Membrane systems and time Petri nets

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Outline

1. Time Petri nets
2. Membrane systems with local time
3. Connections of membrane systems and time Petri nets

Time Petri nets

- The Petri nets are state/transition systems: places are used to convey information and transitions represent events that can modify the information,

- A Petri net is a bipartite graphs: arcs point from places to transitions and from transitions to places. Every arc possesses a multiplicity, which is a positive integer.

Time Petri nets

- A transition is ready to fire, when each of its preplaces, that is the place endpoints of the incoming edges, contains as many tokens as the multiplicity of the arc coming from that preplace.
- Firing a transition means removing as many tokens from the preplaces as prescribed by the multiplicities of the incoming arcs and adding as many tokens to the postplaces as determined by the multiplicities of the outgoing arcs.

Example

In the figures below we illustrate a firing sequence of a Petri net[1]:

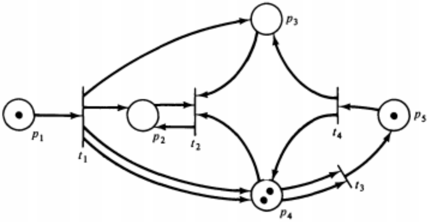

Figure 2.15 A marked Petri net to illustrate the firing rules. Transitions t_1, t_3, and t_4 are enabled.

[1] J. L. Peterson, *Petri Net Theory and the Modelling of Systems*, Prentice Hall, N.J., 1981.

Continuing the example:

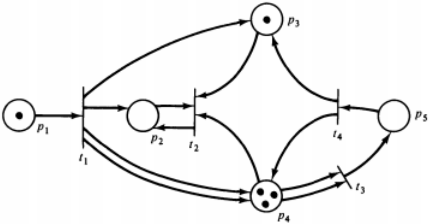

Figure 2.16 The marking resulting from firing transition t_4 in Figure 2.15.

Example

One step more:

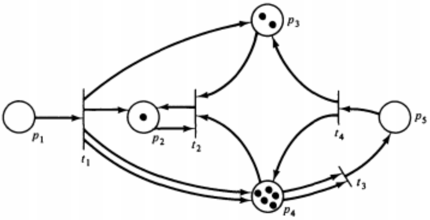

Figure 2.17 The marking resulting from firing transition t_1 in Figure 2.16.

Time Petri nets

Formally: a Petri net is a tuple $N = (P, T, F, V, m_0)$ such that

- P, T, F are finite, where $P \cap T = \emptyset$, $P \cup T \neq \emptyset$ and $F \subseteq (P \times T) \cup (T \times P)$,
- $V : F \to \mathbb{N}_{>0}$,
- $m_0 : P \to \mathbb{N}$.

The elements of P are called places and the elements of T are called transitions. The elements of F are the arcs and F is the flow relation of N. The function V is the multiplicity (weight) of the arcs and m_0 is the initial marking. We may occasionally omit the initial marking and simply refer to a Petri net as the tuple $N = (P, T, F, V)$. We stipulate that, for, every transition t, there is a place p such that $V(p, t) \neq 0$.

Time Petri nets

A Time Petri net (TPN)[2] is a 6-tuple $N = (P, T, F, V, m_0, I)$ such that

- the 5-tuple $S(N) = (P, T, F, V, m_0)$ is a Petri net,
- $I : T \to Q_{\geq 0} \times Q_{\geq 0}$ and, for each $t \in T$, $I(t)_1 \leq I(t)_2$ holds, where $I(t) = [I(t)_1, I(t)_2]$.

We call $I(t)_1$ and $I(t)_2$ earliest and latest firing times belonging to t, respectively. Notation: $eft(t)$, $lft(t)$.

[2] L. Popova-Zeugmann, *Time and Petri Nets*, Springer Verlag, Berlin, 2013.

Time Petri nets

- A transition marking with respect to the strong semantics (or t-marking) is a function $t : T \to \mathbb{R}_{\geq 0} \cup \{\#\}$.
- Let $N = (P, T, F, V, m_o, I)$ be a Time Petri net, m a p-marking and h a t-marking in N. A state in N is a pair $u = (m, h)$ such that
  - $(\forall t \in T)(t^{-} \not\leq m \supset h(t) = \#)$,
  - $(\forall t \in T)(t^{-} \leq m \supset h(t) \in \mathbb{R}_{\geq 0} \wedge h(t) \leq lft(t))$.
- A transition marking with respect to the weak semantics is a function $t : T \to \mathbb{R}_{\geq 0}$. A state in N concerning the weak seamntics is a pair $u = (m, h)$.

Time Petri nets

Let t be a transition and $u = (m, h)$ be a state such that $u \longrightarrow^{t}$. Then the result of the firing of t is a new state $u' = (m', h')$, such that $m' = m + \triangle t(p)$, where $\triangle t(p) = t^{+}(p) - t^{-}(p)$, and

$$h'(\hat{t}) = \begin{cases} h(\hat{t}), & \text{if } \hat{t}^{-} \leq m, \hat{t}^{-} \leq m' \text{ and } {}^{\bullet}\hat{t} \cap {}^{\bullet}t = \emptyset \text{ or } t = \hat{t}, \\ \# & \text{if } \hat{t}^{-} \not\leq m, \\ 0 & \text{otherwise .} \end{cases}$$

Observe that we allow multiple firings of the same transition in a row by demanding $h(\hat{t}) = h(t)$ if t is enabled and $t = \hat{t}$. We adopt a stronger condition for h to annul the value for t: when t and $\hat{t}$ have common preplaces, then $h(\hat{t}) = 0$. This is for ensuring some Church-Rosser properties in the Petri net.

Time Petri nets

Besides the firing of a transition there is another possibility for a state to alter, and this is the time delay step. Let t be a transition and $u = (m, h)$ be a state and $\tau \in \mathbb{R}^+$. Then elapsing of time with τ is possible in the strong semantics for the state u (in notation: $u \longrightarrow^\tau$), if

$$(\forall t \in T)(h(t) \neq \# \supset h(t) + \tau \leq lft(t)).$$

Then the result of the elapsing of time by τ is defined as follows: $u \longrightarrow^\tau u' = (m', h')$, where $m = m'$ and

$$h'(\hat{t}) = \begin{cases} h(\hat{t}) + \tau, & \text{if } \hat{t} + \tau \leq lft(\hat{t}) \text{ for an arbitrary } \hat{t} \in T, \\ \# & \text{otherwise.} \end{cases}$$

Observe that the definition of a time elapse with respect to the strong semantics ensures that we are not able to skip a transition when it is enabled.

Let t be a transition and $u = (m, h)$ be a state and $\tau \in \mathbb{R}^+$. Then elapsing of time with τ is always possible in the weak semantics. Then the result of the elapsing of time at state $u = (m, h)$ by τ is defined as $u \longrightarrow^{\tau} u' = (m', h')$, where $m = m'$ and $h'(t) = h(t) + \tau$ for every transition t.

Correspondence between P systems and Time Petri nets

Petri nets with time can generate recursively enumerable sets, hence it is a reasonable task to represent P systems with time Petri nets. In the Petri net, places represent elements of the membrane compartments and tokens correspond to the multiplicities of elements in the membranes. The transitions correspond to the rules: a maximal parallel step is represented in two phases- the rule application and communication phases. Therefore, two subnets describe the membrane system, which are connected by a third subnet regulating the order of the application and communication phases.

Correspondence between P systems and Time Petri nets

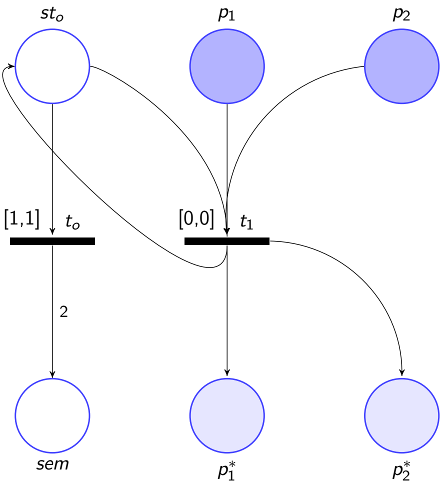

Figure: The Petri net simulating the rule application part of a membrane computational step.

Correspondence between P systems and Time Petri nets

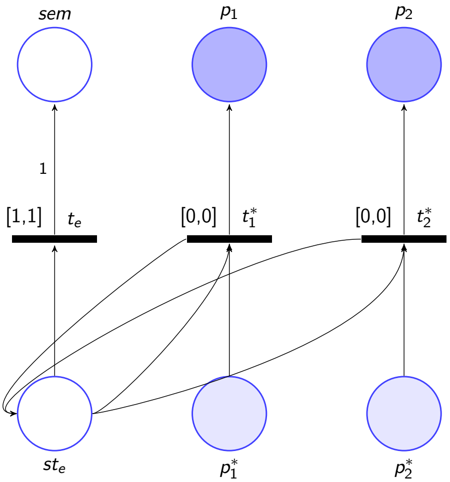

Figure: The Petri net simulating the communication part of a membrane computational step.

The two subparts are merged together by a semaphore subnet which behaves as follows.

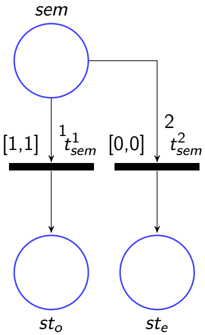

Figure: The semaphore for the Petri nets.

The semaphore ensures that the rule application and communication parts follow each other in the prescribed order.

In the literature,[3] Petri nets are considered with maximal parallelism in order to simulate P systems. In our case, strong timed semantics substitutes for maximal parallelism.

[3] cf. e.g. J. H. C. M. Kleijn and M. Koutny and G. Rozenberg, Towards a Petri Net Semantics for Membrane Systems. *Lecture Notes in Computer Science*, volume 3850

Membrane systems with rule-time

On the analogy of time Petri nets we can assign time intervals to rules in membrane systems. Let $\Pi = (A, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n)$ be a membrane system, where A is the alphabet, μ is the membrane structure, $C_0 = (w_1, \ldots, w_n)$ is the start configuration and $R = (R_1, \ldots, R_n)$ are the set of rules.

Membrane systems with rule-time

Let $\mathcal{I} : R \to Int\mathbb{Q}$ be a function, where $Int\mathbb{Q}$ is the set of closed intervals with non-negative rational endpoints. We call $\Pi = (V, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n, \mathcal{I})$ a membrane system with rule-time. In the construction $\mathcal{I}$ is the time interval for the rules when the rules can be active.

Membrane systems with rule-time

We define a clock for each of the rules in the compartments. The clock together with the multiset contents of the compartment give the timed configuration of the membrane system. Let $\Pi = (A, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n, \mathcal{I})$ be a membrane system with rule-time.

- Let $s : R \to \mathbb{R}_{\geq 0}$ be a function. Then s is the local time for the rules or a rule-marking for the membrane system with respect to the weak semantics.
- Assume $s : R \to \mathbb{R}_{\geq 0} \cup \{\#\}$ be a function. Then s is the local time for the rules or a rule-marking for the membrane system with respect to the strong semantics.

Membrane systems with rule-time

Timed configurations differ according to weak or strong semantics are considered.

- Let $\Pi = (V, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n, \mathcal{I})$ be a membrane system with rule-time. A timed configuration in the strong semantics is a pair (C, s), where C is a, possibly intermediate, configuration and $s(r) \neq \#$ implies $s(r) \leq \mathcal{I}^+(r)$ for every $r \in R$, where $\mathcal{I}^+(r)$ is the upper endpoint of the interval $\mathcal{I}(r)$.
- Let $\Pi = (V, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n, \mathcal{I})$ be a membrane system with rule-time. A timed configuration in the weak semantics is a pair (C, s), where C is a, possibly intermediate, configuration and $s(r) \in \mathbb{R}_{\geq 0}$.

Membrane systems with rule-time

Let $\Pi = (V, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n, \mathcal{I})$ be a membrane system with rule-time. Let (C, s) be a possibly intermediate configuration. Then a rule execution from (C, s) in compartment m_i is defined as usual. Time elapse is distinguished by the different semantics.

- A time step $(C, s) \rightarrow^{\tau} (C, s')$ with τ can be made in compartment m_i w.r.t. the strong semantics, if, for every $r \in R_i$, $s(r) \in \mathbb{R}$ implies $s(r) + \tau \leq \mathcal{I}^{+}(r)$. Moreover, $s'(r) = s(r) + \tau$.
- A time step $(C, s) \rightarrow^{\tau} (C, s')$ can be made in compartment m_i w.r.t. the weak semantics and $s'(r) = s(r) + \tau$.

Membrane systems with rule-time

Let $\Pi = (V, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n, \mathcal{I})$ be a membrane system with rule-time. Let (C, s) be a proper configuration. A computational step from (C, s) is a sequence of complete runs $\{\sigma_1, \ldots, \sigma_n\}$. Let $\mathcal{R}_{i1}, \ldots, \mathcal{R}_{ip}$ be multisets of rules in R_i.

- If $\sigma_i = (\mathcal{R}_{i1}, \tau_1, \ldots, \mathcal{R}_{ip}, \tau_p)$, then σ_i is a run of length p in m_i with respect to maximal parallel execution when $\mathcal{R}_{ij}$ are maximal multisets of rules and τ_j are taken according to the weak or strong semantics.
- If $\sigma_i = (\mathcal{R}_{i1}, \tau_1, \ldots, \mathcal{R}_{ip}, \tau_p)$, then σ_i is a run of length p in m_i with unsynchronized execution provided $\mathcal{R}_{ij}$ are multisets of rules and τ_j are taken according to the weak or strong semantics. In this case no stipulation is made on the maximality of the multisets of rules executed between the time steps.

A run is complete, if no more steps are possible in the strong semantics, or the sum of the time values in the run exceed the upper bound of the function $\mathcal{I}^+$.

Results

- For every membrane system with rule time w. r. t. the weak semantics (and maximal parallelism) we can find a time Petri net w. r. t. the strong semantics giving exactly the same result.
- For every membrane system with rule time w. r. t. the strong semantics (and maximal parallelism) there exists a time Petri net w. r. t. the strong semantics giving exactly the same result.

Corollary

Runs can be integer runs in computations, that is, it is enough to take integer values for every time step in a run. This follows from the corresponding theorem for time Petri nets understood with the strong semantics[4]. This means, a straightforward interpretation of the rule-time system is possible with promoters (in case of maximal parallelism) and priority (in case of the strong semantics).

[4] L. Popova-Zeugmann, *Time and Petri Nets*, Springer Verlag, Berlin, 2013.

Corollary

Let $\Pi = (V, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n, \mathcal{I})$ be a membrane system with rule-time. Assume Π is interpreted with maximal parallel mode w.r.t. the strong semantics. Let $B = max\{\mathcal{I}^+(r) \mid r \in R\}$. Let us assume that each membrane contains the promoters $\{F, C_0\}$ at the start. Then the following rules define a membrane systems that can compute exactly the same set of numbers as Π. In what follows let $r \in R_i$, where $1 \leq i \leq n$.

- $I, lhs(r) \longrightarrow rhs(r), F|_{C_t} \quad (Init_t)$
- $r|_{\{F, C_t\}} \quad (Rule_t)$
- $F \longrightarrow I$
- $C_t \longrightarrow C_{t+1} \quad (Shift_t)$
- $Init_t > Shift_t$ for every $0 \leq t \leq B$

Relating the computational models: remarks, conjectures

- The simulation of symbol object membrane systems with time Petri nets with the strong semantics suggests that maximal parallel execution can be substituted for rule-time with the strong semantics.
- It can happen that all four kinds of P systems with rule time differ in computational strength. It is known by the results for Petri nets that weak and strong semantics are not bisimilar. Obviously, rule-time with the strong semantics is Turing complete either with maximal parallelism or not. Is it true for rule-time system with the weak semantics without maximal parallelism?
- Existing timed membrane systems defined by imposing time delays on rules[5]. Rule-time membrane systems with maximal parallel execution can simulate these models in a straightforward way.

[5] M. Cavaliere and D. Sburlan, Time and synchronization in membrane systems. *Fundamenta Informaticae*, 20 (2007), 1–14.

Relating the computational models: open questions

The power of the timed model does not seem to be restricted to substituting for maximal parallelism. We conjecture that even membrane dissolution and priority can be defined by means of time intervals assigned to rules.

Thank you for your attention!

M. Cavaliere and D. Sburlan, Time and synchronization in membrane systems. *Fundamenta Informaticae*, 20 (2007), 1–14.

J. H. C. M. Kleijn and M. Koutny and G. Rozenberg, Towards a Petri Net Semantics for Membrane Systems. *Lecture Notes in Computer Science*, volume 3850, International Workshop on Membrane Computing, WMC 2005, 292–309, Springer Verlag, Berlin, 2005.

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L. Popova-Zeugmann, *Time and Petri Nets*, Springer Verlag, Berlin, 2013.