of P systems are not complex enough

→In general:

The communication graph is a tree

How about the special case $P_sNPC_{tree}(ETOL)$?

The idea:

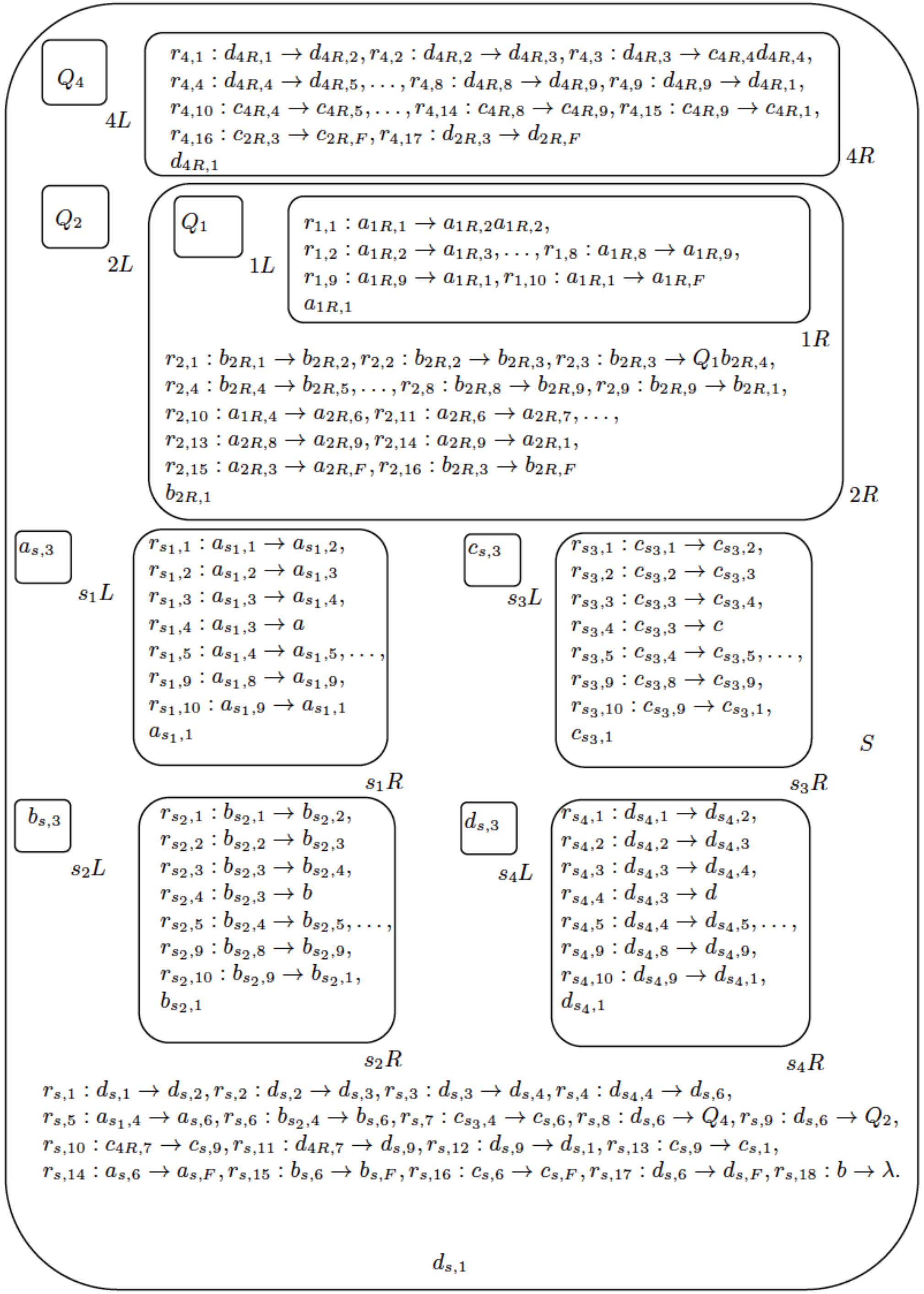


We can prove:

Theorem : $\alpha(NOP(\text{polylogm}, n^{O(1)})) = PSNPC_{tree}(ETOL)$

The earlier example:

G_1	G_2	G_3	G_4
a	b	d	d
aa	$Q_1 b$	Q_4	cd
aa	aab	cd	cd
$aaaa$	$aaQ_1 b$	cQ_4	ccd
$aaaa$	$aa\,aaaa\,b$	$c\,ccd$	ccd
a^8	$aa\,aaaa\,Q_1 b$	$c\,ccQ_2$	$cccd$
a^8	$aa\,aaaa\,a^8 b$	$c\,ccQ_2$	$cccd$
a^8	$aa\,aaaa\,a^8 b$	$c\,cc\,aa\,aaaa\,a^8 b$	$cccd$
a^{16}	$aa\,aaaa\,a^8 Q_1 b$	$c\,cc\,aa\,aaaa\,a^8$	$ccccd$



The earlier example:

G_1	G_2	G_3	G_4
a	b	d	d
aa	$Q_1 b$	Q_4	cd
aa	aab	cd	cd
$aaaa$	$aaQ_1 b$	cQ_4	ccd
$aaaa$	$aa\,aaaa\,b$	$c\,ccd$	ccd
a^8	$aa\,aaaa\,Q_1 b$	$c\,ccQ_2$	$cccd$
a^8	$aa\,aaaa\,a^8 b$	$c\,ccQ_2$	$cccd$
a^8	$aa\,aaaa\,a^8 b$	$c\,cc\,aa\,aaaa\,a^8 b$	$cccd$
a^{16}	$aa\,aaaa\,a^8 Q_1 b$	$c\,cc\,aa\,aaaa\,a^8$	$ccccd$

S	$2R$	$4R$	$1R$	s_4
$d_{s,1}$	$b_{2R,1}$	$d_{4R,1}$	$a_{1R,1}$	$d_{s_4,1}$
$d_{s,2}$	$b_{2R,2}$	$d_{4R,2}$	$a_{1R,2}a_{1R,2}$	$d_{s_4,2}$
$d_{s,3}$	$b_{2R,3}$	$d_{4R,3}$	$a_{1R,3}a_{1R,3}$	$d_{s_4,3}$
$d_{s,4}$	$Q_1 b_{2R,4}$	$c_{4R,4}d_{4R,4}$	$a_{1R,4}a_{1R,4}$	$d_{s_4,4}$
$d_{s_4,4}$	$a_{1R,4}a_{1R,4}b_{2R,5}$	$c_{4R,5}d_{4R,5}$	$a_{1R,5}a_{1R,5}$	$d_{s_4,5}$
$d_{s,6}$	$a_{2R,6}a_{2R,6}b_{2R,6}$	$c_{4R,6}d_{4R,6}$	$a_{1R,6}a_{1R,6}$	$d_{s_4,6}$
Q_4	$a_{2R,7}a_{2R,7}b_{2R,7}$	$c_{4R,7}d_{4R,7}$	$a_{1R,7}a_{1R,7}$	$d_{s_4,7}$
$c_{4R,7}d_{4R,7}$	$a_{2R,8}a_{2R,8}b_{2R,8}$	$c_{4R,8}d_{4R,8}$	$a_{1R,8}a_{1R,8}$	$d_{s_4,8}$
$c_{s,9}d_{s,9}$	$a_{2R,9}a_{2R,9}b_{2R,9}$	$c_{4R,9}d_{4R,9}$	$a_{1R,9}a_{1R,9}$	$d_{s_4,9}$
$c_{s,1}d_{s,1}$	$a_{2R,1}a_{2R,1}b_{2R,1}$	$c_{4R,1}d_{4R,1}$	$a_{1R,1}a_{1R,1}$	$d_{s_4,1}$
$c_{s,2}d_{s,2}$	$a_{2R,2}a_{2R,2}b_{2R,2}$	$c_{4R,2}d_{4R,2}$	$(a_{1R,2})^4$	$d_{s_4,2}$
$c_{s,3}d_{s,3}$	$a_{2R,3}a_{2R,3}b_{2R,3}$	$c_{4R,3}d_{4R,3}$	$(a_{1R,3})^4$	$d_{s_4,3}$
$c_{s,4}d_{s,4}$	$a_{2R,4}a_{2R,4}Q_1 b_{2R,4}$	$c_{4R,4}c_{4R,4}d_{4R,4}$	$(a_{1R,4})^4$	$d_{s_4,4}$
$c_{s,5}d_{s_4,4}$	$(a_{2R,5})^2(a_{1R,4})^4 b_{2R,5}$	$c_{4R,5}c_{4R,5}d_{4R,5}$	$(a_{1R,5})^4$	$d_{s_4,5}$
$c_{s,6}d_{s,6}$	$(a_{2R,6})^2(a_{2R,6})^4 b_{2R,6}$	$c_{4R,6}c_{4R,6}d_{4R,6}$	$(a_{1R,6})^4$	$d_{s_4,6}$
$c_{s,7}Q_4$	$(a_{2R,7})^2(a_{2R,7})^4 b_{2R,7}$	$c_{4R,7}c_{4R,7}d_{4R,7}$	$(a_{1R,7})^4$	$d_{s_4,7}$
$c_{s,8}c_{4R,7}c_{4R,7}d_{4R,7}$	$(a_{2R,8})^2(a_{2R,8})^4 b_{2R,8}$	$c_{4R,8}c_{4R,8}d_{4R,8}$	$(a_{1R,8})^4$	$d_{s_4,8}$
$c_{s,9}c_{s,9}c_{s,9}d_{s,9}$	$(a_{2R,9})^2(a_{2R,9})^4 b_{2R,9}$	$c_{4R,9}c_{4R,9}d_{4R,9}$	$(a_{1R,9})^4$	$d_{s_4,9}$
$c_{s,1}(c_{s,1})^2 d_{s,1}$	$(a_{2R,1})^2(a_{2R,1})^4 b_{2R,1}$	$c_{4R,1}c_{4R,1}d_{4R,1}$	$(a_{1R,1})^4$	$d_{s_4,1}$
$c_{s,2}(c_{s,2})^2 d_{s,2}$	$(a_{2R,2})^2(a_{2R,2})^4 b_{2R,2}$	$c_{4R,2}c_{4R,2}d_{4R,2}$	$(a_{1R,2})^8$	$d_{s_4,2}$
$c_{s,3}(c_{s,3})^2 d_{s,3}$	$(a_{2R,3})^2(a_{2R,3})^4 b_{2R,3}$	$c_{4R,3}c_{4R,3}d_{4R,3}$	$(a_{1R,3})^8$	$d_{s_4,3}$
$c_{s,4}(c_{s,4})^2 d_{s,4}$	$(a_{2R,4})^2(a_{2R,4})^4 Q_1 b_{2R,4}$	$c_{4R,4}c_{4R,4}c_{4R,4}d_{4R,4}$	$(a_{1R,4})^8$	$d_{s_4,4}$
$c_{s_3,4}(c_{s_3,4})^2 d_{s_4,4}$	$(a_{2R,5})^2(a_{2R,5})^4(a_{1R,4})^8 b_{2R,5}$	$c_{4R,5}c_{4R,5}c_{4R,5}d_{4R,5}$	$(a_{1R,5})^8$	$d_{s_4,5}$
$c_{s,6}(c_{s,6})^2 d_{s,6}$	$(a_{2R,6})^2(a_{2R,6})^4(a_{2R,6})^8 b_{2R,6}$	$c_{4R,6}c_{4R,6}c_{4R,6}d_{4R,6}$	$(a_{1R,6})^8$	$d_{s_4,6}$
$c_{s,7}(c_{s,7})^2 Q_2$	$(a_{2R,7})^2(a_{2R,7})^4(a_{2R,7})^8 b_{2R,7}$	$c_{4R,7}c_{4R,7}c_{4R,7}d_{4R,7}$	$(a_{1R,7})^8$	$d_{s_4,7}$
$c_{s,8}(c_{s,8})^2(a_{2R,7})^2(a_{2R,7})^4(a_{2R,7})^8 b_{2R,7}$	$(a_{2R,8})^2(a_{2R,8})^4(a_{2R,8})^8 b_{2R,8}$	$c_{4R,8}c_{4R,8}c_{4R,8}d_{4R,8}$	$(a_{1R,8})^8$	$d_{s_4,8}$
$c_{s,9}(c_{s,9})^2(a_{s,9})^2(a_{s,9})^4(a_{s,9})^8 b_{s,9}$	$(a_{2R,9})^2(a_{2R,9})^4(a_{2R,9})^8 b_{2R,9}$	$c_{4R,9}c_{4R,9}c_{4R,9}d_{4R,9}$	$(a_{1R,9})^8$	$d_{s_4,9}$
$c_{s,1}(c_{s,1})^2(a_{s,1})^2(a_{s,1})^4(a_{s,1})^8 b_{s,1}$	$(a_{2R,1})^2(a_{2R,1})^4(a_{2R,1})^8 b_{2R,1}$	$c_{4R,1}c_{4R,1}c_{4R,1}d_{4R,1}$	$(a_{1R,1})^8$	$d_{s_4,1}$
$c_{s,2}(c_{s,2})^2(a_{s,2})^2(a_{s,2})^4(a_{s,2})^8 b_{s,2}$	$(a_{2R,2})^2(a_{2R,2})^4(a_{2R,2})^8 b_{2R,2}$	$c_{4R,2}c_{4R,2}c_{4R,2}d_{4R,2}$	$(a_{1R,F})^8$	$d_{s_4,2}$
$c_{s,3}(c_{s,3})^2(a_{s,3})^2(a_{s,3})^4(a_{s,3})^8 b_{s,3}$	$(a_{2R,3})^2(a_{2R,3})^4(a_{2R,3})^8 b_{2R,3}$	$c_{4R,3}c_{4R,3}c_{4R,3}d_{4R,3}$	$(a_{1R,F})^8$	$d_{s_4,3}$
$c_{s,4}(c_{s,4})^2(a_{s,4})^2(a_{s,4})^4(a_{s,4})^8 b_{s,4}$	$(a_{2R,F})^2(a_{2R,F})^4(a_{2R,F})^8 b_{2R,F}$	$c_{4R,F}c_{4R,F}c_{4R,F}d_{4R,F}$	$(a_{1R,F})^8$	d
$c(c)^2(a)^2(a)^4(a)^8 b$	$(a_{2R,F})^2(a_{2R,F})^4(a_{2R,F})^8 b_{2R,F}$	$c_{4R,F}c_{4R,F}c_{4R,F}d_{4R,F}$	$(a_{1R,F})^8$	d
$c(c)^2(a)^2(a)^4(a)^8$	$(a_{2R,F})^2(a_{2R,F})^4(a_{2R,F})^8 b_{2R,F}$	$c_{4R,F}c_{4R,F}c_{4R,F}d_{4R,F}$	$(a_{1R,F})^8$	d

Thus…

We have a characterization of non-cooperative polymrphic systems in terms of parallel communicating ET0L systems.

Thank you.