Formal verification and some open problems with Virus Machines

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Introduction

Why Virus Machines?

Why Virus Machines?

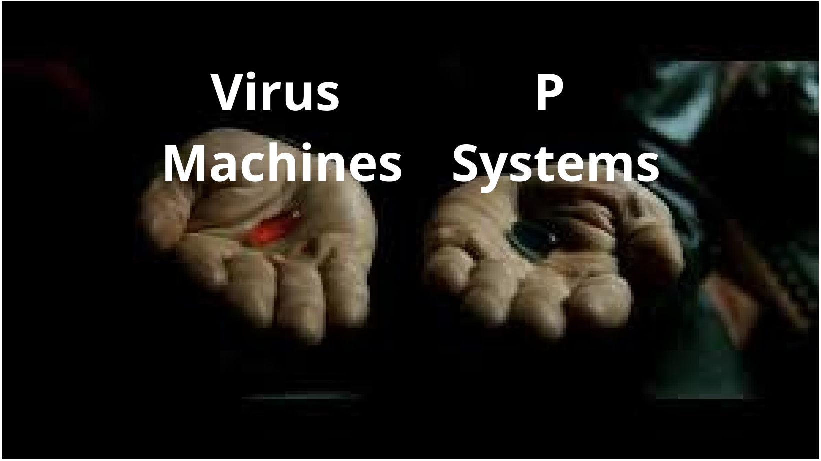

Figure: PhD meeting

Why Virus Machines?

Figure: PhD choose

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- New computing parading.

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    - Solving mathematical problems...

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- New computing parading.
- COVID-19.
- What do all mathematicians love?
    - Solving mathematical problems...
    - And invariants!

Brief definitions

Syntax

Definition (Virus Machine)

A virus machine of degree (p, q), $p \geq 1, q \geq 1$ is a tuple $\Pi = (\Gamma, H, I, D_H, D_I, G_C, n_1, \ldots, n_p, i_1, h_{out})$, where:

- $\Gamma = \{v\}$ is the singleton alphabet:
- $H = \{h_1, \ldots, h_p\}$ (**host**), $I = \{i_1, \ldots, i_q\}$ (**instructions**) are ordered sets and h_{out} represents the **environment**;
- $D_H = (H \cup \{h_{out}\}, E_H, w_H)$ weighted directed graph (WDG);
- $D_I = (I, E_I, w_I)$ is WDG;
- $G_C = (V_C, E_C)$ undirected bipartite graph;
- $n_j \in \mathbb{N}$, for each j, $1 \leq j \leq p$.

X. Chen, M.J. Pérez-Jiménez, L. Valencia-Cabrera, B. Wang, X. Zeng. Computing with viruses. *Theoretical Computer Science*, **623** (2016), 146–159.

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Bipartite Graph

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Number of viruses

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Initial instruction

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Singleton alphabet

WDG

Number of viruses

Environment

Ordered sets

Bipartite Graph

Initial instruction

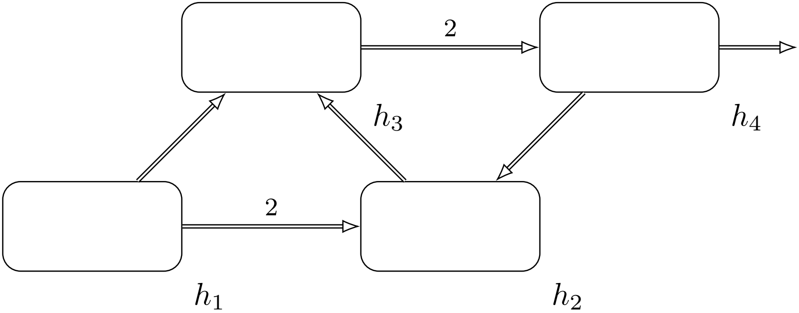

Figure: Host Graph of a Virus Machine

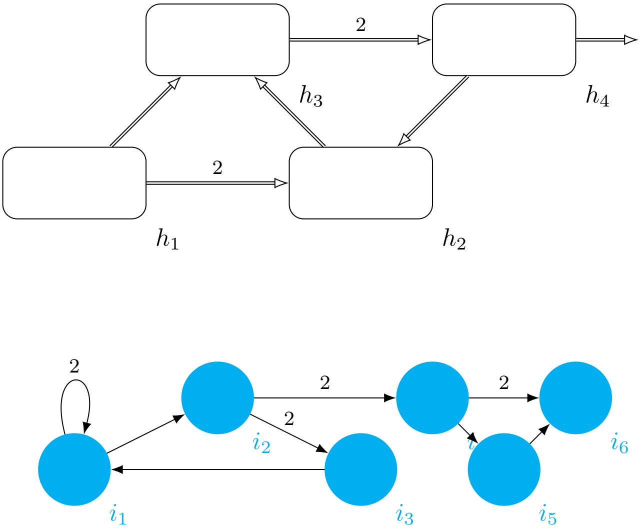

Figure: Host Graph and Instruction Graph of a Virus Machine

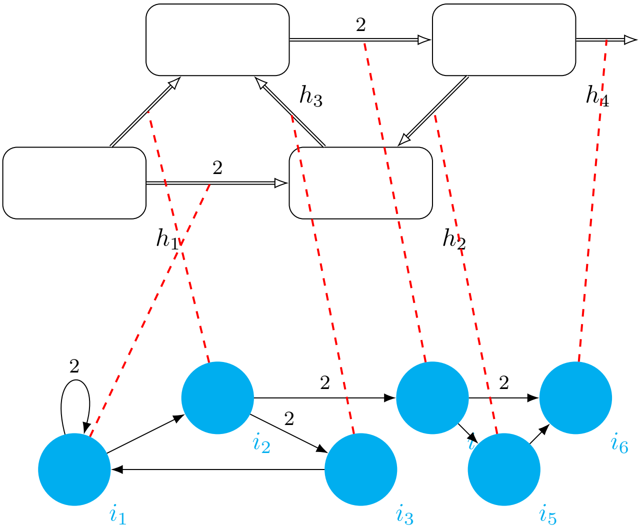

Figure: Heterogeneous network of a Virus Machine

Semantics

Definition

An *instantaneous description* or a *configuration* $\mathcal{C}_t$ at an instant t is a tuple $(a_{1,t}, \ldots, a_{p,t}, u_t, a_{0,t})$ where $a_{0,t}, a_{1,t}, \ldots, a_{p,t}$ are natural numbers and $u_t \in I \cup \{\#\}$.

Semantics

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Definition

A *computation* $\mathcal{C} = (\mathcal{C}_0, \mathcal{C}_1, \dots)$ of a virus machine Π is a (possibly infinite) sequence of configurations such that $\mathcal{C}_0$ is the initial configuration of Π and for each $t \in \mathbb{N}, \mathcal{C}_t \Rightarrow_\Pi \mathcal{C}_{t+1}$. A computation $\mathcal{C} = (\mathcal{C}_0, \mathcal{C}_1, \dots, \mathcal{C}_k)$ is called a *halting computation* if there exists a k such that $\mathcal{C}_k$ is a halting configuration; that is, $u = \#$.

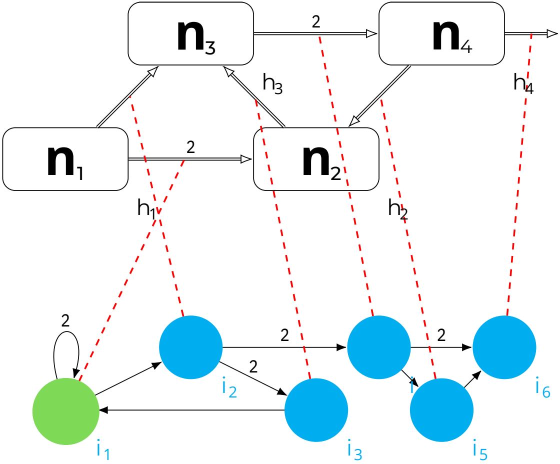

Figure: Configuration: $C_0 = (n_1, n_2, n_3, n_4, i_1, 0)$

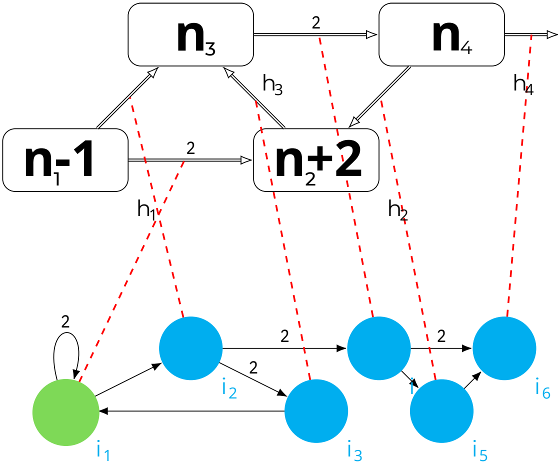

Figure: Configuration: $C_1 = (n_1 - 1, n_2 + 2, n_3, n_4, i_1, 0)$

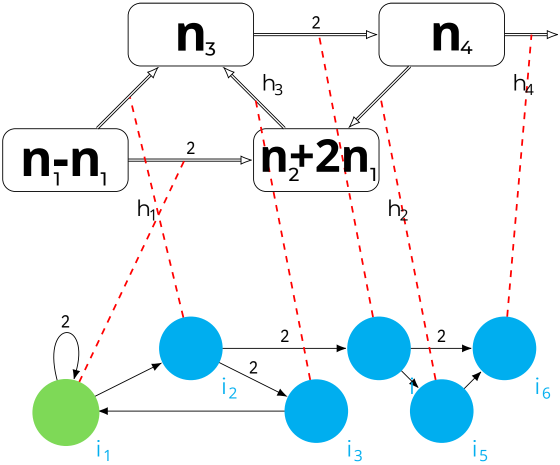

Figure: Configuration: $C_{n_1} = (n_1 - n_1, n_2 + n_1, n_3, n_4, i_1, 0)$

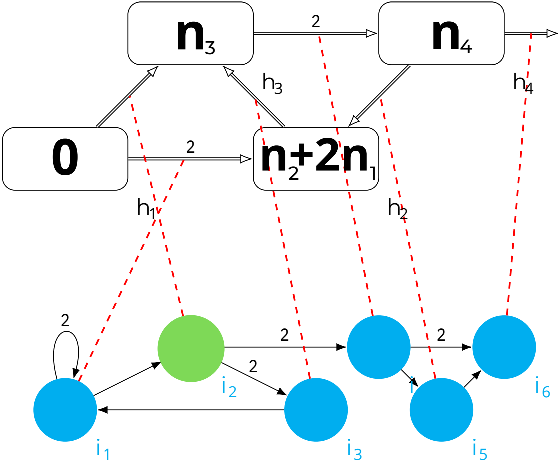

Figure: Configuration: $C_{n_1+1} = (0, n_2 + 2n_1, n_3, n_4, i_2, 0)$

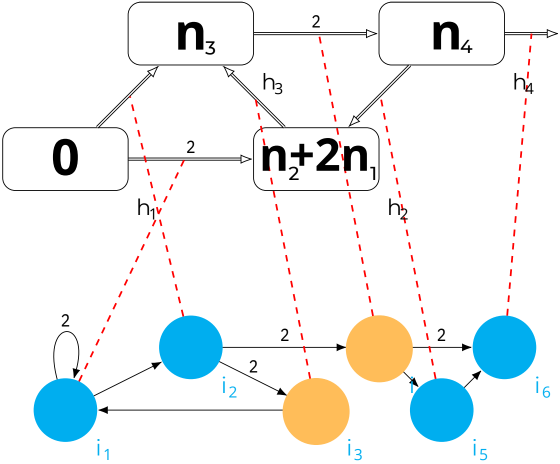

Figure: Non-deterministic behavior

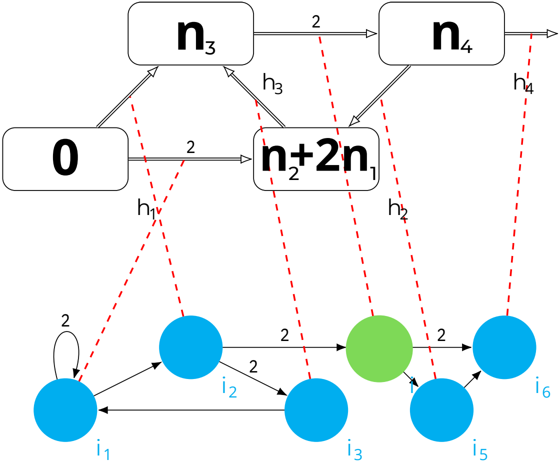

Figure: Configuration: $C_{n_1+2} = (0, n_2 + 2n_1, n_3 - 1, n_4 + 2, i_4, 0)$

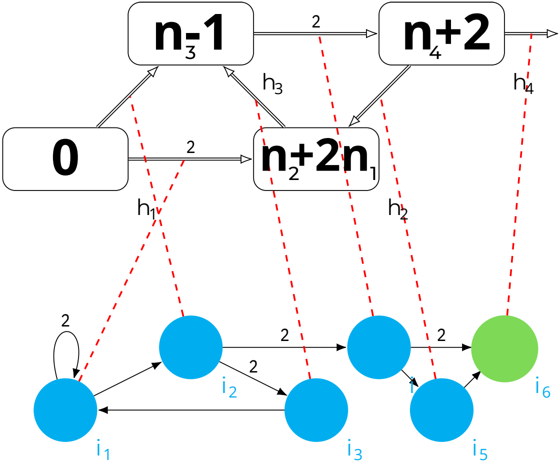

Figure: Configuration: $C_{n_1+3} = (0, n_2 + 2n_1, n_3 - 1, n_4 + 2, i_6, 0)$

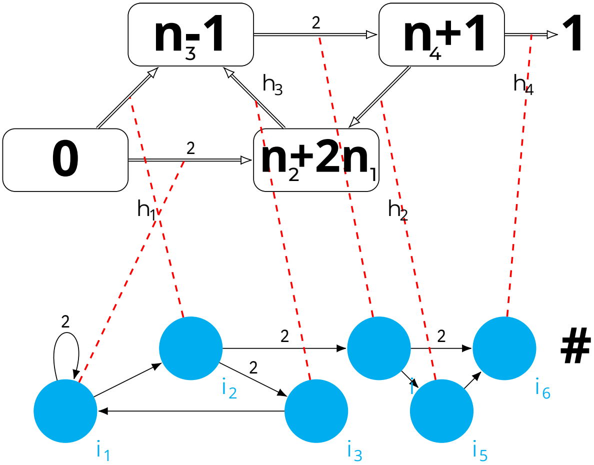

Figure: Configuration: $C_{n_1+3} = (0, n_2 + 2n_1, n_3 - 1, n_4 + 1, \#, 1)$

Computing

Definition

A *virus machine with input of degree* (p, q, r) [4], $p, q, r \geq 1$ is a tuple $\Pi = (\Gamma, H, H_r, I, D_H, D_I, G_C, n_1, \ldots, n_p, i_1, h_{out})$, where:

- $\Pi = (\Gamma, H, I, D_H, D_I, G_C, n_1, \ldots, n_p, i_1, h_{out})$ is a virus machine of degree (p, q); and
- $H_r = \{h_{j_1}, \ldots, h_{j_r}\} \subseteq H$ is the ordered set of r *input host* and $h_{out} \notin H_r$.

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The initial configuration of a virus machine with input $(a_1, \ldots, a_r) \in \mathbb{N}^r$ is given by $(n_1, \ldots, n_{j_1} + a_1 \ldots, n_{j_r} + a_r, \ldots, n_p)$, and it will be denoted by $\Pi + (a_1, \ldots, a_r)$ ($\Pi + a$ for single inputs).

Formal Verification

What is formal verification?

Formal verification[1] is the act of proving or disproving the correctness of intended algorithms underlying a system with respect to a certain formal specification or property, using formal methods of mathematics.

[1] en.wikipedia.org/wiki/Formal_verification

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A method to formally verify that a computational device of a model solves a given problem is to find **invariant formulas** in some relevant loops of the device, in such a way that the veracity of those formulas at the end of the loops provides relevant information.

[1] en.wikipedia.org/wiki/Formal_verification

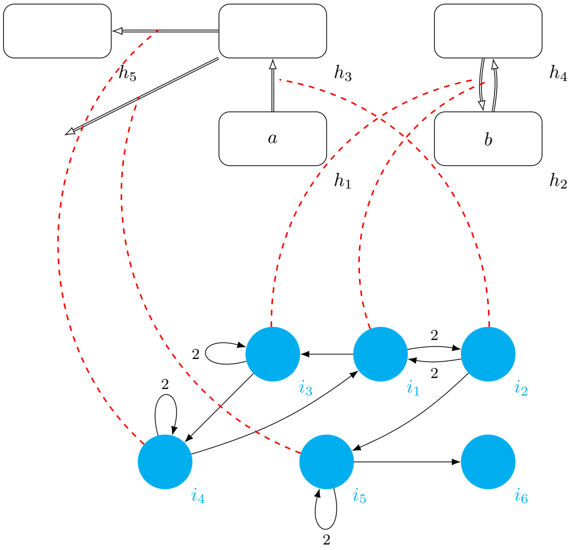

Figure: Virus machine with input $\Pi_{rem} + (a, b)$ computing the remainder of the integer division

Formal verification

Let (a, b) be the input of Π and $a = q \cdot b + r$. Then the following invariant holds:

$$\phi(k) \equiv \mathcal{C}_{k(4b+3)} = (a - b \cdot k, b, 0, 0, b \cdot k, i_1, 0), \text{ for } 0 \leq k \leq q$$

Formal verification

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$$\phi(k) \equiv \mathcal{C}_{k(4b+3)} = (a - b \cdot k, b, 0, 0, b \cdot k, i_1, 0), \text{ for } 0 \leq k \leq q$$

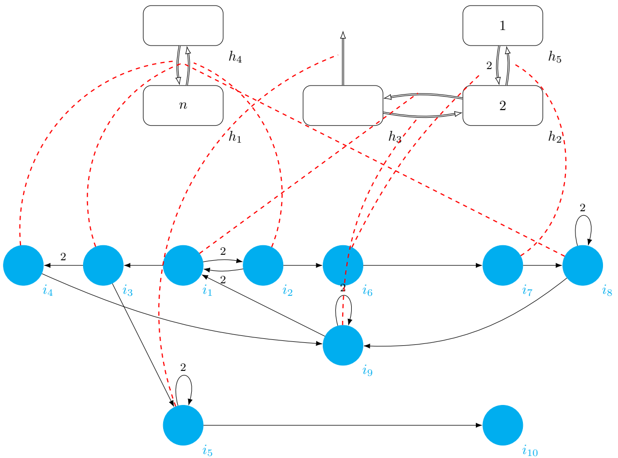
In particular, $\phi(q)$ is true, that is

$$\phi(q) \equiv \mathcal{C}_{q(4b+3)} = (\underbrace{a - b \cdot k}_{r}, b, 0, 0, b \cdot q, i_1, 0),$$

The halting configuration is

$$\mathcal{C}_{q \cdot (4b+3)+r+3} = (0, b - (r+1), 0, r+1, b \cdot q, \#, r)$$

Computing the Least Prime Divisor



i_1 activates $h_2 \to h_3$

i_2 activates $h_1 \to h_4$

i_3 activates $h_1 \to h_4$

i_4 activates $h_4 \to h_1$

i_5 activates $h_3 \to env$

i_6 activates $h_5 \to h_2$ (2)

i_7 activates $h_2 \to h_5$

i_8 activates $h_4 \to h_1$

i_9 activates $h_3 \to h_2$

Figure: Virus machine solving the LPD problem

In the case that the input n of the virus machine is any odd natural number, that is $n = p_1 \cdot q_{p_1}$ where $p_1 \neq 2$ is the minimum prime factor and $q_{p_1} = \dfrac{n}{p_1}$. Let us consider the following notation:

1. For every $j \in \mathbb{N}$, such that $j \geq 2$, we consider:

$$\alpha_j = q_j(3 \cdot j + 4) + 3r_j + n + 7$$

   where q_j and r_j are the quotient and the reminder of the integer division between n and j, i.e. they satisfy $n = j \cdot q_j + r_j$.

2. Let $\beta_k = \alpha_2 + \cdots + \alpha_k$, for every natural number $k \geq 2$.

The following invariant holds:

$$\psi(k) \equiv C_{\beta_k} = (n, k+1, 0, 0, 1, i_1, 0), \text{ for } 2 \leq k \leq p_1 - 1$$

Formal verification

In particular $\psi(p_1 - 1)$ is true, that is

$$\psi(p_1 - 1) \equiv C_{\beta_{p_1 - 1}} = (n, p_1, 0, 0, 1, i_1, 0),$$

The halting configuration is

$$C_{\beta_{p_1 - 1} + (q_{p_1})(3p_1 + 4)} = (0, 0, 0, n, 1, i_1, p_1),$$

Some open problems

Efficiency

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- Soft Parallelism: A set of instructions can be activated a the same transition step.

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- MC ingredients:
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  - Dead hosts, threshold in the number of viruses,mutation...

Soft Parallelism

Definition

A *parallel virus machine of degree* (p, q) , $p, q \geq 1$ is a tuple $\Pi = (\Gamma, H, I, D_H, D_I, G_C, n_1, \ldots, n_p, I_0, h_{out})$, where:

- $I_0 = \{i_{j_1}, \ldots i_{j_k}\} \subseteq I$ is the ordered set of k initial instructions; and
- $\Pi = (\Gamma, H, I, D_H, D_I, G_C, n_1, \ldots, n_p, i_{j_r}, h_{out})$ is a virus machine of degree (p, q) for every $i_{j_r} \in I_0$.

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What about semantics? Let represent the semantics of a virus machine as s, that is, if u_t is the instruction that will be activated at ant step t, then $s(u_t) = u_{t+1}$.

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$$I_{t+1} = \{s(i_k) \ : \ i_k \in I_t\}$$

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This has to be well defined

Let's go to the black(green)board!

Thank you for your attention!

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